Phys 103

Chapter 4

Motion in Two Dimensions

By

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LECTURE OUTLINE

4.1 The Position, Velocity, and Acceleration Vectors

4.2 Two-Dimensional Motion with Constant Acceleration

4.3 Projectile Motion

4.4 Uniform Circular Motion

4.5 Tangential and Radial Acceleration

Introduction

Constant Acceleration motion of a particle in 2-D:

$v_{xf} = v_{xi} + a_x t$	$v_{yf} = v_{yi} + a_y t$
$\boldsymbol{x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$	$\boldsymbol{y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2}$
${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$	${v_{yf}}^2 = {v_{yi}}^2 + 2a_y(y_f - y_i)$

Velocity and position in Vector form in 2-D motion:

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$
$$\overrightarrow{v_f} = (v_{xi} + a_x t)\hat{\imath} + (v_{yi} + a_y t)\hat{\jmath}$$
$$\overrightarrow{v_f} = \left(v_{xi}\hat{\imath} + v_{yi}\hat{\jmath}\right) + (a_{xi}\hat{\imath} + a_{yi}\hat{\jmath})$$
$$\overrightarrow{v_f} = \overrightarrow{v_i} + at$$

$$\boldsymbol{r_f = \left(v_{xi} t + \frac{1}{2} a_x t^2\right)\hat{\imath} + \left(v_{yi} t + \frac{1}{2} a_y t^2\right)\hat{\jmath}}$$
$$\boldsymbol{= (v_{xi}\,\hat{\imath} + v_{yi}\,\hat{\jmath})t + \frac{1}{2}(a_x\,\hat{\imath} + a_y\,\hat{\jmath})t^2}$$
$$\boldsymbol{= v_i t + \frac{1}{2} a\, t^2}$$

PROBLEMS

- **Section 4.1 The Position, Velocity, and Acceleration Vectors**

1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the total vector displacement, (b) the average speed, and (c) the average velocity. Let the positive x axis point east.

SOLUTIONS TO PROBLEM:

Net displacement=$\sqrt{x^2 + y^2}$

Average speed=$\frac{\sum x_i}{\sum t_i}$= =$\frac{\sum v_i \times t_i}{\sum t_i}$

Average velocity=$\frac{\text{Net displacement}}{\sum t_i}$

PROBLEMS

- **Section 4.1 The Position, Velocity, and Acceleration Vectors**

3. When the Sun is directly overhead, a hawk dives toward the ground with a constant velocity of 5.00 m/s at 60.0° below the horizontal Calculate the speed of her shadow on the level ground.

SOLUTIONS TO PROBLEM:

The sun projects onto the ground the *x*-component of her velocity:

$5.00 \cos(-60.0°)$ m /s

PROBLEMS

- **Section 4.2 Two-Dimensional Motion with Constant Acceleration**

5. At $t = 0$, a particle moving in the xy plane with constant acceleration has a velocity of $\mathbf{v}_i = (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s and is at the origin. At $t = 3.00$ s, the particle's velocity is $\mathbf{v} = (9.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

SOLUTIONS TO PROBLEM:

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

$$\boldsymbol{r_f = r_i} + v_i \boldsymbol{t} + \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{a}\,\mathbf{t}^{\mathbf{2}}$$

X=?

Y=?

PROBLEMS

- **Section 4.2 Two-Dimensional Motion with Constant Acceleration**

6. The vector position of a particle varies in time according to the expression **r =** (3.00ˆ**i** -6.00*t*2ˆ**j**) m. (a) Find expressions for the velocity and acceleration as functions of time.

(b) Determine the particle's position and velocity at $t = 1.00$ s.

SOLUTIONS TO PROBLEM:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

PROBLEMS

- **Section 4.2 Two-Dimensional Motion with Constant Acceleration**

8. A particle initially located at the origin has an acceleration of **a=**3.00ˆ**j** m/s2 and an initial velocity of **v***i* =500ˆ**i** m/s. Find (a) the vector position and velocity at any time *t* and (b) the coordinates and speed of the particle at *t* = 2.00 s.

SOLUTIONS TO PROBLEM:

$$\boldsymbol{r_f} = \boldsymbol{r_i} + v_i\boldsymbol{t} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{a}\,\mathbf{t}^{\mathbf{2}}$$

$$\boldsymbol{v_f} = v_i + at$$

At t=2.00 s $\boldsymbol{r_f} =?$ so $\boldsymbol{x_f}$ =? and $\boldsymbol{y_f}$=?

$$\vec{v} = ?$$

$$\boldsymbol{v}_{\boldsymbol{f}=}|\vec{v}|$$

PROBLEMS

- **Section 4.2 Two-Dimensional Motion with Constant Acceleration**

14. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free-fall acceleration on the planet?

SOLUTIONS TO PROBLEM:

$R = \frac{v^2{}_i \, sin^2 2\theta_i}{g}$=15 m and $v_i = 3m/s$

θ $_{max}$=45

g= $\frac{v^2{}_i}{R}$

PROBLEMS

- **Section 4.2 Two-Dimensional Motion with Constant Acceleration**

15. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

SOLUTIONS TO PROBLEM:

$h = \frac{v^2{}_i \, sin^2\theta_i}{2g}$ $R = \frac{v^2{}_i \, sin^2 2\theta_i}{g}$

3h=R

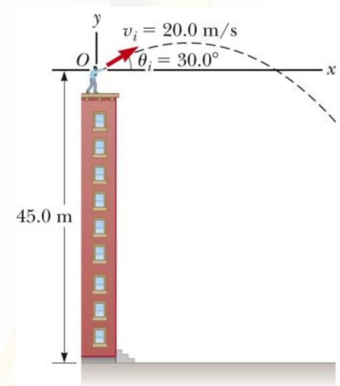
$\theta_i = ?$

PROBLEMS

- **Section 4.2 Two-Dimensional Motion with Constant Acceleration**

17. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later.

