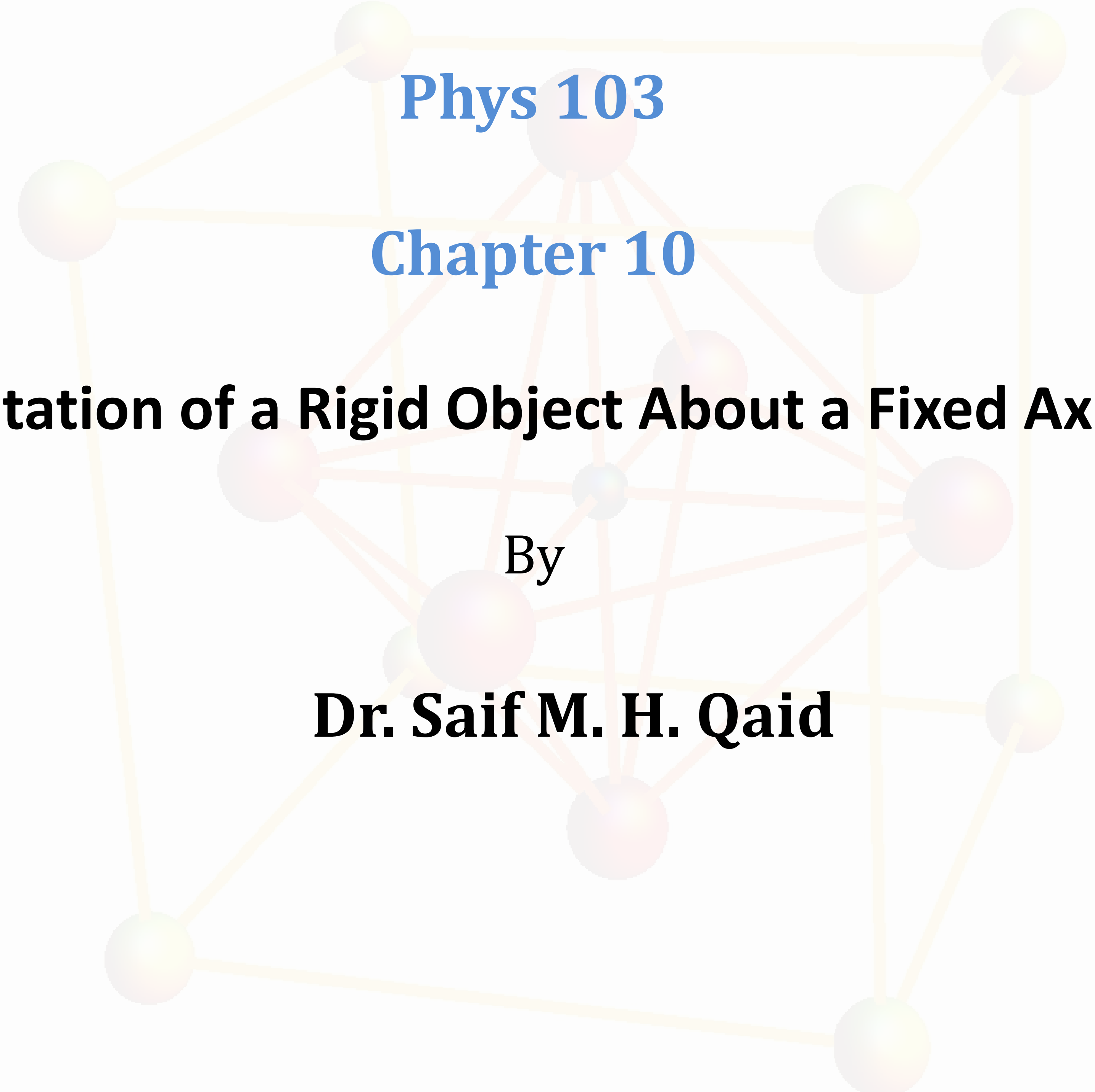

Phys 103

Chapter 10

Rotation of a Rigid Object About a Fixed Axis

By

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LECTURE OUTLINE

10.1 Angular Position, Velocity, and Acceleration

10.2 Rotational Kinematics:

Rotational Motion with

Constant Angular Acceleration

10.3 Angular and Linear Quantities

10.4 Rotational Kinetic Energy

10.5 Calculation of Moments of Inertia

10.6 Torque

10.7 Relationship Between Torque and Angular Acceleration

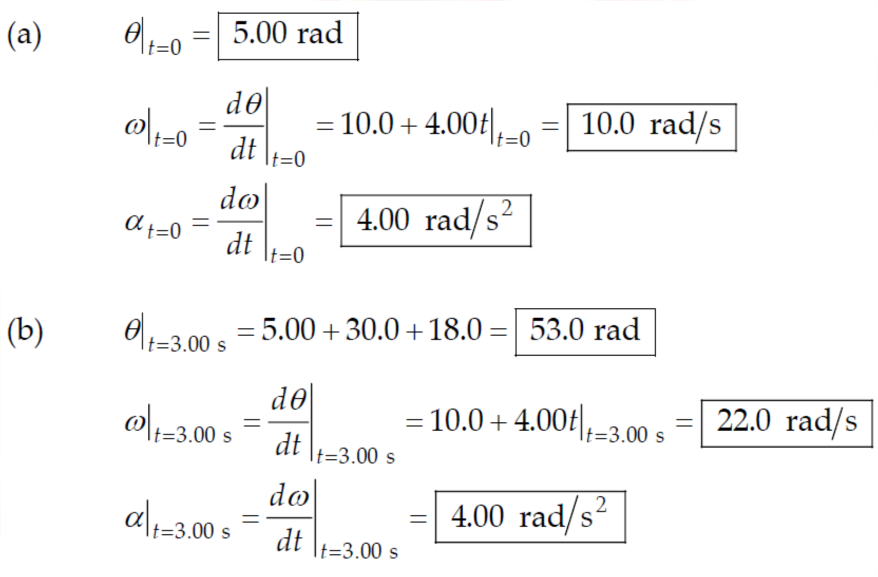
10.8 Work, Power, and Energy in Rotational Motion

PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

1. During a certain period of time, the angular position of a swinging door is described by $\boldsymbol{\theta} = 5 + 10t + 2t^2$, where $\boldsymbol{\theta}$ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ (b) at $t = 3.00$ s.

