Phys 103

Chapter 7

Energy and Energy Transfer

By

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LECTURE OUTLINE

7.2 Work Done by a Constant Force

7.3 The Scalar Product of Two Vectors

7.4 Work Done by a Varying Force

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

7.6 The Nonisolated System—Conservation of Energy

7.7 Situations Involving Kinetic Friction

7.8 Power

7.2 Work Done by a Constant Force

The work W done on a system by an agent exerting a constant force on the system is the product of the magnitude **F**of the force, the magnitude Δ**r** of the displacement of the point of application of the force, and cosθ, where θ is the angle between the force and displacement vectors:

$$W = F\Delta r \cos\theta$$

if θ= 90°, then W = 0 because cos 90o= 0

If an applied force **F** is in the same direction as the displacement Δ**r**, then θ= 0 and cos 0 = 1. In this case, Equation 7.1 gives:

$$W = F\Delta r$$

Work is a *scalar quantity*, and its units are force multiplied by length. Therefore, the SI unit of work is the newton.meter (N. m). This combination of units is used so frequently that it has been given a name of its own: the *joule*(J).

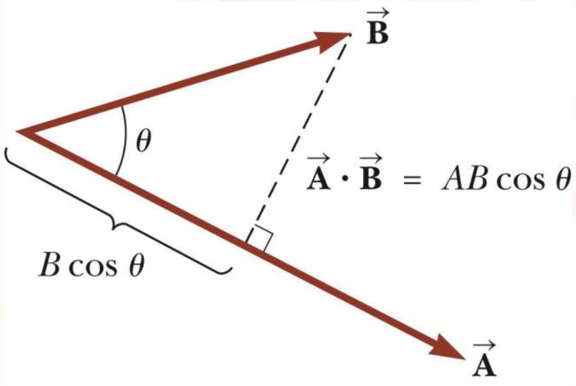
7.3 The Scalar Product of Two Vectors

Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the scalar product of two vectors.

In general; for any two vectors **A**and **B**; Scalar product is defined as:

$$A.B = AB\cos\theta$$

$$W = F\Delta r\ \cos\theta = F.\Delta r$$



In other words, **F.Δr**("F dot Δr") is a shorthand notation for $F\Delta r \cos\theta$.

7.3 The Scalar Product of Two Vectors

Dot Products

Note that the scalar product is commutative.

That is:

$$A.B = B.A$$

Although (7.3) defines the work in terms of two vectors, *work is a scalar*. *All types of* energy and energy transfer are scalars. This is a major advantage of the energy approach. We don't need vector calculations!

Dot Products of Unit Vectors

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$$

Using component form with vectors:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$

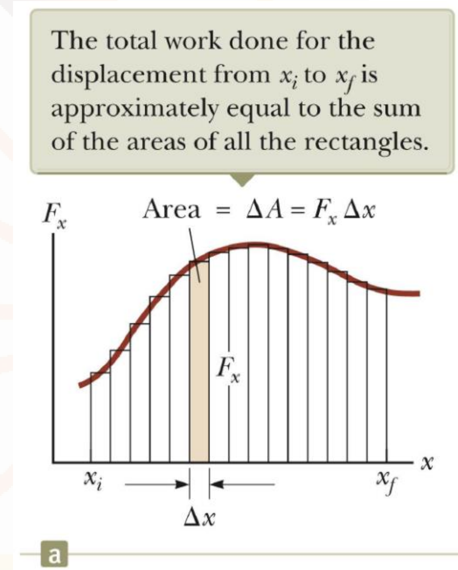
In the special case where

$$\vec{\mathbf{A}} = \vec{\mathbf{B}};$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

7.4 Work Done by a Varying Force

If a force $\mathbf{F}_x$ is varying with position, *x*, we can express the work done by $\mathbf{F}_x$ as the particle moves from x_i to x_f as:

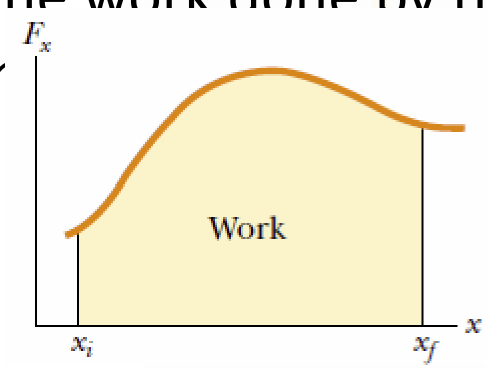


$$W \approx \sum_{x_i}^{x_f} F_x \, \Delta x$$

or

$$W = \int_{x_i}^{x_f} F_x \, dx$$

The work done by the component $\mathbf{F}_x$ *of* the varying force as the particle m



xactly equal to the area under this curve.

$$W = \int_{x_i}^{x_f} F_x \, dx$$

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

When work (W) is applied on a system; its kinetic energy (K) changes from initial value (K_i) to final value (K_f) so that:

$$W = K_f - K_i$$

We define k as: $K = \frac{1}{2}mv^2$

$$W = \frac{1}{2}m({v^2}_f - {v^2}_i)$$

The **work–kinetic energy theorem** is defined as:

$$W = K_f - K_i = \Delta K$$

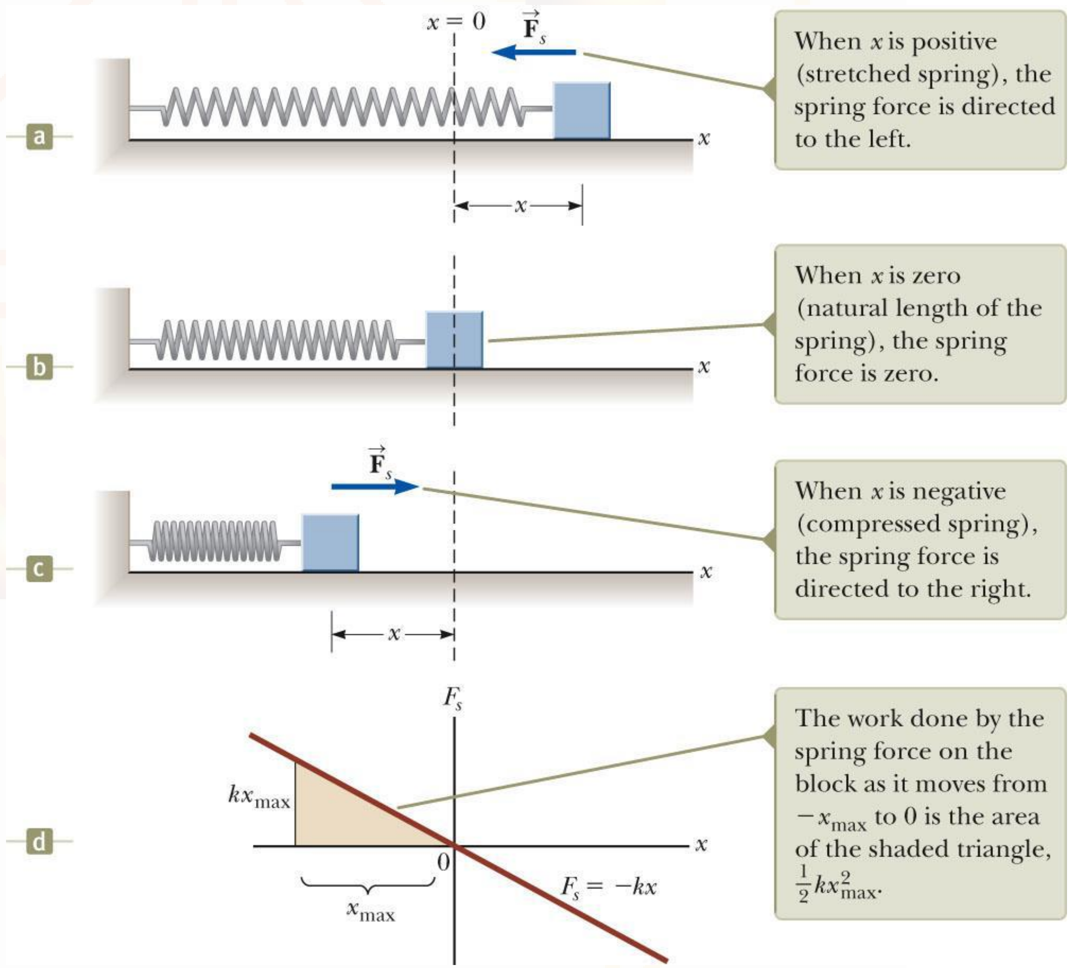
This theorem indicates that the speed of a particle will **increase** if the net work done on it is **positive**, because the final kinetic energy will be greater than the initial kinetic energy. The speed will **decrease** if the net work is **negative.**

Remember work is a scalar, so this is the algebraic sum.