(a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?



SOLUTIONS TO PROBLEM:

$$x_f = v_{xi}t = v_i\cos(\theta_i)t$$

Taking y positive downwards, v_{yi}=$v_i\sin(\theta_i)$ $\quad y_f = v_{yi}t + \frac{1}{2}g\,t^2$

y_f=10.0m t=?

PROBLEMS

Section 4.3 Projectile Motion

19. A place-kicker must kick a football from a point 36.0 m (about 40 yards) from the goal, and half the crowd hopes the ball will clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of 20.0 m/s at an angle of 53.0° to the horizontal. (a) By how much does the ball clear or fall short of clearing the crossbar?

(b) Does the ball approach the crossbar while still rising or while falling?

SOLUTIONS TO PROBLEM:

x_f=36, v_i=20 and θ = 53.0°

We use the trajectory equation $y_f = x_f \tan \theta_i - \frac{g{x_f}^2}{2{v_i}^2 \cos {\theta_i}^2}$ The ball clears the bar by $y_{f-3.05}$

The time the ball takes to reach the maximum height is $t_1 = \frac{v_i \sin\theta_i}{g}$=1.63s

The time to travel 36.0 m horizontally is $t_2 = \frac{x_f}{v_{xi}}$=2.99s

Since $t2 > t1$ the ball clears the goal on its way down .

PROBLEMS

Section 4.3 Projectile Motion

20. A firefighter, a distance *d* from a burning building, directs a stream of water from a fire hose at angle θi above the horizontal as in Figure P4.20. If the initial speed of the stream is *vi,* at what height *h* does the water strike the building?

SOLUTIONS TO PROBLEM:

The horizontal component of displacement is $x_f = v_{xi} t = v_i\,(cos\,\theta_\mathrm{i}\,)\mathrm{t}$

Therefore, the time required to reach the building a distance *d* away is *t=?*

At this time, the altitude of the water is

$$y_f = v_{yi} t + \frac{1}{2} a_y \mathrm{t}^2 \;so\;\; y_f = v_i\, sin\,\theta_\mathrm{i}\, \frac{x_f}{v_i \cos\theta_\mathrm{i}} - \frac{1}{2} g (\frac{x_f}{v_i \cos\theta_\mathrm{i}})^2$$

$$y_f = tan\,\theta_\mathrm{i}\; x_f - \frac{g}{2 v_i{}^2 \cos\theta_\mathrm{i}{}^2} x_f{}^2$$

Therefore the water strikes the building at a height *h* above ground level of $h = y_f$

PROBLEMS

Section 4.3 Projectile Motion

22. A dive bomber has a velocity of 280 m/s at an angle θ below the horizontal. When the altitude of the aircraft is 2.15 km, it releases a bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle θ.

SOLUTIONS TO PROBLEM:

When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{3.25^2 - 2.15^2}$$

$$-2150m = y_f = tan\,\theta_\mathrm{i}\; x_f - \frac{g}{2v_i^2\cos\theta_\mathrm{i}^2}\, x_f^{\,2}$$

PROBLEMS

Section 4.3 Projectile Motion

23. A soccer player kicks a rock horizontally off a 40.0-m high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

SOLUTIONS TO PROBLEM:

The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from $y_f = v_{yi}t + \frac{1}{2}a_y\mathrm{t}^2$ y_f **=-40m and** a_y **=-g**

t=?

The extra time 3.00 s − 2.86 s = 0.143 s is the time required for the sound she hears to travel straight back to the player.

It covers distance 343/0.143=49= $\sqrt{x^2 + 40^2}$

where x represents the horizontal distance the rock travels.

$$x_f = 28.3 = v_{xi}t + 0\mathrm{t}^2$$

$$v_{xi} = 28.3/2.86$$

PROBLEMS

Section 4.3 Projectile Motion

25. An archer shoots an arrow with a velocity of 45.0 m/s at an angle of 50.0° with the horizontal. An assistant standing on the level ground 150 m downrange from the launch point throws an apple straight up with the minimum initial speed necessary to meet the path of the arrow. (a) What is the initial speed of the apple? (b) At what time after the arrow launch should the apple be thrown so that the arrow hits the apple?

SOLUTIONS TO PROBLEM:

The arrow's flight time to the collision point is

$$\boldsymbol{t = \frac{x_f - x_i}{v_{xi}}}$$

The arrow's altitude at the collision is $y_f = v_{yi}t + \frac{1}{2}a_y \mathrm{t}^2$ **.** The required launch speed for the apple is given by $v_{\mathrm{y}\boldsymbol{f}}{}^2 = v_{\mathrm{yi}}{}^2 + 2a_y(y_f - y_i)$ v_{yi}=?

The time of flight of the apple is given by $0 = v_{yf} = v_{yi} + a_y t$ t=?

So the apple should be launched after the arrow by $\boldsymbol{\frac{x_f - x_i}{v_{xi}}} - \frac{v_{yi}}{a_y}$

PROBLEMS

Section 4.4 Uniform Circular Motion

29. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

SOLUTIONS TO PROBLEM:

$$\boldsymbol{v_t} = \frac{2\pi r}{T}$$

$$a = \frac{v^2}{r}$$

Thank You


ACKNOWLEDGEMENTS