SOLUTIONS TO PROBLEM:



(a) $$\theta\big|_{t=0} = \boxed{5.00 \text{ rad}}$$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $$\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$$

$$\omega\big|_{t=3.00\text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{ s}} = 10.0 + 4.00t\big|_{t=3.00\text{ s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

PROBLEMS

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

3. A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12.0 rad/s in 3.00 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time.

SOLUTIONS TO PROBLEM:

(a) $$\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0\ \text{rad/s}}{3.00\ \text{s}} = \boxed{4.00\ \text{rad/s}^2}$$

(b) $$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(4.00\ \text{rad/s}^2\right)(3.00\ \text{s})^2 = \boxed{18.0\ \text{rad}}$$

PROBLEMS

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

5. An electric motor rotating a grinding wheel at 100 rev/min is switched off. With constant negative angular acceleration of magnitude 2.00 rad/s2, (a) how long does it take the wheel to stop? (b) Through how many radians does it turn while it is slowing down?

SOLUTIONS TO PROBLEM:

$$\omega_i = \frac{100\text{ rev}}{1.00\text{ min}}\left(\frac{1\text{ min}}{60.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1.00\text{ rev}}\right) = \frac{10\pi}{3}\text{ rad/s},\ \omega_f = 0$$

(a) $$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - \frac{10\pi}{3}}{-2.00}\ s = \boxed{5.24\text{ s}}$$

(b) $$\theta_f = \bar{\omega}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6}\text{ rad/s}\right)\left(\frac{10\pi}{6}\text{ s}\right) = \boxed{27.4\text{ rad}}$$

PROBLEMS

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

6. A centrifuge in a medical laboratory rotates at an angular speed of 3 600 rev/min. When switched off, it rotates 50.0 times before coming to rest. Find the constant angular acceleration of the centrifuge.

SOLUTIONS TO PROBLEM:

$$\omega_i = 3\,600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$$

$$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad and } \omega_f = 0$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$0 = \left(3.77 \times 10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14 \times 10^2 \text{ rad}\right)$$

$$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$$

PROBLEMS

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

8. A rotating wheel requires 3.00 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.00-s interval is 98.0 rad/s. What is the constant angular acceleration of the wheel?

SOLUTIONS TO PROBLEM:

$\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$\omega_i = \omega_f - \alpha t$: $\quad \theta_f - \theta_i = \left(\omega_f - \alpha t\right)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$

$$37.0\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 98.0\text{ rad/s}(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$232\text{ rad} = 294\text{ rad} - \left(4.50\text{ s}^2\right)\alpha$: $\quad \alpha = \frac{61.5\text{ rad}}{4.50\text{ s}^2} = \boxed{13.7\text{ rad/s}^2}$

PROBLEMS

Section 10.3 Angular and Linear Quantities

12. A racing car travels on a circular track of radius 250 m. If the car moves with a constant linear speed of 45.0 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

SOLUTIONS TO PROBLEM:

(a) $$v = r\omega;\ \omega = \frac{v}{r} = \frac{45.0\ \text{m/s}}{250\ \text{m}} = \boxed{0.180\ \text{rad/s}}$$

(b) $$a_r = \frac{v^2}{r} = \frac{(45.0\ \text{m/s})^2}{250\ \text{m}} = \boxed{8.10\ \text{m/s}^2\ \text{toward the center of track}}$$

PROBLEMS

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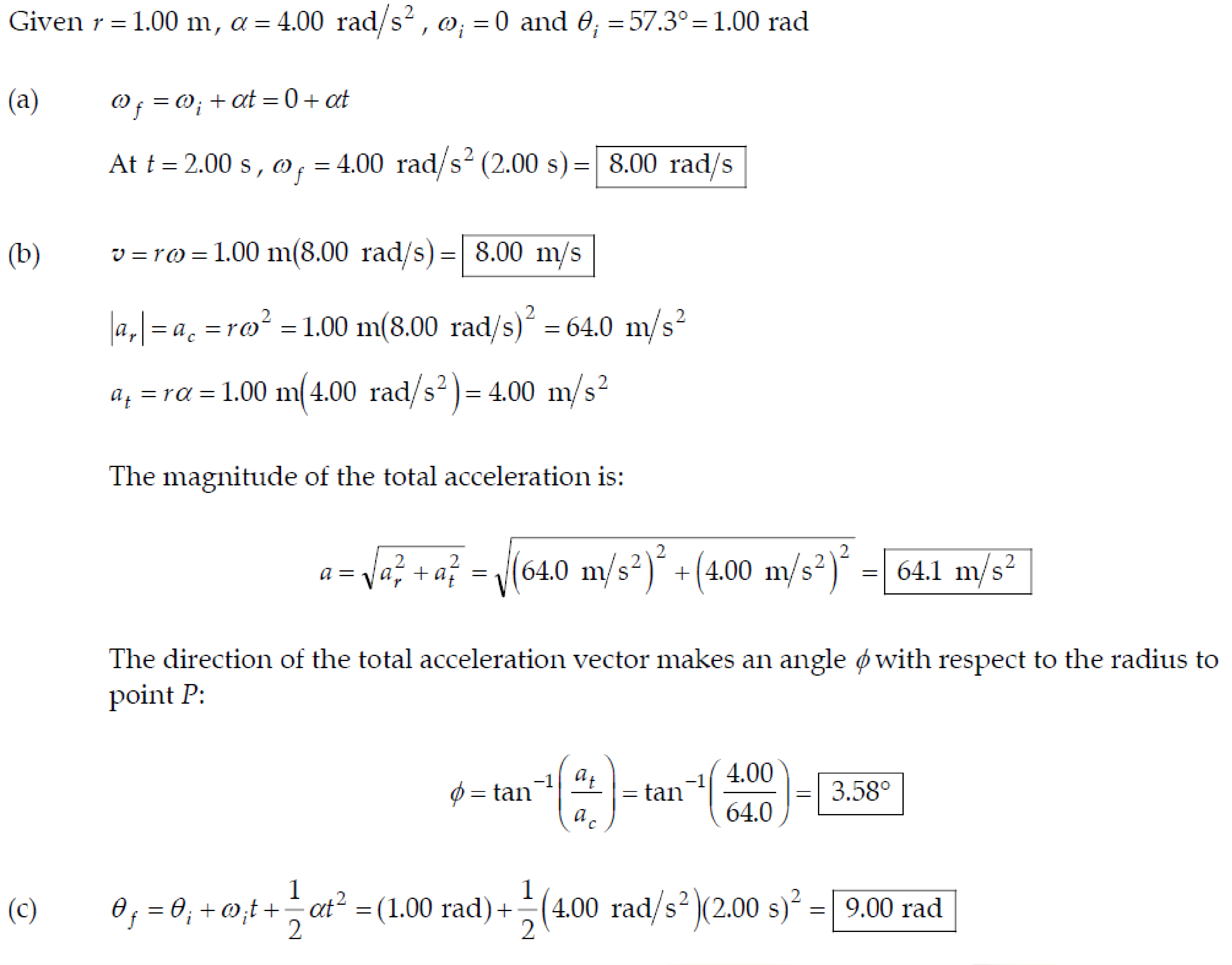


