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$$1) (x^2 + y^2 + 1) dx + x(x-2y) dy = 0$$

$$\frac{\partial M}{\partial y} = 2y \quad \frac{\partial N}{\partial x} = 2x - 2y$$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ D.E is not exact

$$\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{1}{x(x-2y)} (-2x + 4y)$$

$$I.F = \mu(x) = \int \frac{-2}{x} dx = -2 \ln|x|$$

$$= -2 \ln|x| \Rightarrow \mu(x) = x^{-2}$$

Exact = $\frac{\partial}{\partial x} \left(\frac{1}{x^2} (x^2 + y^2 + 1) \right) = \frac{\partial}{\partial y} \left(\frac{1}{x^2} x(x-2y) \right)$

$$(1 + x^2 y^2 + x^{-2}) dx + x^{-1} (x-2y) dy = 0$$

$$\frac{\partial M}{\partial y} = 2x^{-1}y, \quad \frac{\partial N}{\partial x} = 2x^{-2}y$$

$$f = \int M dx + \int N dy + h(x)$$

$$2) y(2x-y+1) dx + x(3x-4y+2) dy = 0$$

$$M = 2xy - y^2 + y, \quad N = 3x^2 - 4xy + 2x$$

$$\frac{\partial M}{\partial y} = 2x - 2y + 1, \quad \frac{\partial N}{\partial x} = 6x - 4y + 2$$

$$\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{1}{x(3x-4y+2)} (-4x + 2y - 1)$$

$$\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = \frac{4x - 2y + 1}{y(2x-y+1)} = \frac{2(2x-y+1)}{y(2x-y+1)}$$

$$I.F = \mu(y) = \int \frac{2}{y} dy = 2 \ln|y|$$

$$y^2(2x-y+1) dx + xy^2(3x-4y+2) dy = 0$$

$$M = 2xy^3 - y^4 + y^3, \quad N = 3x^2y^2 - 4xy^3 + 2xy^2$$

$$\frac{\partial M}{\partial y} = 6xy^2 - 4y^3 + 3y^2, \quad \frac{\partial N}{\partial x} = 6xy^2 - 4y^3 + 2y^2$$

$$f = \int M dx + \int N dy + h(x)$$

$$x^2 y^3 - xy^4 + xy^3 = C \quad (G.S)$$

$$17) N = x^2 y \cos x$$

$$\frac{\partial N}{\partial x} = 2xy \cos x - 2x^2 y \sin x$$