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Work Done By A Spring

A model of a common physical system for which the force varies with position. The block is on a horizontal, frictionless surface. Observe the motion of the block with various values of the spring constant.



7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Spring Force (Hooke's Law)

The force exerted by the spring is $F_s = -kx$

x is the position of the block with respect to the equilibrium position (x = 0).

k is called the spring constant or force constant and measures the stiffness of the spring.

k measures the stiffness of the spring. This is called Hooke's Law.

When x is positive (spring is stretched), F is negative

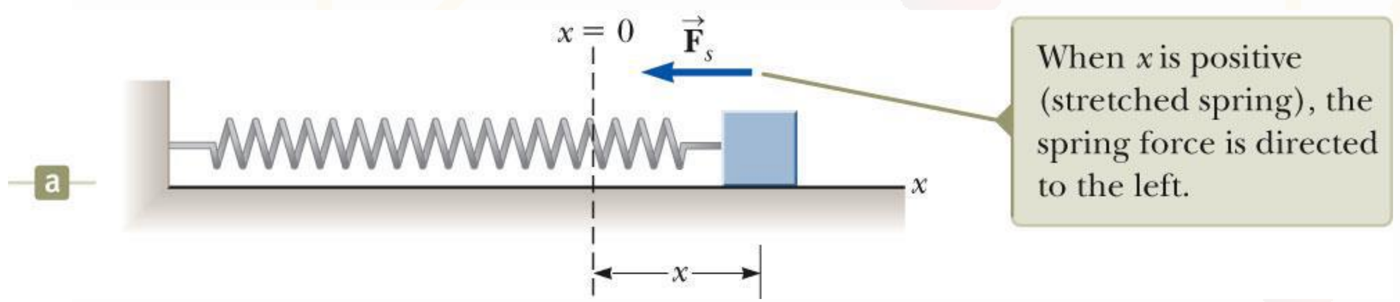
When x is 0 (at the equilibrium position), F is 0

When x is negative (spring is compressed), F is positive

The force exerted by the spring is always directed opposite to the displacement from equilibrium.

The spring force is sometimes called the restoring force.

If the block is released it will oscillate back and forth between –x and x.



PROBLEMS

Section 7.2 Work Done by a Constant Force

1. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the total work done on the block.

SOLUTIONS TO PROBLEM:

$W = F\Delta r \cos\theta$ = a16.0 Nfa2.20 mfcos 25.0°=31.9 J

The normal force and the weight are both at 90° to the displacement in any time interval. Both do 0 work.

PROBLEMS

Section 7.2 Work Done by a Constant Force

4. A raindrop of mass 3.35 * 10&5 kg falls vertically at constant speed under the influence of gravity and air resistance.

Model the drop as a particle. As it falls 100 m, what is the work done on the raindrop (a) by the gravitational force and (b) by air resistance?

SOLUTIONS TO PROBLEM:

$W = mgh$

Since $R = mg$

$W_{\text{air resistance}} = -W$

PROBLEMS

Section 7.3 The Scalar Product of Two Vectors

7. A force **F** = (6ˆ**i** & 2ˆ**j**) N acts on a particle that undergoes a displacement Δ **r =** (3ˆ**i**)ˆ**j**) m. Find (a) the work done by the force on the particle and (b) the angle between **F** and Δ **r**.

SOLUTIONS TO PROBLEM:

$W = \mathbf{F}\cdot\Delta\mathbf{r} = F_x x + F_y y = 6.00(3.00)\ \text{N}\cdot\text{m} + (-2.00)(1.00)\ \text{N}\cdot\text{m} = 16.0\ \text{J}$

$$\boldsymbol{\theta} = \cos^{-1}\left(\frac{\boldsymbol{F}.\Delta\boldsymbol{r}}{F\Delta r}\right) = \cos^{-1}\left(\frac{\mathbf{16}}{\sqrt{\boldsymbol{F_x}^2 + \boldsymbol{F_y}^2}\sqrt{\boldsymbol{x}^2 + \boldsymbol{y}^2}}\right)$$