Given $r = 1.00\ \text{m}$, $\alpha = 4.00\ \text{rad/s}^2$, $\omega_i = 0$ and $\theta_i = 57.3° = 1.00\ \text{rad}$

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00\ \text{s}$, $\omega_f = 4.00\ \text{rad/s}^2\,(2.00\ \text{s}) = \boxed{8.00\ \text{rad/s}}$

(b) $v = r\omega = 1.00\ \text{m}(8.00\ \text{rad/s}) = \boxed{8.00\ \text{m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00\ \text{m}(8.00\ \text{rad/s})^2 = 64.0\ \text{m/s}^2$

$a_t = r\alpha = 1.00\ \text{m}\left(4.00\ \text{rad/s}^2\right) = 4.00\ \text{m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(64.0\ \text{m/s}^2\right)^2 + \left(4.00\ \text{m/s}^2\right)^2} = \boxed{64.1\ \text{m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

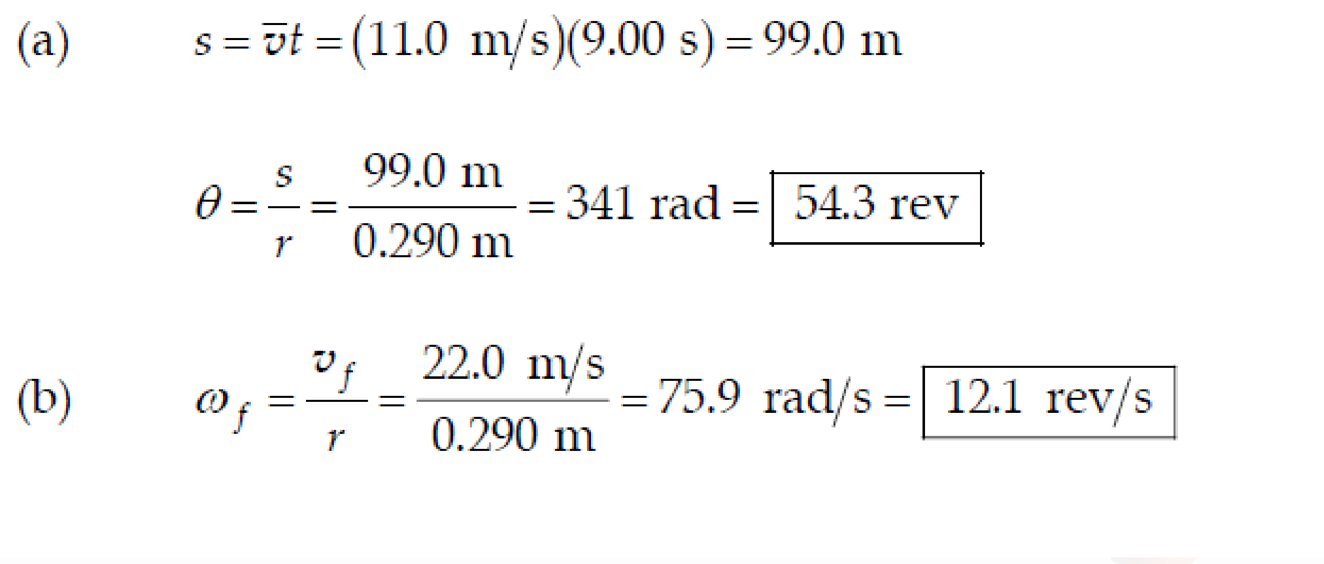
(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\ \text{rad}) + \frac{1}{2}\left(4.00\ \text{rad/s}^2\right)(2.00\ \text{s})^2 = \boxed{9.00\ \text{rad}}$

PROBLEMS

Section 10.3 Angular and Linear Quantities

16. A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. If the diameter of a tire is 58.0 cm, find (a) the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

SOLUTIONS TO PROBLEM:



(a) $$s = \bar{v}t = (11.0\ \text{m/s})(9.00\ \text{s}) = 99.0\ \text{m}$$

$$\theta = \frac{s}{r} = \frac{99.0\ \text{m}}{0.290\ \text{m}} = 341\ \text{rad} = \boxed{54.3\ \text{rev}}$$

(b) $$\omega_f = \frac{v_f}{r} = \frac{22.0\ \text{m/s}}{0.290\ \text{m}} = 75.9\ \text{rad/s} = \boxed{12.1\ \text{rev/s}}$$

PROBLEMS

Section 10.3 Angular and Linear Quantities

17. A disk 8.00 cm in radius rotates at a constant rate of 1 200 rev/min about its central axis. Determine (a) its angular speed, (b) the tangential speed at a point 3.00 cm from its center, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2.00 s.

