# Phys 103

# Chapter 8

## Potential Energy

By

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# LECTURE OUTLINE

- 8.1 Potential Energy of a System
- 8.2 The Isolated System—Conservation of Mechanical Energy
- 8.3 Conservative and Nonconservative Forces
- 8.4 Changes in Mechanical Energy for Nonconservative Forces
- 8.5 Relationship Between Conservative Forces and Potential Energy

# Introduction

- In Chapter 7 we introduced the concepts of kinetic energy associated with the motion of members of a system and internal energy associated with the temperature of a system.
- In this chapter we introduce ***potential energy***, the energy associated with the configuration of a system of objects that exert forces on each other.
- The potential energy concept can be used only when dealing with a special class of forces called ***conservative forces***. When only conservative forces act within an isolated system,
- the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy. This balancing of the two forms of energy is known as the ***principle of conservation of mechanical energy***.

# 8.1 Potential Energy of a System

- Let us now derive an expression for the **gravitational potential energy** ($U_g$) associated with an object (m) at a given location (y) above the surface of the Earth

$$U_g = mgy$$

- Mathematical description of the work done on a system that changes the gravitational potential energy of the system is give by:

$$W = \triangle U_g$$

- The gravitational potential energy depends only on the vertical height of the object above the surface of the Earth. The same amount of work must be done on an object–Earth system whether the object is lifted vertically from the Earth or is pushed starting from the same point up a frictionless incline, ending up at the same height

# 8.2 The Isolated System— Conservation of Mechanical Energy

- As the book, shown in the figure, falls back to its original height, from $y_b$ to $y_a$, the work done by the gravitational force on the book is:

$$W = mgy_b - mgy_a = \triangle K = -\triangle U_g$$

So

$$\triangle K + \triangle U_g = 0$$

- Mechanical energy is defined as:

$$E_{mech} = K + U_g$$

- Or, in general:

$$E_{mech} = K + U$$

# 8.2 The Isolated System— Conservation of Mechanical Energy

- Let us now write the changes in energy in Equation 8.7 explicitly:

$$\left(K_f - K_i\right) + \left(U_f - U_i\right) = 0$$
$$\left(K_f + U_f\right) - \left(K_i + U_i\right) = 0$$
$$K_f + U_f = K_i + U_i$$

- Equation 8.9 is a statement of conservation of mechanical energy for an isolated system.
- An isolated system is one for which there are no energy transfers across the boundary.
- The energy in such a system is conserved—the sum of the kinetic and
- Potential energies remains constant.
- This statement assumes that no nonconservative forces act within the system.

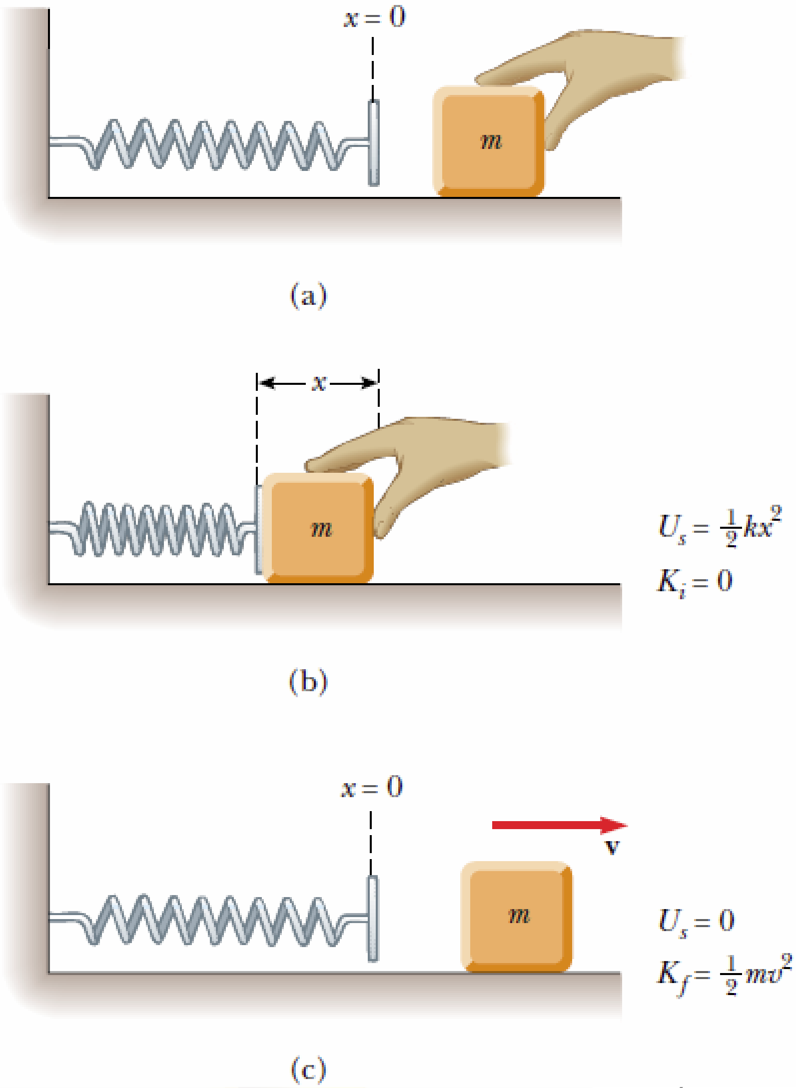
# 8.2 The Isolated System— Conservation of Mechanical Energy

