Phys 103

Chapter 4

Motion in Two Dimensions

By

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LECTURE OUTLINE

4.1 The Position, Velocity, and Acceleration Vectors

4.2 Two-Dimensional Motion with Constant Acceleration

4.3 Projectile Motion

4.4 Uniform Circular Motion

4.5 Tangential and Radial Acceleration

Introduction

Constant Acceleration motion of a particle in 2-D:

$v_{xf} = v_{xi} + a_x t$ $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ $v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$	$v_{yf} = v_{yi} + a_y t$ $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ $v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i)$

Velocity and position in Vector form in 2-D motion:

$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ $\overrightarrow{v_f} = (v_{xi} + a_x t)\hat{\imath} + (v_{yi} + a_y t)\hat{\jmath}$ $\overrightarrow{v_f} = (v_{xi}\hat{\imath} + v_{yi}\hat{\jmath}) + (a_{xi}\hat{\imath} + a_{yi}\hat{\jmath})$ $\overrightarrow{v_f} = \overrightarrow{v_i} + at$	$r_f = \left(v_{xi}t + \frac{1}{2}a_x t^2\right)\hat{\imath} + \left(v_{yi}t + \frac{1}{2}a_y t^2\right)\hat{\jmath}$ $= (v_{xi}\hat{\imath} + v_{yi}\hat{\jmath})t + \frac{1}{2}(a_x\hat{\imath} + a_y\hat{\jmath})t^2$ $= v_i t + \frac{1}{2}a\,t^2$

4.3 Projectile Motion

Projectile Motion

An object may move in both the x and y directions simultaneously. The form of two-dimensional motion we will deal with is called **projectile motion.**

4.3 Projectile Motion

Assumptions of Projectile Motion

The free-fall acceleration is constant over the range of motion.

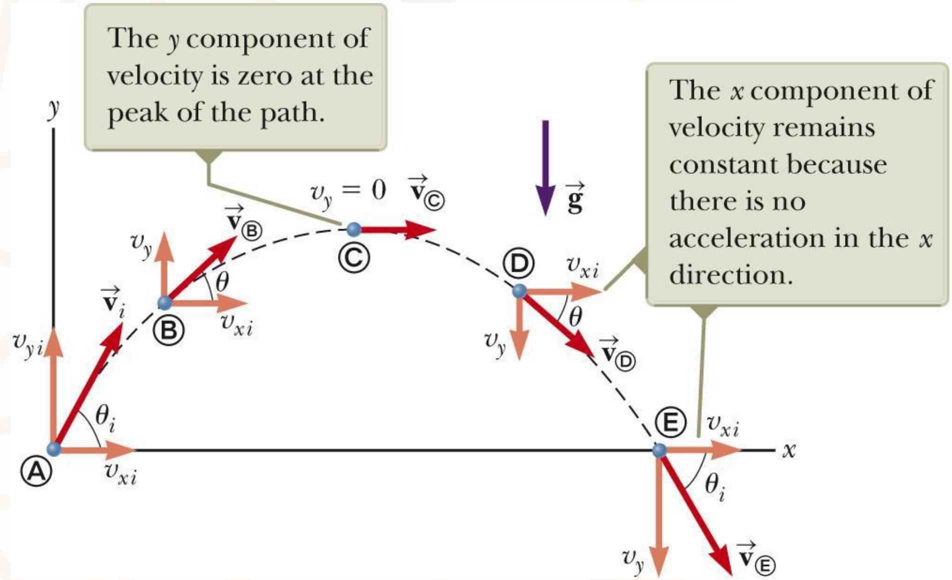
- It is directed downward.
- This is the same as assuming a flat Earth over the range of the motion.
- It is reasonable as long as the range is small compared to the radius of the Earth. The effect of air friction is negligible. With these assumptions, an object in projectile motion will follow a parabolic path.
- This path is called the **trajectory.**

4.3 Projectile Motion

Projectile Motion Diagram

Anyone who has observed a baseball in motion has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration g is constant over the range of motion and is directed downward, and (2) the effect of air resistance is negligible.

- We find that the path of a projectile, which we call its trajectory, is always a ***parabola***



The parabolic path of a projectile that leaves the origin with a velocity *vi*. The x component of v remains constant in time. The y component of velocity is zero at the peak of the path.