SOLUTIONS TO PROBLEM:

(a) $\omega = 2\pi f = \dfrac{2\pi\,\text{rad}}{1\text{ rev}}\left(\dfrac{1\,200\text{ rev}}{60.0\text{ s}}\right) = \boxed{126\ \text{rad/s}}$

(b) $v = \omega r = (126\ \text{rad/s})(3.00 \times 10^{-2}\ \text{m}) = \boxed{3.77\ \text{m/s}}$

(c) $a_c = \omega^2 r = (126)^2(8.00 \times 10^{-2}) = 1\,260\ \text{m/s}^2$ so $\mathbf{a}_r = \boxed{1.26\ \text{km/s}^2\ \text{toward the center}}$

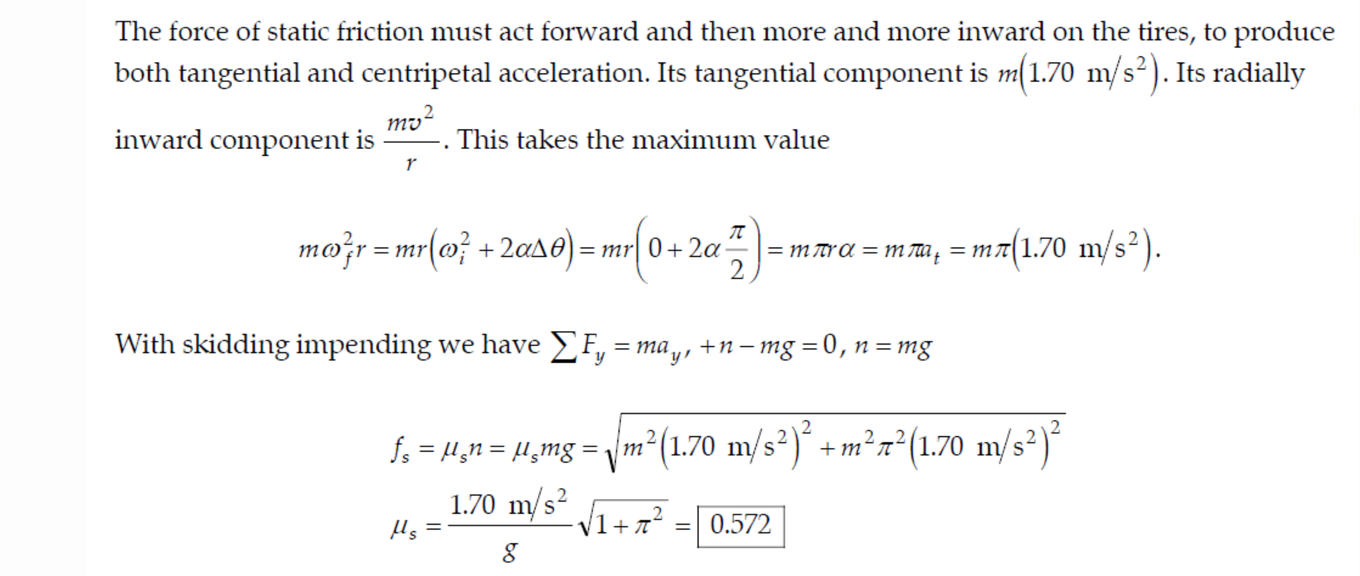
(d) $s = r\theta = \omega r t = (126\ \text{rad/s})(8.00 \times 10^{-2}\ \text{m})(2.00\ \text{s}) = \boxed{20.1\ \text{m}}$

PROBLEMS

Section 10.3 Angular and Linear Quantities

18. A car traveling on a flat (unbanked) circular track accelerates uniformly from rest with a tangential acceleration of 1.70 m/s2. The car makes it one quarter of the way around the circle before it skids off the track. Determine the coefficient of static friction between the car and track from these data.

SOLUTIONS TO PROBLEM:



The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m\left(1.70\ \text{m/s}^2\right)$. Its radially inward component is $\frac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr\left(\omega_i^2 + 2\alpha\Delta\theta\right) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi\left(1.70\ \text{m/s}^2\right).$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2\left(1.70\ \text{m/s}^2\right)^2 + m^2\pi^2\left(1.70\ \text{m/s}^2\right)^2}$$

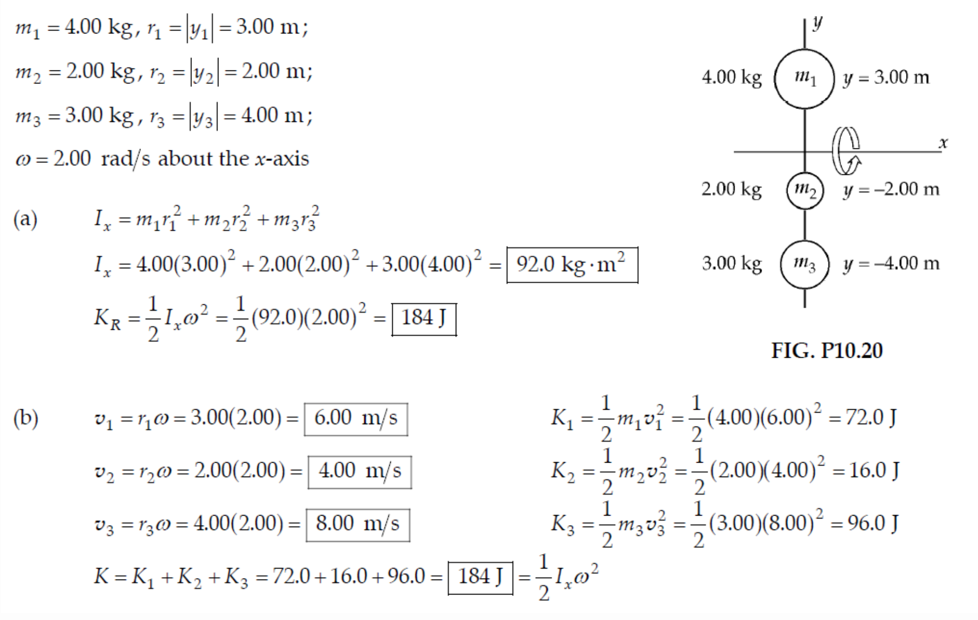
$$\mu_s = \frac{1.70\ \text{m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

PROBLEMS

Section 10.4 Rotational Kinetic Energy

20. Rigid rods of negligible mass lying along the y axis connect three particles (Fig. P10.20). If the system rotates about the x axis with an angular speed of 2.00 rad/s, find (a) the moment of inertia about the x axis and the total rotational kinetic energy evaluated from $\frac{1}{2} I\,\omega^2$ and (b) the tangential speed of each particle and the total kinetic energy evaluated from $\sum_i \frac{1}{2} m_i v_i{}^2$.

