Phys 103

Chapter 3

Vectors

By

Dr. Saif M. H. Qaid

LECTURE OUTLINE

- 3.1 Coordinate Systems
- 3.2 Vector and Scalar Quantities
- 3.3 Some Properties of Vectors
- 3.4 Components of a Vector and Unit Vectors

Lecture Summary

Scalar quantities are those that have only a numerical value and no associated direction.

Vector quantities have both magnitude and direction and obey the laws of vector

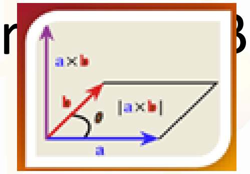
The magnitude of a vector is always a positive number.

When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity. We can add two vectors A and B graphically. In this method (Fig. 3.6), the resultant vector R =A + B runs from the tail of A to the tip of B.

A second method of adding vectors involves components of the vectors. The x component Ax of the vector A is equal to the projection of A along the x axis of a coordinate system, as shown in Figure 3.13, where $A_x = A\ cos\theta$. The y component A_y of A is the projection of A along the y axis, where $A_y = A\ sin\theta$.

Lecture Summary

- Be sure you can determine which trigonometric functions you should use in all situations, especially when ! is defined as something other than the counterclockwise angle from the positive x axis.
- If a vector **A** has an *x* component *Ax* and a *y* component *Ay*, the vector can be expressed in unit–vector form as $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$. In this notation, ˆ**i** is a unit vector pointing in the positive *x* direction, and ˆ**j** is a unit vector pointing in the positive *y* dir… Because $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors, $|\hat{\imath}| = |\hat{\jmath}| = 1$.



- We can find the resultant of two or more vectors by resolving all vectors into their *x* and *y* components, adding their resultant *x* and *y* components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. We can find the angle that the resultant vector makes with respect to the *x* axis by using a suitable trigonometric function.

PROBLEMS

- **Section 3.1 Coordinate Systems**

1. The polar coordinates of a point are r = 5.50 m and θ= 240°. What are the Cartesian coordinates of this point?

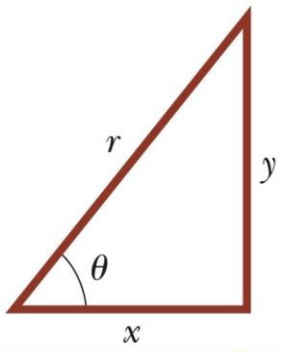
SOLUTIONS TO PROBLEM:

- $x = r \cos \theta$
- $y = r \sin \theta$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



PROBLEMS

- **Section 3.1 Coordinate Systems**

4. Two points in the *xy* plane have Cartesian coordinates (2.00, -4.00) m and (-3.00, 3.00) m. Determine (a) the distance between these points and (b) their polar coordinates.

SOLUTIONS TO PROBLEM:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

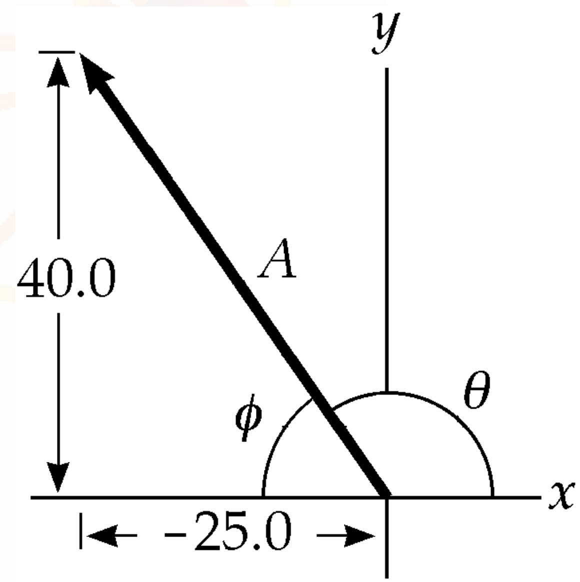
$$r_1 = \sqrt{{x_1}^2 + {y_1}^2} \textbf{ and } r_2 = \sqrt{{x_2}^2 + {y_2}^2}$$

$$\theta_1 = tan^{-1}\frac{y_1}{x_1} \textbf{ and } \theta_2 = tan^{-1}\frac{y_2}{x_2}$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

19. A vector has an *x* component of -25.0 units and a *y* component of 40.0 units. Find the magnitude and direction of this vector.