4.3 Projectile Motion

Acceleration at the Highest Point

The vertical velocity is zero at the top. The acceleration is not zero anywhere along the trajectory.

- If the projectile experienced zero acceleration at the highest point, its velocity at the point would not change.
- The projectile would move with a constant horizontal velocity from that point on.

4.3 Projectile Motion

Analyzing Projectile Motion

Consider the motion as the superposition of the motions in the x- and y-directions. The actual position at any time is given by:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}\, t^2$$

The initial velocity can be expressed in terms of its components.

$$v_{xi} = v_i \cos\theta_\mathrm{i} \; and \; v_{yi} = v_i \, sin\, \theta_\mathrm{i}$$

$$x_f = v_{xi} t \; and \;\; y_f = v_{yi} t + \frac{1}{2} a_y \mathrm{t}^2$$

The x-direction has constant velocity.

$a_x = 0$

The y-direction is free fall.

- $a_y = -g$

4.3 Projectile Motion

Analyzing Projectile Motion

We will be having 2 sets of equations: 1 for x and 1 for y directions*:*

$$x_f = v_{xi} t$$

$$t = \frac{x_f}{v_i \cos \theta_i}$$

$$y_f = v_{yi} t + \frac{1}{2} a_y t^2$$

$$y_f = v_i \sin \theta_i \frac{x_f}{v_i \cos \theta_i} - \frac{1}{2} g \left(\frac{x_f}{v_i \cos \theta_i}\right)^2$$

$$y_f = \tan \theta_i \, x_f - \frac{g}{2 v_i^2 \cos \theta_i^2} x_f^2$$

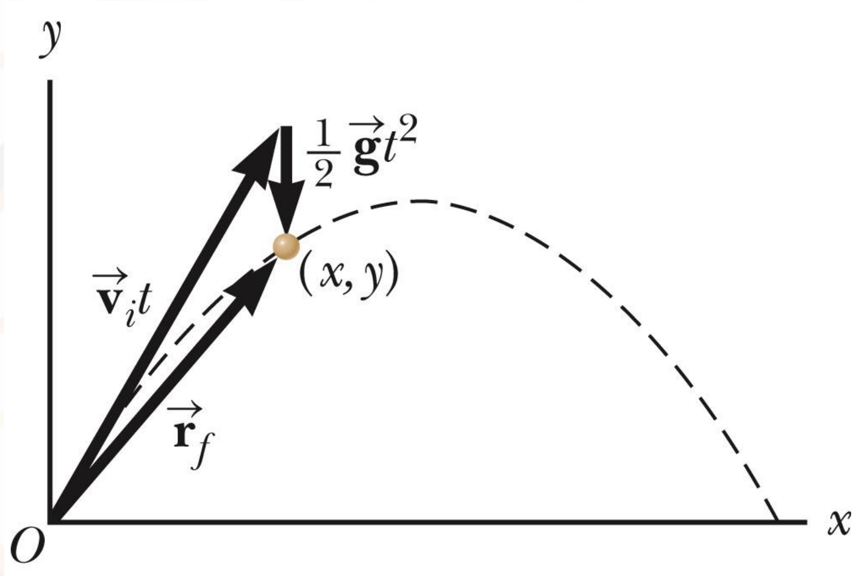
Or

$$y = ax - bx^2$$

4.3 Projectile Motion

Projectile Motion Vectors

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}\,\mathbf{t^2}$$



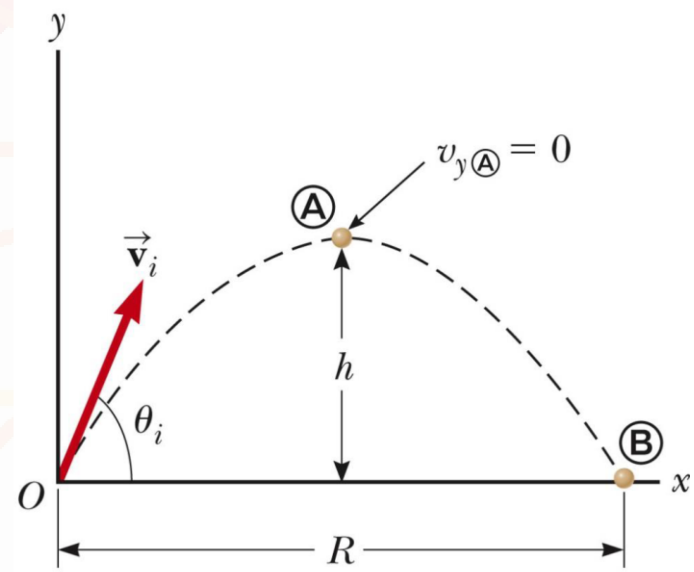
- The final position is the vector sum of the initial position, the position resulting from the initial velocity and the position resulting from the acceleration.