SOLUTIONS TO PROBLEM:



$m_1 = 4.00\ \text{kg},\ r_1 = |y_1| = 3.00\ \text{m};$

$m_2 = 2.00\ \text{kg},\ r_2 = |y_2| = 2.00\ \text{m};$

$m_3 = 3.00\ \text{kg},\ r_3 = |y_3| = 4.00\ \text{m};$

$\omega = 2.00\ \text{rad/s}$ about the x-axis

(a)

$$I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\ \text{kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2} I_x \omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\ \text{J}}$$

(b)

$$v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\ \text{m/s}} \qquad K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\ \text{J}$$

$$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\ \text{m/s}} \qquad K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\ \text{J}$$

$$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\ \text{m/s}} \qquad K_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\ \text{J}$$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\ \text{J}} = \frac{1}{2} I_x \omega^2$$

FIG. P10.20

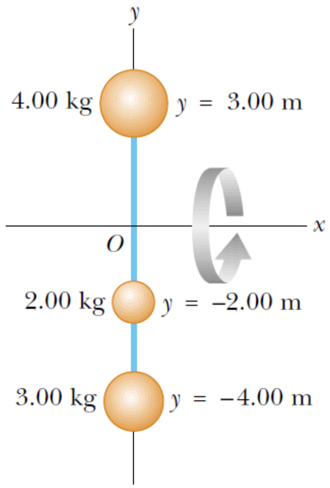


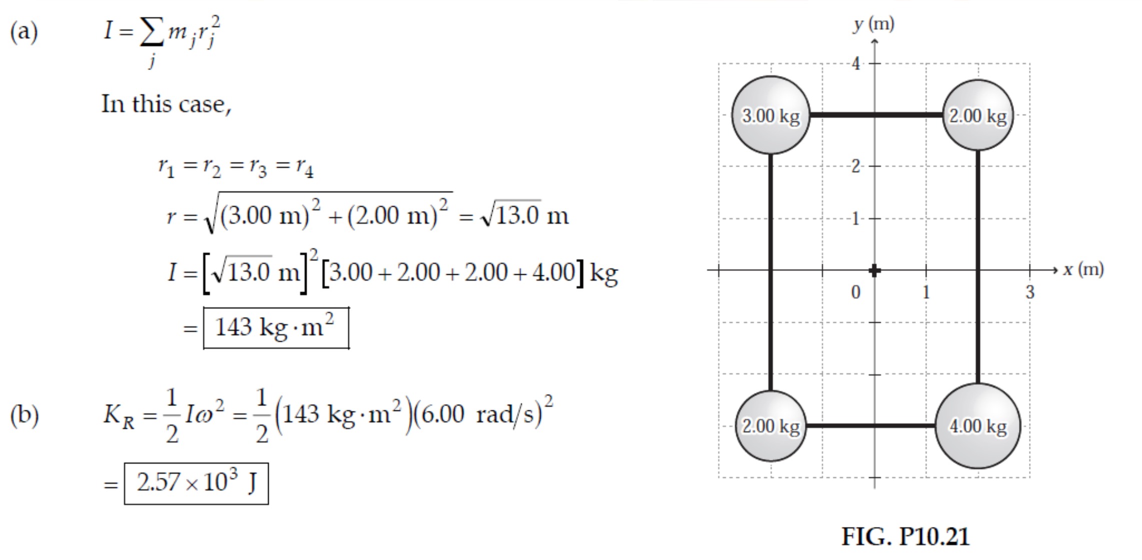
Figure P10.20

PROBLEMS

Section 10.4 Rotational Kinetic Energy

21. The four particles in Figure P10.21 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. If the system rotates in the *xy* plane about the *z* axis with an angular speed of 6.00 rad/s, calculate (a) the moment of inertia of the system about the *z* axis and (b) the rotational kinetic energy of the system.

SOLUTIONS TO PROBLEM:



(a) $I=\sum_j m_j r_j^2$

In this case,

$$r_1=r_2=r_3=r_4$$

$$r=\sqrt{(3.00\text{ m})^2+(2.00\text{ m})^2}=\sqrt{13.0}\text{ m}$$

$$I=\left[\sqrt{13.0}\text{ m}\right]^2[3.00+2.00+2.00+4.00]\text{ kg}$$

$$=\boxed{143\text{ kg}\cdot\text{m}^2}$$

(b) $K_R=\frac{1}{2}I\omega^2=\frac{1}{2}(143\text{ kg}\cdot\text{m}^2)(6.00\text{ rad/s})^2$

$$=\boxed{2.57\times10^3\text{ J}}$$

FIG. P10.21

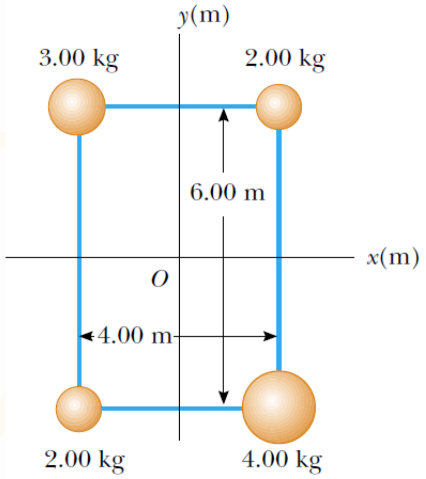


Figure P10.21

PROBLEMS

Section 10.6 Torque

31. Find the net torque on the wheel in Figure P10.31 about the axle through *O* if *a* =10.0 cm and *b*= 25.0 cm.

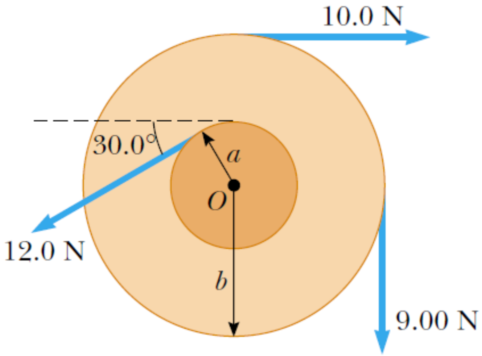


Figure P10.31

SOLUTIONS TO PROBLEM:

$$\sum \tau = 0.100\ \text{m}(12.0\ \text{N}) - 0.250\ \text{m}(9.00\ \text{N}) - 0.250\ \text{m}(10.0\ \text{N}) = \boxed{-3.55\ \text{N}\cdot\text{m}}$$

The thirty-degree angle is unnecessary information.