SOLUTIONS TO PROBLEM:

$$\text{A} = \sqrt{{A_x}^2 + {A_y}^2}$$

$$\boldsymbol{tan}\emptyset = \frac{|A_y|}{|A_x|} \rightarrow \emptyset = \boldsymbol{tan}^{-1}\left(\frac{|A_y|}{|A_x|}\right)$$

$$\theta = 180 - \emptyset$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

21. Obtain expressions in component form for the position vectors having the following polar coordinates: (a) 12.8 m, 150° (b) 3.30 cm, 60.0° (c) 22.0 in., 215°.

SOLUTIONS TO PROBLEM:

$$x = r\cos\theta \text{ and } \quad y = r\sin\theta$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

27. Given the vectors

$$\boldsymbol{A}=2.00\,\hat{\imath} + 6.00\hat{\jmath}$$

And

$$\mathbf{B}=3.00\,\hat{\imath} + 2.00\hat{\jmath}$$

(a) draw the vector sum **C** = **A** + **B** and the vector difference **D** = **A** - **B**.
(b) Calculate **C** and **D**, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the + *x* axis.

SOLUTIONS TO PROBLEM:

$$\boldsymbol{C} = (A_x+B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$\boldsymbol{D} = (A_x-B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath}$$

$C_x = A_x + B_x$ and $C_y = A_y + B_y$ C= $\sqrt{{C_x}^2 + {C_y}^2}$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

30. Vector **A** has x and y components of -8.70 cm and 15.0 cm, respectively; vector **B** has x and y components of 13.2 cm and -6.60 cm, respectively. If **A** - **B** +3**C** =0, what are the components of **C**?

SOLUTIONS TO PROBLEM:

$C_x = A_x + B_x$ and $C_y = A_y + B_y$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

31. Consider the two vectors $\boldsymbol{A}$=$3\ \hat{\imath} - 2\hat{\jmath}$ and $\mathbf{B}$=$-\ \hat{\imath} - 4\hat{\jmath}$.

Calculate

(a) **A** + **B**,

(b) **A** - **B**,

(c) | **A** + **B** |,

(d) | **A** - **B** |,

and (e) the directions of **A** + **B** and **A** – **B**.

SOLUTIONS TO PROBLEM:

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

$$\mathbf{A} - \mathbf{B} = (A_x - B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath}$$

$$|\ \mathbf{A} + \mathbf{B}\ | = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

33. A particle undergoes the following consecutive displacements: 3.50 m south, 8.20 m northeast, and 15.0 m west.

What is the resultant displacement?

SOLUTIONS TO PROBLEM:

$$\boldsymbol{d_1} = (-3.5\hat{\boldsymbol{j}})\boldsymbol{m}$$
$$\boldsymbol{d_2} = (\mathbf{8.20}\ \boldsymbol{cos}\mathbf{45}\hat{\boldsymbol{\imath}} + \mathbf{8.20}\ \boldsymbol{sin}\mathbf{45})\boldsymbol{m}$$
$$\boldsymbol{d_1} = (-\mathbf{15}\hat{\boldsymbol{\imath}})\boldsymbol{m}$$
$$\boldsymbol{R} = \boldsymbol{d_1} + \boldsymbol{d_2} + \boldsymbol{d_3}$$

The magnitude of the resultant displacement is $\mathrm{R} = \left|\boldsymbol{R}\right| = \sqrt{R_x{}^2 + R_y{}^2}$ **and** The direction is

$\theta = \boldsymbol{tan^{-1}}\left(\frac{R_y}{R_x}\right)$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

39. Vector **B** has *x*, *y*, and *z* components of 4.00, 6.00, and 3.00 units, respectively. Calculate the magnitude of **B** and the angles that **B** makes with the coordinate axes.