4.3 Projectile Motion

Time of Flight of a Projectile

We will consider the maximum height reached by a projectile:

1st: time of flight: at maxi. height $v_{yf} = 0$



$$v_{yf} = v_{yi} + a_y t = 0$$

$$\therefore \; 0 = v_i \sin \boldsymbol{\theta}_i - g t_{max}$$

$$t_{max} = \frac{v_i \sin \boldsymbol{\theta}_i}{g}$$

$$t_{flight} = \frac{2\, v_i \sin \boldsymbol{\theta}_i}{g}$$

Time of flight is twice the time required to reach to the maximum point. We call this Time-of –flight and is true only if the projecile final destination is on the same level as its starting point.

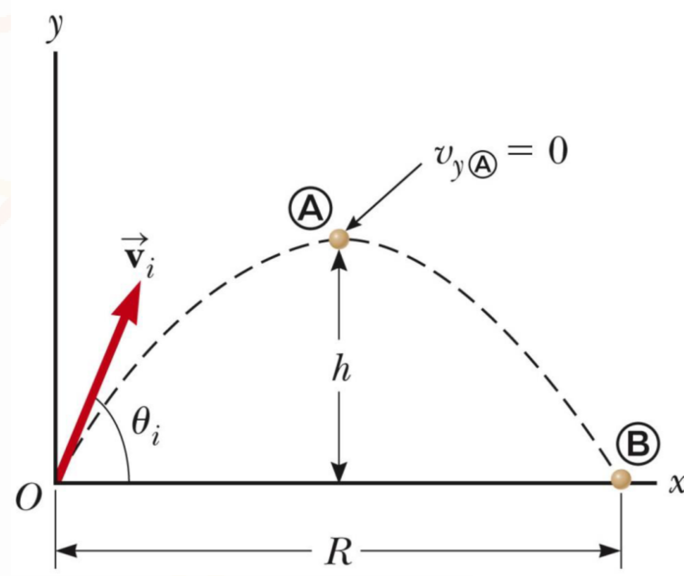
4.3 Projectile Motion

Range and Maximum Height of a Projectile

When analyzing projectile motion, two characteristics are of special interes.t The range, R, is the horizontal distance of the projectile. The maximum height the projectile reaches is h.

The maximum height of the projectile can be found in terms of the initial velocity vector:

$$h = \frac{v^2{}_i \, sin^2\theta_i}{2g}$$



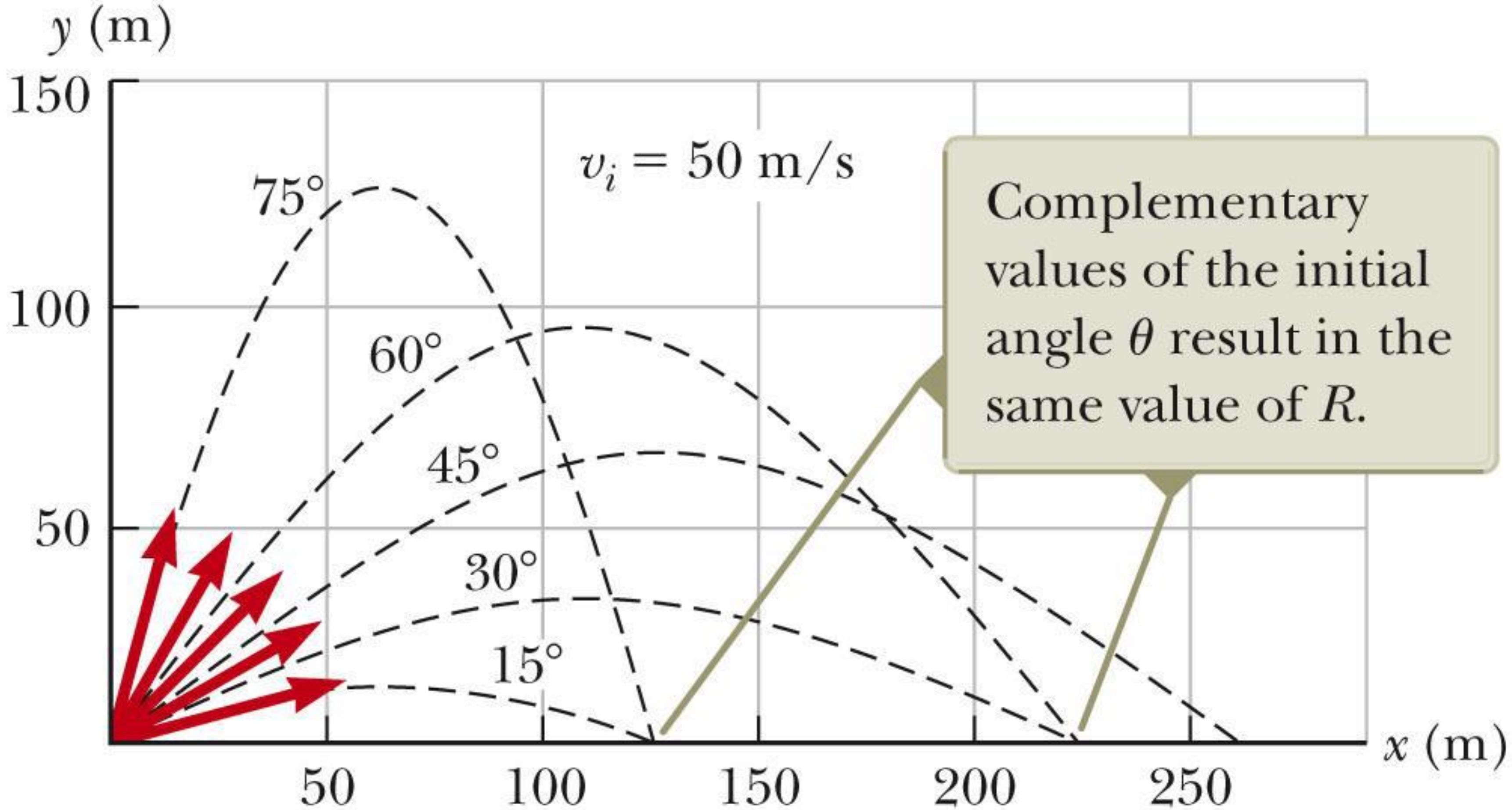
The range of a projectile can be expressed in terms of the initial velocity vector:

$$R = \frac{v^2{}_i \, sin^2 2\theta_i}{g}$$

This equation is valid only for symmetric

4.3 Projectile Motion

More About the Range of a Projectile



4.3 Projectile Motion

More About the Range of a Projectile

Range of a Projectile, final The maximum range occurs at $\theta_i = 45^o$. Complementary angles will produce the same range.

- The maximum height will be different for the two angles.
- The times of the flight will be different for the two angles.

4.3 Projectile Motion

Projectile Motion – Problem Solving Hints

Conceptualize

- Establish the mental representation of the projectile moving along its trajectory. Categorize
- Confirm air resistance is neglected.
- Select a coordinate system with x in the horizontal and y in the vertical direction. Analyze
- If the initial velocity is given, resolve it into x and y components.
- Treat the horizontal and vertical motions independently.

4.3 Projectile Motion

Projectile Motion – Problem Solving Hints, cont.

Analysis, cont.

- Analyze the horizontal motion with the particle-under-constant-velocity model.
- Analyze the vertical motion with the particle-under-constant-acceleration model.
- Remember that both directions share the same time.

