Evaluation of Mohr-Coulomb Failure Criterion Using Unconfined Compressive Strength

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Abstract

Several rock deformation mechanisms are involved in the oil and gas reservoir formations and the overlaying strata. These deformations may occur during drilling, production, fracturing, stimulation, or enhanced oil recovery (secondary and tertiary). The evaluation of Mohr-Coulomb failure criterion as well as other mechanical properties for reservoir rocks is essential for well planning, development and characterization of oil and gas reservoirs. This is because the understanding of the rock-stress relationship can solve many reservoir problems and avoid cost of remedial work. For example, a Mohr-Coulomb failure criterion may be used for borehole instability analysis, water injection design, production optimization techniques, compaction and sand production prediction, etc. A Mohr-Coulomb failure criterion is a function of the apparent cohesion and the angle of internal friction. The evaluation of these two parameters requires testing of many rock samples using an expensive and time-consuming triaxial testing set-up. In this study, a correlation between the apparent cohesion and the unconfined compressive strength was developed based on laboratory measurement data of more than 300 rock samples of different types obtained from the literature. The correlation coefficient of the developed correlation equals to 0.88. Verification of the developed correlation using literature data from sources other than those used in the correlation development has shown average error of estimation around 10%. Therefore, the Mohr-Coulomb failure criterion's parameters can be roughly estimated using the developed correlation based on fast and cheap measurements of the unconfined compressive strength.

Introduction

Several problems occur repeatedly during oil and gas well drilling operations such as sloughing shale, stuck drillpipe or casing, etc. Normally, the first attempt to solve such problems is based on experience. If all methods fail to solve such problems, a rock mechanics study is considered to be the last hope. At this stage, rock mechanics analysis will be difficult due to the lack of data and rock samples, therefore, back analysis may be the only solution if new offset wells are not drilled.

Thus, rock mechanics must be considered from the beginning and a rock mechanical database must be established to assist in solving new problems (Al-Awad, M. N. J., 1992). For example, borehole instability during drilling costs the industry around $400–500 millions per year (Bol, G.M., 1998). When producing reservoir fluids over a long period, several problems related to rock mechanics may be encountered such as sand production, perforation instability, subsidence, mechanical damage or payzone permeability, etc. In order to solve any of the previously mentioned problems that may be encountered, a rock failure criterion and rock mechanical properties must be implemented. To evaluate the required mechanical properties, sufficient number of rock specimens are needed in addition to an expensive triaxial testing set-up. Therefore, simple correlations are needed to estimate rock mechanical properties using a limited number of rock samples and inexpensive and rapid tests.

Objectives

The objective of this paper is to develop a general correlation between the unconfined (uniaxial) compressive strength and the apparent cohesion of rocks. Using this correlation, several important mechanical parameters can be estimated including, apparent cohesion, angle of internal friction, and Poisson's ratio.

Theoretical Background

Mohr-Coulomb failure criterion was introduced to rock mechanics by Jaeger (Jaeger, J.C. and Cook, N.G.W., 1979) by combining the work of Mohr and Coulomb. This criterion stated that shear failure across a plane is restricted by the cohesion of the material. This criterion can be expressed mathematically as follows:

$$\left|\tau_f\right| = \tau_o + \sigma \tan \phi \quad (1)$$

Where, τ_f and σ are the shear and normal stresses respectively, τ_o is the apparent or inherent cohesion and ϕ is the angle of internal friction.

The evaluation of the Mohr-Coulomb failure criterion needs to carry out adequate discrete triaxial compression tests on rock specimens at various confining pressures. Alternativelly, when limited number of core samples are avilable, multi-stage triaxial compression test can be used to establish the failure criterion with limited accurecy. Using data series obtained from either discerete or multi-stage compression tests, Mohr’s circles can be plotted and the locus of the tangent points of circles is drawn, developing the failure envelope for the tested rock which defines the boundary between stable and unstable stress states. Once the failure criterion is established, the failure state (instability) at any other conditions can be predicted.

Poisson’s ratio (ν) is another important rock property which is defined as the ratio of the lateral strain to the axial strain in an axially stressed sample. If the axial and lateral strains are measured during triaxial testing to determine the Mohr-Coulomb failure criterion, Poisson’s ratio can be estimated as well. Alternatively, Poisson’s ratio can also be estimated, if the angle of internal friction is known, using the following relationship (Turk, N. and Dearman, W.R., 1986):

$$\phi = \sin^{-1}\left[\frac{1-\nu}{1+\nu}\right] \quad (2)$$

Equation 2 provides only estimated values of Poisson’s ratio since it was developed based on non destructive unconfined compression tests. However, for more accurate values of Poisson’s ratio, data obtained using the triaxial tests must be used.