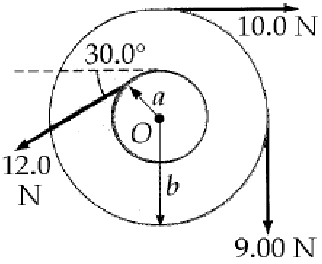


FIG. P10.31

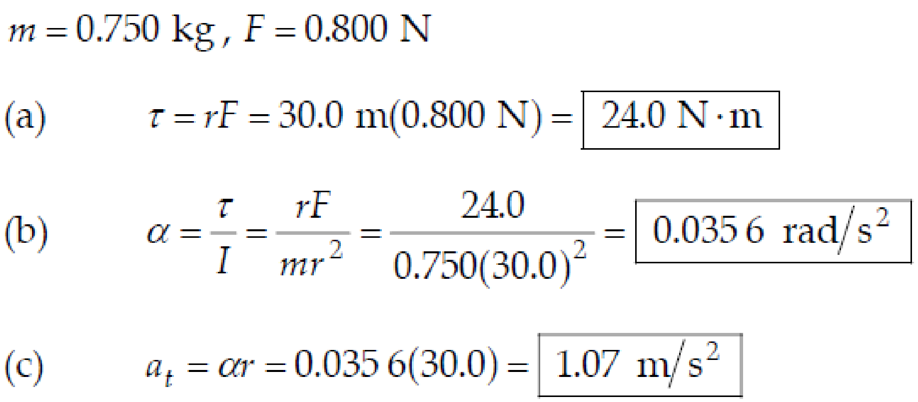
PROBLEMS

Section 10.7 Relationship between Torque and Angular Acceleration

35. A model airplane with mass 0.750 kg is tethered by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire. (a) Find the torque the net thrust produces about the center of the circle. (b) Find the angular acceleration of the airplane when it is in level flight.

(c) Find the linear acceleration of the airplane tangent to its flight path.

SOLUTIONS TO PROBLEM:



$m = 0.750\ \text{kg}$, $F = 0.800\ \text{N}$

(a) $\tau = rF = 30.0\ \text{m}(0.800\ \text{N}) = \boxed{24.0\ \text{N}\cdot\text{m}}$

(b) $\alpha = \dfrac{\tau}{I} = \dfrac{rF}{mr^2} = \dfrac{24.0}{0.750(30.0)^2} = \boxed{0.0356\ \text{rad/s}^2}$

(c) $a_t = \alpha r = 0.0356(30.0) = \boxed{1.07\ \text{m/s}^2}$

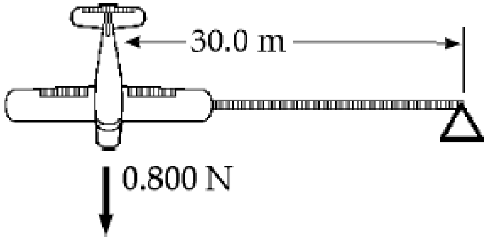


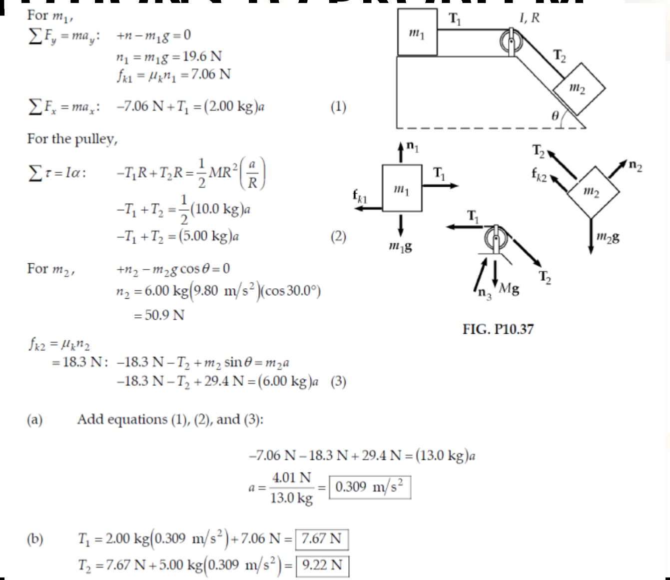
FIG. P10.35

PROBLEMS

Section 10.7 Relationship between Torque and Angular Acceleration

37. A block of mass *m*1=2 kg and a block of mass *m*2= 6 kg are connected by a massless string over a pulley in the shape of a solid disk having radius *R* =0.250 m and mass *M* =10 kg. These blocks are allowed to move on a fixed block-wedge of angle $\boldsymbol{\theta}$=30.0° as in Figure P10.37. The coefficient of kinetic friction is 0.360 for both blocks. Draw free-body diagrams of both blocks and of the pulley. Determine (a) the acceleration of the two blocks and (b) the tensions in the string on both sides of the pulley.

SOLUTIONS TO PROBLEM:



For m_1,

$$\sum F_y = ma_y: \quad +n - m_1 g = 0$$

$$n_1 = m_1 g = 19.6\ \text{N}$$

$$f_{k1} = \mu_k n_1 = 7.06\ \text{N}$$

$$\sum F_x = ma_x: \quad -7.06\ \text{N} + T_1 = (2.00\ \text{kg})a \qquad (1)$$

For the pulley,

$$\sum \tau = I\alpha: \quad -T_1 R + T_2 R = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0\ \text{kg})a$$

$$-T_1 + T_2 = (5.00\ \text{kg})a \qquad (2)$$

For m_2,

$$+n_2 - m_2 g\cos\theta = 0$$

$$n_2 = 6.00\ \text{kg}\left(9.80\ \text{m/s}^2\right)(\cos 30.0°)$$

$$= 50.9\ \text{N}$$

$$f_{k2} = \mu_k n_2$$

$$= 18.3\ \text{N}: \quad -18.3\ \text{N} - T_2 + m_2\sin\theta = m_2 a$$

$$-18.3\ \text{N} - T_2 + 29.4\ \text{N} = (6.00\ \text{kg})a \qquad (3)$$

FIG. P10.37

(a) Add equations (1), (2), and (3):

$$-7.06\ \text{N} - 18.3\ \text{N} + 29.4\ \text{N} = (13.0\ \text{kg})a$$

$$a = \frac{4.01\ \text{N}}{13.0\ \text{kg}} = \boxed{0.309\ \text{m/s}^2}$$

(b)

$$T_1 = 2.00\ \text{kg}\left(0.309\ \text{m/s}^2\right) + 7.06\ \text{N} = \boxed{7.67\ \text{N}}$$

$$T_2 = 7.67\ \text{N} + 5.00\ \text{kg}\left(0.309\ \text{m/s}^2\right) = \boxed{9.22\ \text{N}}$$