**Potential Energy of a Spring**

Potential Energy of a Spring is given by:

$$U_s = \frac{1}{2}kx^2$$

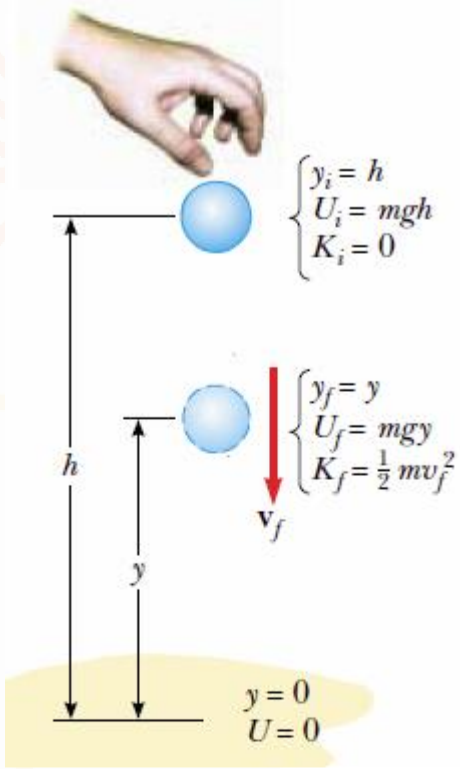
- When the block is released from rest, t
  block and returns.
- to its original length. The stored elastic
  transformed into kinetic energy of the
- The elastic potential energy stored in a
- Energy is stored in the spring only whe
  or compressed.



# 8.2 The Isolated System— Conservation of Mechanical Energy

**Example 8.2 Ball in Free Fall**

A ball of mass m is dropped from a height h above the ground, as shown in Figure . *Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.*




- **Solution**:

$$K_f + U_f = K_i + U_f$$

$$\frac{1}{2}mv^2_f + mgy = 0 + mgh$$

$$v_f = \sqrt{2g(h-y)}$$

# 8.2 The Isolated System— Conservation of Mechanical Energy

- **Example 8.3 The Pendulum**

A pendulum consists of a sphere of mass m = 200 gm attached to a light cord of length L =50 cm, as shown in Figure. The sphere is released from rest at point A when the cord makes an angle θA=37owith the vertical.

