page 54 :

1) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = q$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{q}{-x} = h(x)$

$M(x) = e^{\int \frac{q}{-x} dx} = e^{-q\int \frac{dx}{x}} = e^{-q \ln x} = e^{\ln x^{-q}} = x^{-q}$

I.F =

نضرب بالمعامل التكاملي

Another solution: $(x^5 + 3y)dx - x\,dy = 0$

$(x^5 + 3y)dx = x\,dy$

$\frac{x^5 + 3y}{x} = \frac{dy}{dx}$

$x^4 + \frac{3}{x}y = y'$ linear: $y' + P(x)y = Q(x)$

$x^4 = y' - \frac{3}{x}y$ linear ... (*)

$\int P(x)dx = \int \frac{-3}{x} dx = -3 \ln x$

$u = e^{-3\ln x} = e^{\ln x^{-3}} = x^{-3}$

نضرب بالمعامل (*)

$x = x^{-3}y' - 3x^{-4}y$

$y = e^{-\int P(x)dx}\left(\int e^{\int P(x)dx} \cdot Q(x)dx + c\right)$

$= e^{-\int \frac{-3}{x}dx}\left(\int e^{\int \frac{-3}{x}dx} \cdot x^4 dx + c\right)$

$= e^{3\ln x}\left(\int e^{-3\ln x} \cdot x^4 dx + c\right)$

$y = x^3\left[\int x^{-3} \cdot x^4 dx + c\right]$

$y = x^3\left[\int x\,dx + c\right]$

$= x^3\left[\frac{x^2}{2} + c\right]$

$y = \frac{x^5}{2} + cx^3$

9) Linear: $P(x) = \frac{x}{1+x^2}$, $Q(x) = \frac{x}{1+x^2}$

$y = e^{-\int \frac{x}{1+x^2}dx}\left(\int e^{\int \frac{x}{1+x^2}dx} \cdot \frac{x}{1+x^2}dx + c\right)$

$= e^{-\frac{1}{2}\ln(1+x^2)} \cdot \left(\int e^{\frac{1}{2}\ln(1+x^2)} \cdot \frac{x}{1+x^2}dx + c\right)$

$y = \frac{1}{\sqrt{1+x^2}}\left(\int \sqrt{1+x^2} \cdot \frac{x}{1+x^2}dx + c\right)$

$y = \frac{1}{\sqrt{1+x^2}}\left(\int \frac{x}{\sqrt{1+x^2}}dx + c\right)$

$y = \frac{1}{\sqrt{1+x^2}}\left(\frac{1}{2}\int 2x(1+x^2)^{-\frac{1}{2}}dx + c\right)$

$= \frac{1}{\sqrt{1+x^2}}\left(\frac{1}{2}\frac{(1+x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c\right)$

$y = \frac{1}{\sqrt{1+x^2}}\left(\sqrt{1+x^2} + c\right)$

$y = 1 + \frac{c}{\sqrt{1+x^2}}$