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$$2) (1 + y^2 + xy^2) dx + (x^2y + y + 2xy) dy = 0$$

$$M = 1 + y^2 + xy^2 \quad | \quad N = x^2y + y + 2xy$$

$$\frac{\partial M}{\partial y} = 2y + 2xy \quad | \quad \frac{\partial N}{\partial x} = 2xy + 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{D.E. is exact}$$

$$f = \int M dx + g(y) \checkmark$$

$$f = \int N dy + h(x)$$

$$f = \int (1 + y^2 + xy^2) dx + \textcircled{g(y)} ?$$

$$f = x + xy^2 + \frac{x^2}{2} y^2 + g(y) \dots \dots (*)$$

$$\frac{\partial f}{\partial y} = 2xy + x^2y + g'(y)$$

$$N = 2xy + x^2y + g'(y)$$

$$\cancel{x^2y} + y + \cancel{2xy} = \cancel{2xy} + \cancel{x^2y} + g'(y)$$

$$y = g'(y) \Rightarrow \frac{y^2}{2} = g(y)$$

$$f = x + xy^2 + \frac{x^2}{2} y^2 + \frac{y^2}{2}$$

$$\text{G.S. is } f = c$$

$$\underline{x + xy^2 + \frac{x^2}{2} y^2 + \frac{y^2}{2} = c \quad (\text{G.S.})}$$