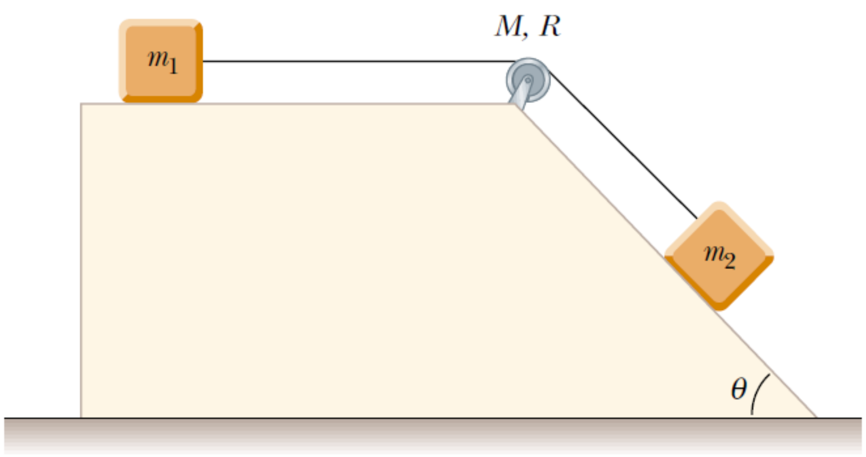


Figure P10.37

PROBLEMS

Section 10.8 Work, Power, and Energy in Rotational Motion

46. A 15.0-kg object and a 10.0-kg object are suspended, joined by a cord that passes over a pulley with a radius of 10.0 cm and a mass of 3.00 kg (Fig. P10.46). The cord has a negligible mass and does not slip on the pulley. The pulley rotates on its axis without friction. The objects start from rest 3.00 m apart. Treat the pulley as a uniform disk, and determine the speeds of the two objects as they pass each other.

SOLUTIONS TO PROBLEM:

Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

$$\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2 = 0 + m_1 g h_{1i} + m_2 g h_{2i} + 0$$

$$\frac{1}{2}(15.0 + 10.0)v^2 + \frac{1}{2}\left[\frac{1}{2}(3.00)R^2\right]\left(\frac{v}{R}\right)^2 = 15.0(9.80)(1.50) + 10.0(9.80)(-1.50)$$

$$\frac{1}{2}(26.5\ \text{kg})v^2 = 73.5\ \text{J} \Rightarrow v = \boxed{2.36\ \text{m/s}}$$

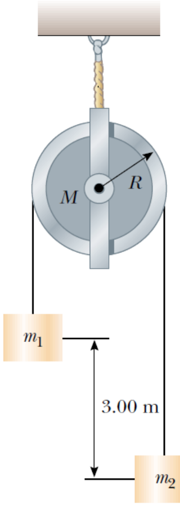


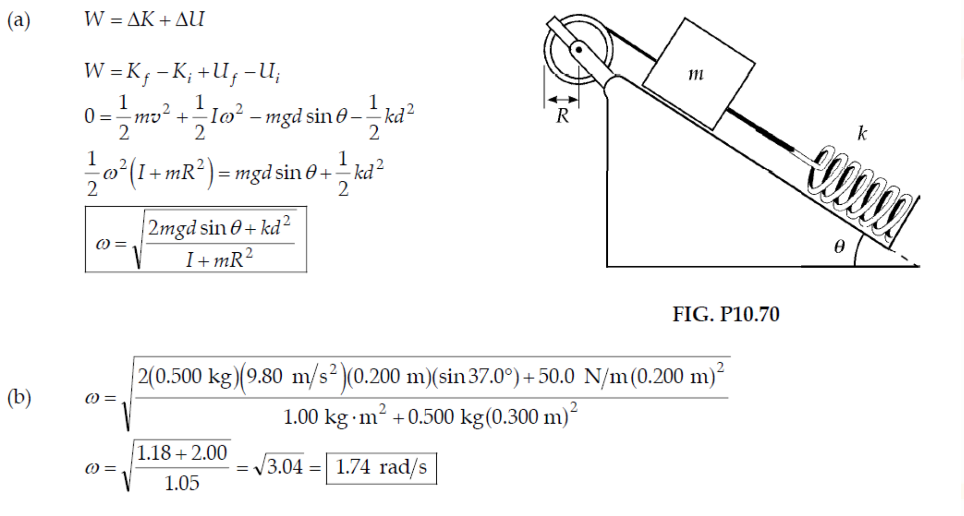
Figure P1

PROBLEMS

Additional Problems

70. The reel shown in Figure P10.70 has radius *R* and moment of inertia *I*. One end of the block of mass *m* is connected to a spring of force constant *k*, and the other end is fastened to a cord wrapped around the reel. The reel axle and the incline are frictionless. The reel is wound counterclockwise so that the spring stretches a distance *d* from its unstretched position and is then released from rest. (a) Find the angular speed of the reel when the spring is again unstretched. (b) Evaluate the angular speed numerically at this point if *I* =1 kg ·m2, *R* =0.3 m, *k* =50 N/m, *m* =0.5 kg, *d* =0.2 m, and ***θ***=37.0°

SOLUTION



(a)

$$W = \Delta K + \Delta U$$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

FIG. P10.70

(b)

$$\omega = \sqrt{\frac{2(0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m})(\sin 37.0°) + 50.0\text{ N/m}(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + 0.500\text{ kg}(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

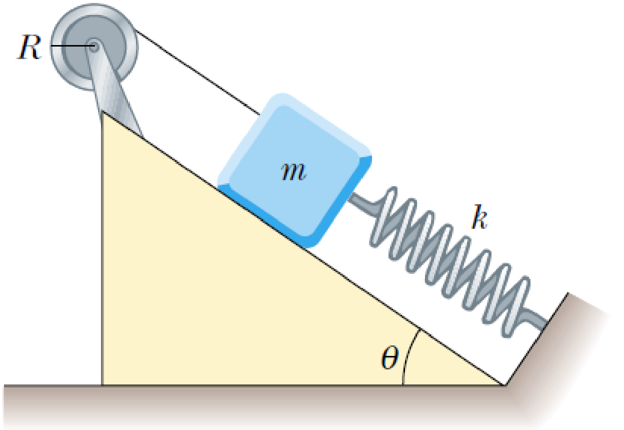


Figure P10.70

PROBLEMS

Additional Problems

71. Two blocks, as shown in Figure P10.71, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia *I*. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s2. (a) Determine *T*1 and *T*2, the tensions in the two parts of the string. (b) Find the moment of inertia of the pulley.