Results and Discussion

Frictional properties, elastic properties, and failure criterion evaluated using experimental data of more than 300 rock samples of different types were obtained from the literature shown at the end of this article. Rock apparent cohesion is a measure of the degree of grain-to-grain bonding. Therefore, it might correlate with the uniaxial (unconfined) compressive strength which is also a measure of grain-to-grain bonding magnitude. For this reason, the uniaxial compressive strength was plotted versus the apparent cohesion of the literature-cited data. A well-defined trend was obtained from this plot as shown in Figure 1.

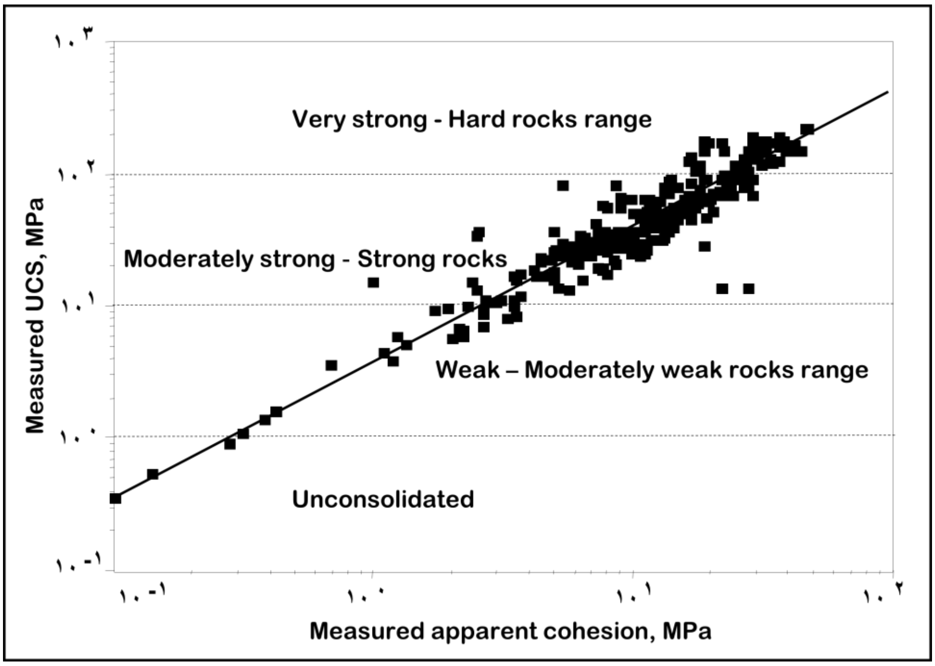

Figure 1: Cross-Plot between Measured Apparent Cohesion and UCS

Thus, a relationship between the apparent cohesion and the unconfined compressive strength for various types of rocks ranging from soft to hard is represented by the following equation with a coefficient of correlation (r^2) equal to 0.88:

$$Log\ \boldsymbol{\sigma_o} = 0.61 + 0.98\ Log\ \boldsymbol{\tau}_o \quad (3)$$

Figure 2 represents the relationship between the literature obtained (based on laboratory measurements) and correlation predicted apparent cohesion values. It can be seen that most of the data points are located on the 45-degree line. Similar result was obtained for the uniaxial compressive strength as shown in Figure 3 indicating the existence of a relationship between rock uniaxial compressive strength and apparent cohesion.

Since, rock apparent cohesion is estimated using **Equation 3**, rock angle of internal friction also can be estimated using the uniaxial compressive strength and the apparent cohesion based on the following fundamental relationship (Fjaer, E. et al, 1992):

At this stage, the two independent values namely the apparent cohession and the angle of internal friction required to defined Mohr-Coulomb failure criterion are defined.