PROBLEMS

Section 7.4 Work Done by a Varying Force

13. A particle is subject to a force F_x that varies with position as in Figure P7.13. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 5.00$ m, (b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

SOLUTIONS TO PROBLEM:

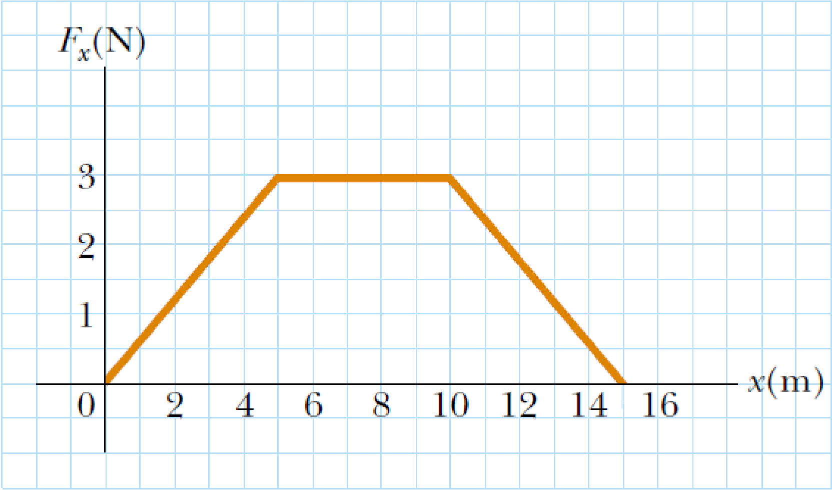


Figure P7.13 Problems 13 and 28.

PROBLEMS

Section 7.4 Work Done by a Varying Force

14. A force **F** = $(4x\hat{\mathbf{i}}\ )\ 3y\hat{\mathbf{j}})$ N acts on an object as the object moves in the x direction from the origin to $x = 5.00$ m. Find the work

$$W = \int F\, dr$$

done on the object by the force.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

15. When a 4.00-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it, and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

16. An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work does the archer do in pulling the bow?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

19. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.4 Work Done by a Varying Force

21. A light spring with spring constant 1 200 N/m is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant 1 800 N/m.

An object of mass 1.50 kg is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as *in series*.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

24. A 0.600-kg particle has a speed of 2.00 m/s at point A And kinetic energy of 7.50 J at point B*.* What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

25. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) **What If?** If its speed were doubled, what would be its kinetic energy?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

26. A 3.00-kg object has a velocity (6.00ˆi - 2.00ˆj) m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to (8.00ˆi + 4.00ˆj) m/s. (*Note:* From the definition of the dot product, *v*2 = **v** . **v**.)

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System—Conservation of Energy

28. A 4.00-kg particle is subject to a total force that varies with position as shown in Figure P7.13. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m, (b) $x = 10.0$ m, (c) $x = 15.0$ m?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

31. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

32. A 2.00-kg block is attached to a spring of force constant 500 N/m as in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

33. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate–incline system due to friction. (c) How much work is done by the 100-N force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.7 Situations Involving Kinetic Friction

35. A sled of mass *m* is given a kick on a frozen pond. The kick imparts to it an initial speed of 2.00 m/s. The coefficient of kinetic friction between sled and ice is 0.100. Use energy considerations to find the distance the sled moves before it stops.

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.8 Power

37. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

SOLUTIONS TO PROBLEM:

PROBLEMS

Section 7.8 Power

40. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

SOLUTIONS TO PROBLEM:

Lecture Summary

The work W done on a system by anagentexertinga constant force on the system is the product of the magnitude **F** of the force, the magnitude Δr of the displacement of the point of application of the force, and cosθ, where θ is the angle between the force and displacement vectors:

$$W = F\Delta r \cos\theta = F.\Delta r$$

The scalar product (dot product) of two vectors A and B is defined by the relationship:

$$A.B = AB\cos\theta$$

If aforce $\mathbf{F_x}$ is varying with position, x, we can express the work done by $\mathbf{F_x}$ as the particle moves from x_i to x_f as:

$$W = \int_{x_i}^{x_f} F_x \, dx$$

Thank You


ACKNOWLEDGEMENTS