Finalize

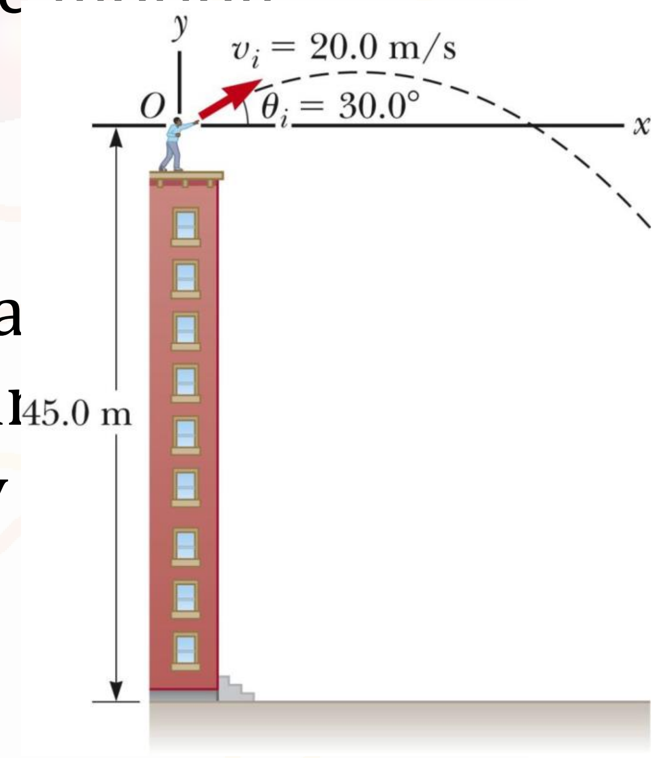
- Check to see if your answers are consistent with the mental and pictorial representations.
- Check to see if your results are realistic.

4.3 Projectile Motion

Non-Symmetric Projectile Motion

Follow the general rules for projectile motion

- Break the y-direction into parts.
- up and down or
- symmetrical back to initial height a the height Apply the problem solvi determine and solve the necessary non-symmetric in other ways



4.4 Uniform Circular Motion

Uniform circular motion occurs when an object moves in a circular path with a constant speed. The associated **analysis model** is a particle in uniform circular motion. An acceleration exists since the direction of the motion is changing .

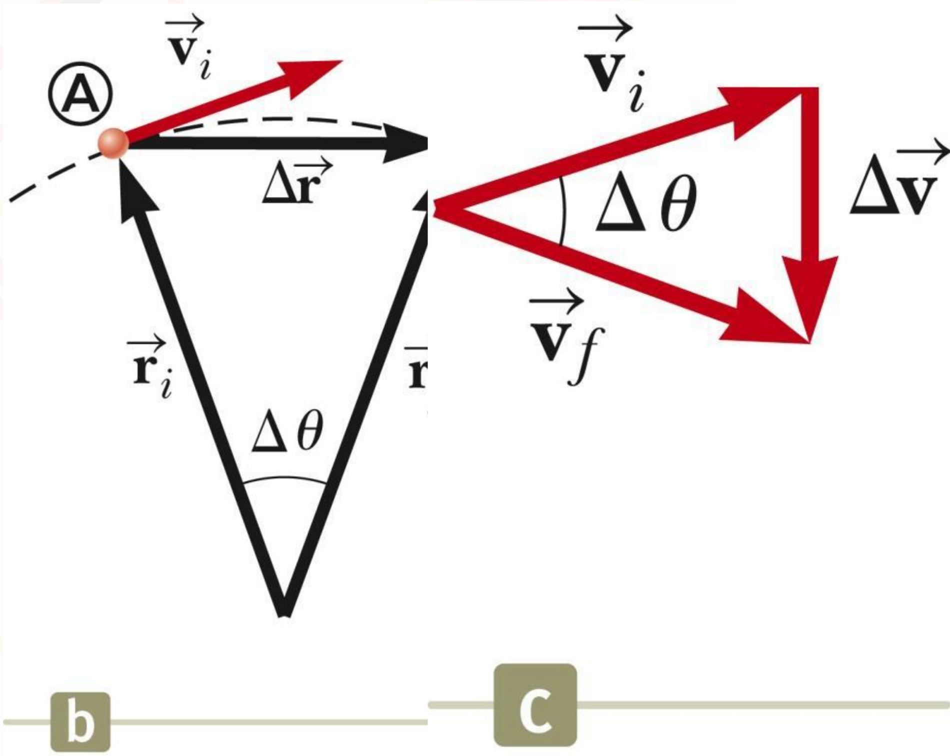
- This change in velocity is related to an acceleration. The constant-magnitude velocity vector is always tangent to the path of the object.

4.4 Uniform Circular Motion

Changing Velocity in Uniform Circular Motion

The change in the velocity vector is due to the change in direction.