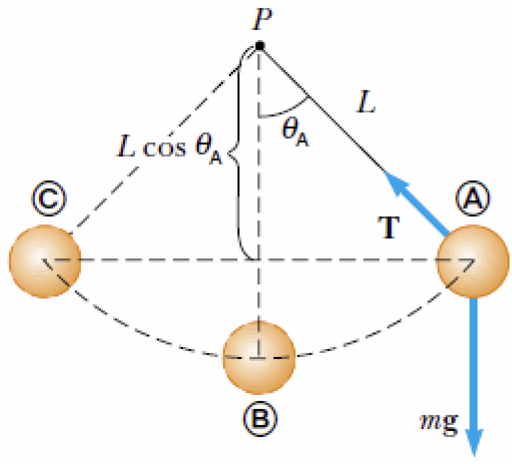
- (A) Find the speed of the sphere when it is at the lowest point B.
- **Solution**:

$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2}mv^2{}_B - mgL = 0 - mgL\,cos\theta$$

$$v_B = \sqrt{2gL(1 - cos\boldsymbol{\theta}_A)}$$

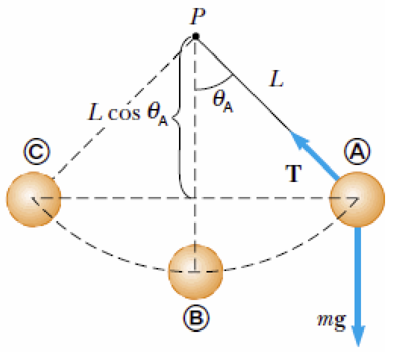
$$v_{B=1.4\,m/s}$$



# 8.2 The Isolated System— Conservation of Mechanical Energy

- **Example 8.3 The Pendulum**

 (B) What is the tension TB in the cord at B?




**Solution:**

Newton’s second law gives:

$$\sum F_r = mg - T_B = ma_r = -m\frac{v^2{}_B}{L}$$

$$T_B = mg + m\frac{v^2{}_B}{L}$$

# 8.2 The Isolated System— Conservation of Mechanical Energy

**Example 8.5 The Spring-Loaded Popgun**

The launching mechanism of a toy gun consists of a spring constant. When the spring is compressed 0.1 fired vertically, is able to launch a 35.0-g projectile t of 20.0 m above the position of the projectile befor

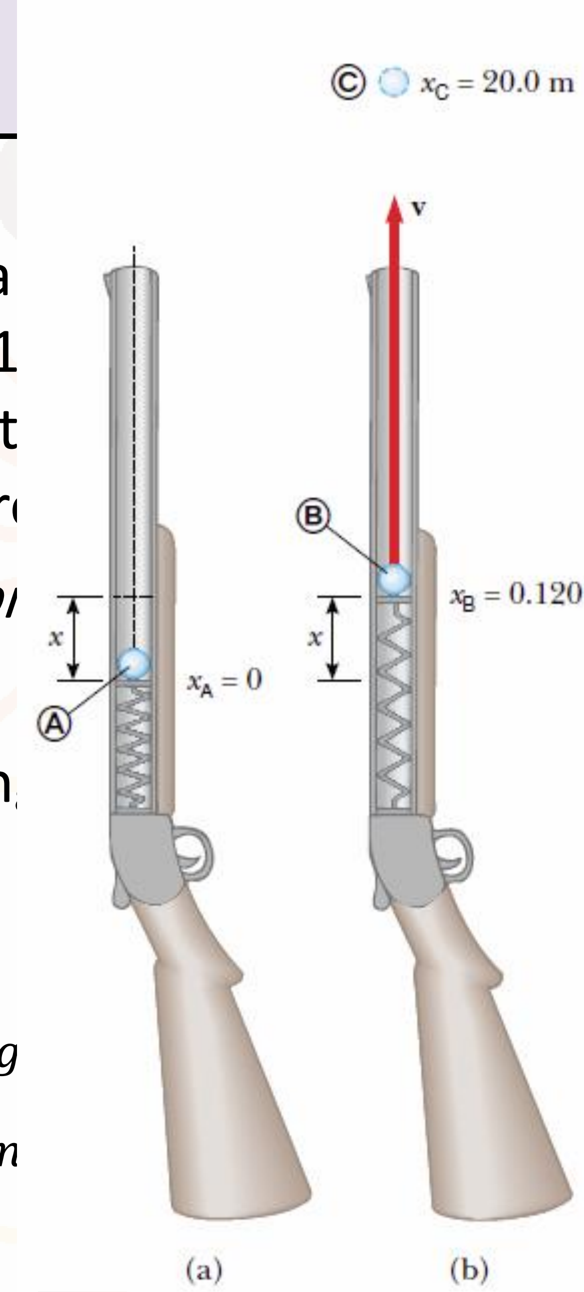
*(A) Neglecting all resistive forces, determine the spr*