SOLUTIONS TO PROBLEM:

$$\mathbf{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$|\boldsymbol{B}| = \sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2}$$

$$\alpha = \cos^{-1}\frac{\mathbf{B_x}}{|\boldsymbol{B}|};\ \beta = \cos^{-1}\frac{\mathbf{B}_{\boldsymbol{y}}}{|\boldsymbol{B}|}\ and\ \gamma = \cos^{-1}\frac{\mathbf{B}_{\boldsymbol{z}}}{|\boldsymbol{B}|}$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

49. Three displacement vectors of a croquet ball are shown in Figure P3.49, where |**A** |= 20.0 units, |**B** | =40.0 units, and |**C** |=30.0 units. Find (a) the resultant in unit–vector notation and (b) the magnitude and direction of the resultant displacement.

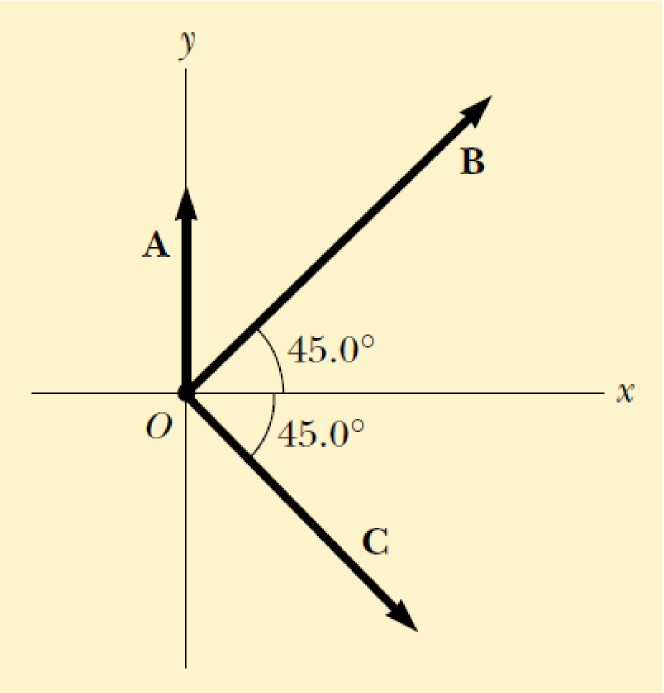


Figure P3.49

SOLUTIONS TO PROBLEM:

$$R_x = (40\ cos45\hat{\imath} + 30\ cos45)$$
$$R_y = (40\ sin45\hat{\imath} - 30\ sin45)$$

$$\mathrm{R} = |\boldsymbol{R}| = \sqrt{{R_x}^2 + {R_y}^2}\ \textbf{and}\ \theta = \boldsymbol{tan^{-1}}\left(\frac{R_y}{R_x}\right)$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

50. If **A** = (6.00ˆ**i** - 8.00ˆ**j**) units, **B** = (-8.00ˆ**i** + 3.00ˆ**j**) units, and **C** = (26.0ˆ**i** +19.0ˆ**j**) units, determine *a* and *b* such that *a***A** + *b***B** + **C=** 0.

SOLUTIONS TO PROBLEM:

Taking components along ˆ**i** and ˆ**j** , we get two equations:

$6.00a - 8.00b + 26.0 = 0$ and $-8.00a + 3.00b + 19.0 = 0$.

Solving simultaneously,

$a = 5.00$, $b = 7.00$.

Therefore, $5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0$.

Thank You

ACKNOWLEDGEMENTS