Page 58 :

1) $2x^3 y' = y(y^2+3x^2)$

not sep

$2x^3 dy = y(y^2+3x^2)dx$

it is Hom

$M dx + N dy = 0$ exact

$\frac{\partial M}{\partial y} = 3y^2 + 3x^2$

$\frac{\partial N}{\partial x} = -6x$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 3y^2 + 9x^2$

$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -(3y^2+9x^2)$

$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{-(3y^2+9x^2)}{y(y^2+3x^2)} = \frac{-3(y^2+3x^2)}{y(y^2+3x^2)} = \frac{-3}{y} = h(y)$

$I.F = \mu(y) = e^{\int -\frac{3}{y} dy} = \cdots$

طريقة ثانية : $2x^3 y' = y(y^2+3x^2)$

$y' = \frac{y}{2x^3}(y^2+3x^2)$

$y' = \frac{1}{2x^3} y^3 + \frac{3}{2x} y$

$y' - \frac{3}{2x} y = \frac{1}{2x^3} y^3$ Bernoulli eq

$w = y^{1-3} = y^{-2} \leftarrow n = 3$

$y' + p(x) y = Q(x) y^n$

put $w = y^{1-n}$

$\frac{dw}{dx} = -2y^{-3} \frac{dy}{dx}$

$-\frac{1}{2} y^3 \frac{dw}{dx} = \frac{dy}{dx}$

$-\frac{1}{2} y^3 \frac{dw}{dx} - \frac{3}{2x} y = \frac{1}{2x^3} y^3$

$w' + p(x) w = Q(x)$ (linear)

$\frac{dw}{dx} + \frac{3}{x} \frac{1}{y^2} = -\frac{1}{x^3}$

$\frac{dw}{dx} + \frac{3}{x} w = -\frac{1}{x^3}$ linear

$w = e^{-\int \frac{3}{x} dx} \left(\int e^{\int \frac{3}{x} dx} \cdot -\frac{1}{x^3} dx + c \right)$

$w = e^{-3\ln x} \left(\int e^{3\ln x} \cdot -\frac{1}{x^3} dx + c \right)$

$w = x^{-3} \left(\int x^3 \left(-\frac{1}{x^3}\right) dx + c \right)$

$w = x^{-3} \left[\int -dx + c \right]$

$w = x^{-3} \left[-x + c \right]$

$y' = \text{——}$

$w = -x^{-2} + \frac{c}{x^3}$

$\frac{1}{y^2} = -\frac{1}{x^2} + \frac{c}{x^3}$

(14) Final answer :

$\frac{1}{y^2} = \frac{e^{2x}}{2} + \frac{3}{2e^{2x}}$ Particular solution