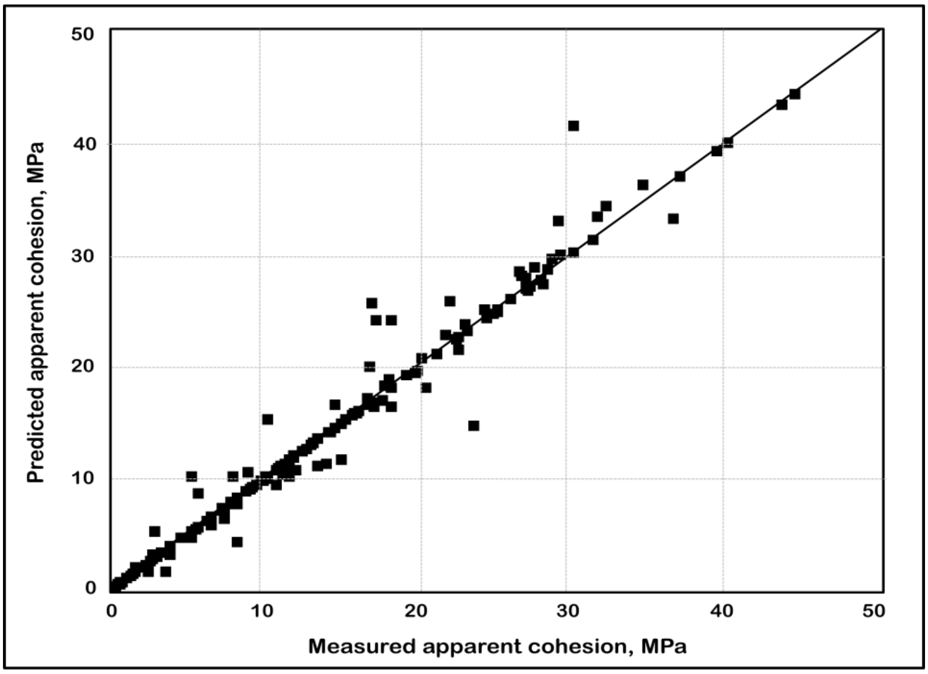

Figure 2: Cross-Plot between Measured and Predicted Apparent Cohesion

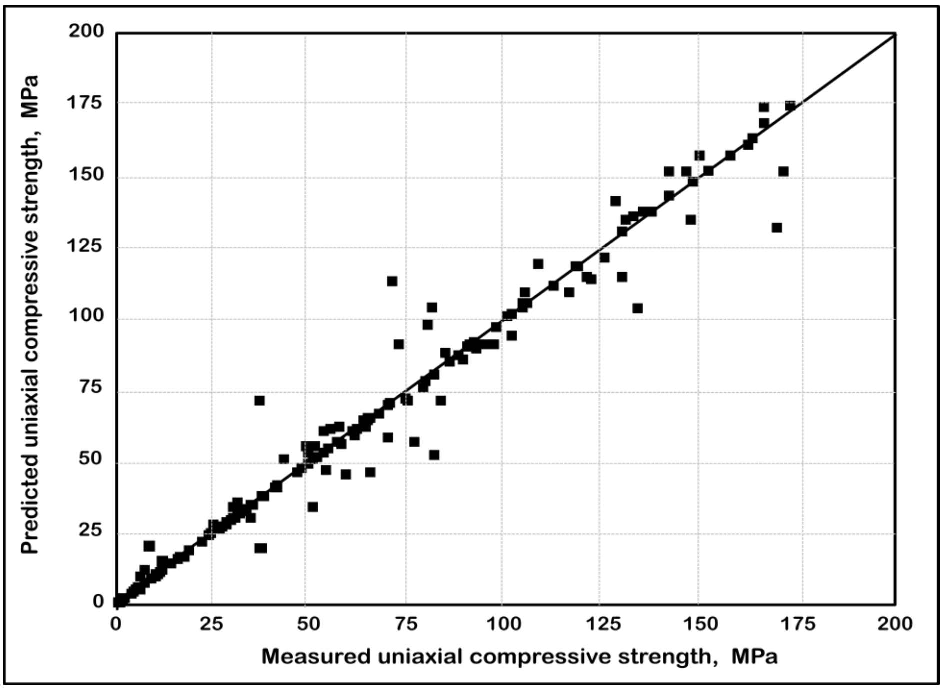

Figure 3: Cross-Plot between Measured and Predicted UCS

To verify the developed correlation, data from references other than those used in the development of the correlation were used.

$$\frac{\sigma_o}{\tau_o} = \frac{2\cos\phi}{1-\sin\phi} \quad (3)$$

The results showed an average error of estimation equal to 10% which is considered as a good confirmation of the validity of the developed correlation (see Table 1).

Table 1: Verification of the developed correlation

Parameters	Crawford, B.R. et al, 1995			Hoek, E., 1990		
	Literature	Prediction	% Error	Literature	Prediction	% Error
UCS, MPa	105.8	106.7	0.85	60	69.2	13.3
Cohesion, MPa	28	27.8	0.71	18	15.6	13.3
Friction Angle, Degree	34	34.3	0.88	26	28	7.14
Parameters	Wilson, A.H., 1988			Farquhar, R.A., 1993		
	Literature	Prediction	% Error	Literature	Prediction	% Error
UCS, MPa	50	48.41	3.18	41.3	49.93	17.28
Cohesion, MPa	12.5	12.92	3.25	12.9	10.63	17.60
Friction Angle, Degree	37	36.9	0.27	25.6	26	1.54

If accurate rock failure criteria are required, triaxial tests using sufficient number of rock samples should be performed. Therefore, this work provides a cheap and rapid estimation of rock failure criterion and some other mechanical properties using inexpensive laboratory measurements.

Conclusions

- A simple general correlation between rock apparent cohesion and unconfined (uniaxial) compressive strength was developed.
- The developed correlation can be used to evaluate Mohr-Coulomb failure criterion’s parameters as well as Poisson's ratio using simple and cheap deformation tests.
- The developed correlation provides only estimated values of rock mechanical properties (within an average error of estimation around 10%).
- For accurate rock mechanical properties, triaxial tests using satisfactory rock samples must be performed.

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