**Solution**:

Total energy at position (c) for the projectile + sprin position (A)

- Hence:

$$\left[K_{projectile} + U_{projectile} + U_{spring}\right.$$
$$\left[K_{projectile} + U_{projectile} + U_{sprin}\right.$$



# 8.2 The Isolated System— Conservation of Mechanical Energy

**Example 8.5 The Spring-Loaded Popgun**

$$K_c + U_{gc} + U_{sc} = K_A + U_{gA} + U_{sA}$$

$$0 + mgh + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$k = \frac{2mgh}{x^2}$$

- Find the speed of the projectile as it moves through the equilibrium position of the spring at $x_B$

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$\frac{1}{2}m{v_B}^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

# 8.3 Conservative and Nonconservative Forces

**Conservative Forces**

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

Examples of Conservative Forces:

- 1.gravitational force
- 2.Spring force

# 8.4 Changes in Mechanical Energy for Nonconservative Forces

A force is nonconservative if it does not satisfy properties 1 and 2 for conservative forces.

Nonconservative forces acting within a system cause a change in the *mechanical energy* Emechof the system.

As an example of the path dependence of the work, consider moving a book between two points on a table. If the book is moved in a straight line along the path between points A and B; a certain amount of work against the kinetic friction force must be spent to keep the book moving at a constant speed.

Now, imagine that the book was pushed along a semicircular path. More work must have been performed against friction along this longer path than along the straight path.

Hence, The work done depends on the path, so the friction force cannot

- be conservative force.

# 8.4 Changes in Mechanical Energy for Nonconservative Forces

Consider a body sliding across a surface. As the body moves through a distance *d,* the only force that does work on it is the force of kinetic friction. This force causes a decrease in the kinetic energy of the body. This decrease was calculated in Chapter 7, leading to Equation 7.20, which we repeat here:

$$\triangle K = -f_k d$$

If there is also a change in potential energy then:

$$E_{mech} = \triangle K + \triangle U_g$$

Or in general, for any potential:

$$E_{mech} = \triangle K + \triangle U = -f_k d$$

where ΔU is the change in all forms of potential energy.

# 8.5 Relationship Between Conservative Forces and Potential Energy

The work done by a cons. force F as a particle moves along the *x axis is:*

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\triangle U$$

$$Or \triangle U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx$$

Therefore, ΔU is negative when $F_x$ and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

We can then define the potential energy function as:

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i$$

# 8.5 Relationship Between Conservative Forces and Potential Energy

If the point of application of the force undergoes an infinitesimal displacement $d_x$, *we can express the infinitesimal change in the potential energy of the system dU as*

$$dU = -F_x \, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship

$$F_x = -\frac{dU}{dx}$$

That is, the x component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to x.

# Lecture Summary

If a particle of mass *m is at a distance y above the Earth's surface, the gravitational potential* energy of the particle–Earth system is

$$U_g = mgy$$

The elastic potential energy stored in a spring of force constant *k is*

$$U_s = \frac{1}{2}kx^2$$

*Total Energy of A system is:*

$$K_f + U_f = K_i + U_i$$

# Lecture Summary

- A force is conservative if the work it does on a particle moving between two points is independent of the path the particle takes between the two points, Or if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be nonconservative.
- The total mechanical energy of a system is defined as the sum of the kinetic energy and the potential energy:

$$E_{mech} = K + U$$

- If a system is isolated and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_f + U_f = K_i + U_i$$

# Thank You


ACKNOWLEDGEMENTS