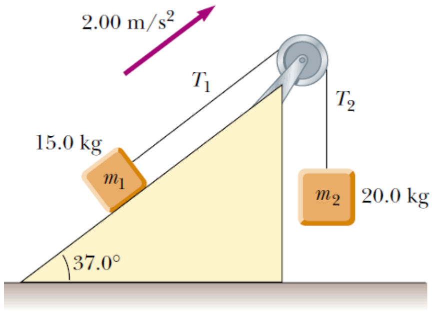


Figure P10.71

SOLUTIONS TO PROBLEM:

(a) $m_2 g - T_2 = m_2 a$

$$T_2 = m_2(g-a) = 20.0\text{ kg}\left(9.80\text{ m/s}^2 - 2.00\text{ m/s}^2\right) = \boxed{156\text{ N}}$$

$$T_1 - m_1 g\sin 37.0° = m_1 a$$

$$T_1 = (15.0\text{ kg})(9.80\sin 37.0° + 2.00)\text{ m/s}^2 = \boxed{118\text{ N}}$$

(b) $(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$

$$I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156\text{ N} - 118\text{ N})(0.250\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{1.17\text{ kg}\cdot\text{m}^2}$$

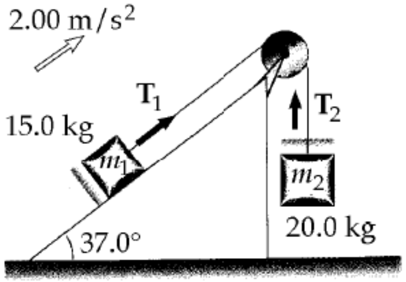
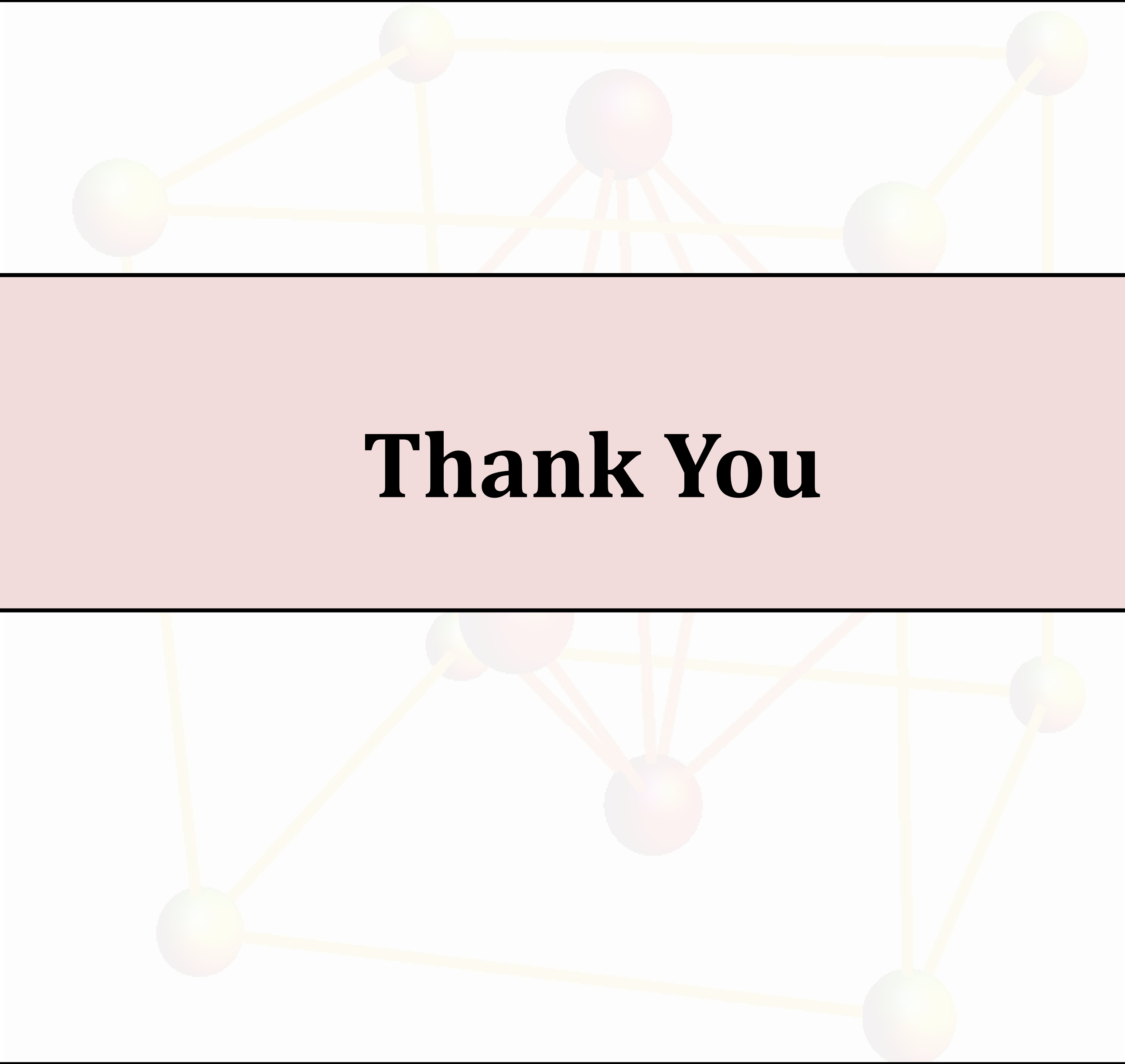


FIG. P10.71



Thank You

ACKNOWLEDGEMENTS