- The direction of the change in velocity is toward the center of the circle.
- The vector diagram shows $\vec{v}_f = \vec{v}_i + \Delta v_i$



4.4 Uniform Circular Motion

Centripetal Acceleration

The acceleration is always perpendicular to the path of the motion. The acceleration always points toward the center of the circle of motion. This acceleration is called the **centripetal acceleration.**

The magnitude of the centripetal acceleration vector is given by

$$a_c = \frac{V^2}{r}$$

The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion.

4.4 Uniform Circular Motion

Period

The **period**, T, is the time required for one complete revolution. The speed of the particle would be the circumference of the circle of motion divided by the period. Therefore, the period is defined as

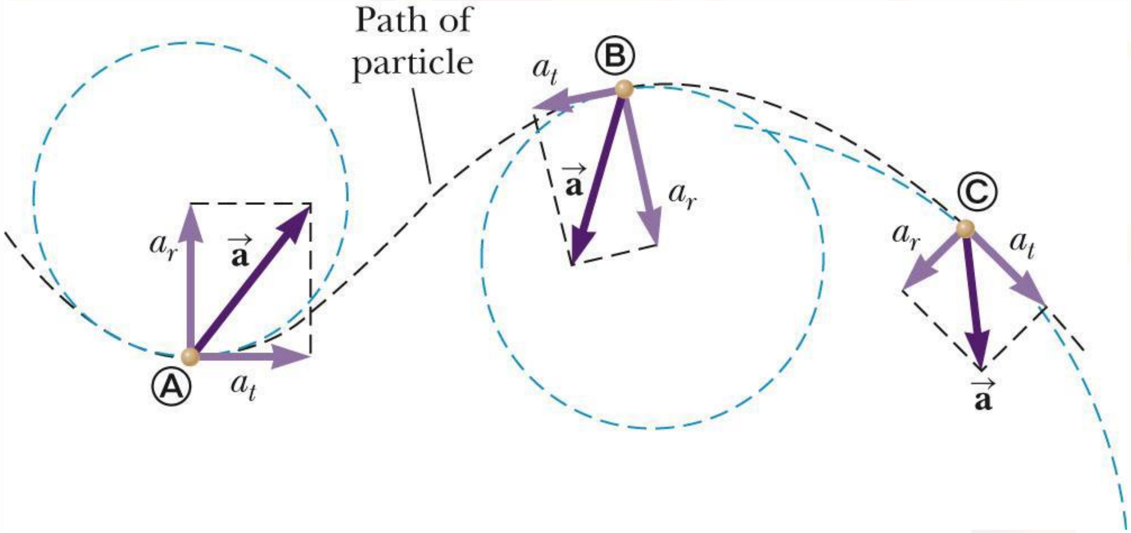
$$T = \frac{2\pi r}{V}$$

4.5 Tangential and Radial Acceleration

Tangential Acceleration

The magnitude of the velocity could also be changing. In this case, there would be a **tangential acceleration.** The motion would be under the influence of both tangential and centripetal accelerations.

- Note the changing acceleration vectors



4.5 Tangential and Radial Acceleration

Total Acceleration

The tangential acceleration causes the change in the speed of the particle. The radial acceleration comes from a change in the direction of the velocity vector.

4.5 Tangential and Radial Acceleration

Total Acceleration, equations

The tangential acceleration:

$$a_t = \left|\frac{dv}{dt}\right|$$

The radial acceleration:

$$a_r = -a_c = -\frac{v^2}{r}$$

The total acceleration:

- Magnitude

$$a = \sqrt{a^2{}_r + a^2{}_t}$$

Lecture Summary

Displacement of a particle in 2-D is:

$$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i$$

The average velocity is defined as:

$$\vec{\boldsymbol{v}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}$$

Instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{r}}}{\boldsymbol{d}t}$$

The average acceleration is defined as:

$$\vec{\boldsymbol{a}}_{avg} \equiv \frac{\Delta\vec{\boldsymbol{v}}}{\Delta \boldsymbol{t}} = \frac{\vec{\boldsymbol{v}}_f - \vec{\boldsymbol{v}}_i}{t_f - t_i}$$

The instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{d}t}$$

Thank You

ACKNOWLEDGEMENTS