Phys 103

Chapter 8

Potential Energy

By

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LECTURE OUTLINE

- 8.1 Potential Energy of a System
- 8.2 The Isolated System—Conservation of Mechanical Energy
- 8.3 Conservative and Nonconservative Forces
- 8.4 Changes in Mechanical Energy for Nonconservative Forces
- 8.5 Relationship Between Conservative Forces and Potential Energy

Lecture Summary

If a particle of mass *m is at a distance y above the Earth's surface, the gravitational potential* energy of the particle–Earth system is

$$U_g = mgy$$

The elastic potential energy stored in a spring of force constant *k is*

$$U_s = \frac{1}{2}kx^2$$

Total Energy of A system is:

$$K_f + U_f = K_i + U_i$$

Lecture Summary

- A force is conservative if the work it does on a particle moving between two points is independent of the path the particle takes between the two points, Or if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be nonconservative.
- The total mechanical energy of a system is defined as the sum of the kinetic energy and the potential energy:

$$E_{mech} = K + U$$

- If a system is isolated and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_f + U_f = K_i + U_f$$

PROBLEMS

Section 8.1 Potential Energy of a System

2. A 400-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

SOLUTIONS TO PROBLEM:

$$U_g = mgy$$

$$y = 2$$

$$y = 2(1 - \cos\boldsymbol{\theta})$$

$$y = 0$$

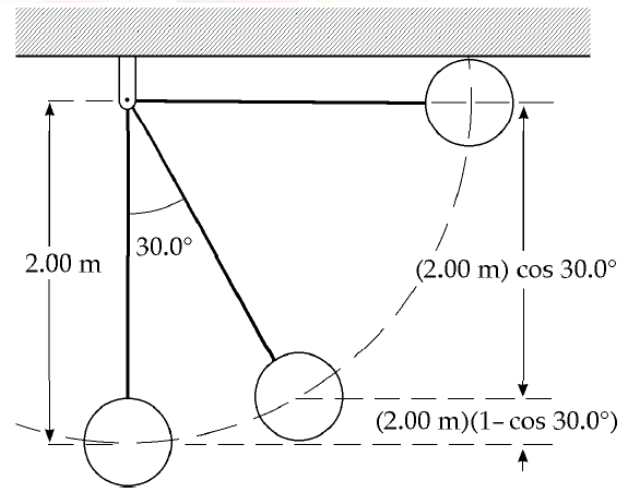

FIG. P8.2

PROBLEMS

Section 8.1 Potential Energy of a System

5. A bead slides without friction around a loop-the-loop (Fig. P8.5). The bead is released from a height *h* " 3.50*R.*

(a) What is its speed at point A?

(b) How large is the normal force on it if its mass is 5.00 g?

SOLUTIONS TO PROBLEM:

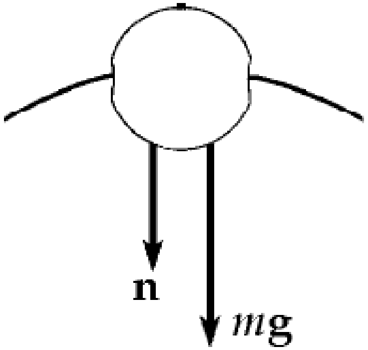

FIG. P8.5

$$K_i + U_i = K_f + U_f$$

$$mgh + 0 = mg(2R) + \frac{1}{2}mv^2$$

$$\sum F = m\frac{v^2}{R} = n + mg$$

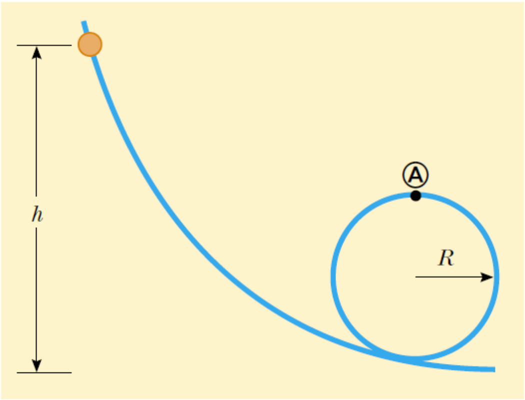

Figure P8.5

PROBLEMS

Section 8.1 Potential Energy of a System

6. Dave Johnson, the bronze medalist at the 1992 Olympic decathlon in Barcelona, leaves the ground at the high jump with vertical velocity component 6.00 m/s. How far does his center of mass move up as he makes the jump?

SOLUTIONS TO PROBLEM:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgy$$

PROBLEMS

Section 8.1 Potential Energy of a System

11. A block of mass 0.250 kg is placed on top of a light vertical spring of force constant 5 000 N/m and pushed downward so that the spring is compressed by 0.100 m. After the block is released from rest, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

SOLUTIONS TO PROBLEM:

From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250\ \text{kg})(9.80\ \text{m/s}^2)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$

This gives a maximum height $h = \boxed{10.2\ \text{m}}$.

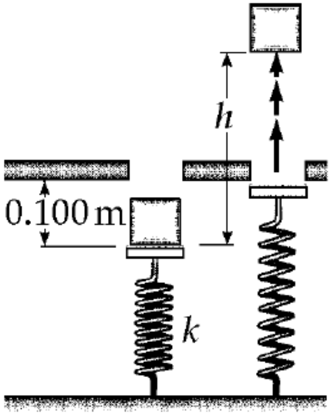

FIG. P8.11

PROBLEMS

Section 8.1 Potential Energy of a System

13. Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure P8.13. The object of mass 5.00 kg is released from rest. Using the principle of conservation of energy, (a) determine the speed of the 3.00-kg object just as the 5.00-kg object hits the ground. (b) Find the maximum height to which the 3.00-kg object rises.

SOLUTIONS TO PROBLEM:

Using conservation of energy for the system of the Earth and the two objects

(a) $$(5.00 \text{ kg})g(4.00 \text{ m}) = (3.00 \text{ kg})g(4.00 \text{ m}) + \frac{1}{2}(5.00 + 3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43 \text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$

$$\Delta y = 1.00 \text{ m}$$

$$y_{max} = 4.00 \text{ m} + \Delta y = \boxed{5.00 \text{ m}}$$

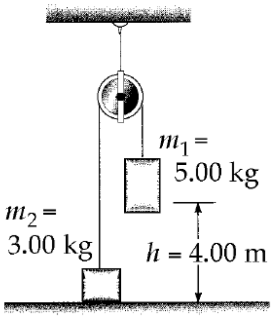

FIG. P8.13

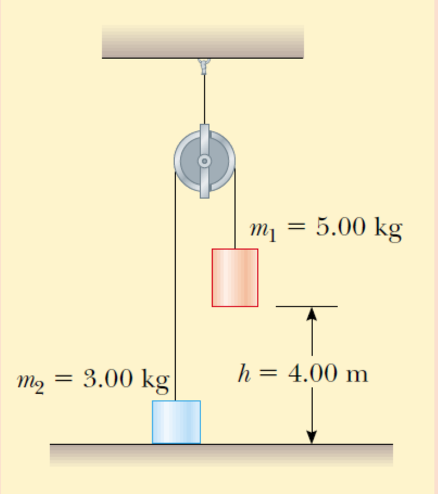

Figure P8.13 Problems 13 and 14.

PROBLEMS

Section 8.1 Potential Energy of a System

17. A 20.0-kg cannon ball is fired from a cannon with muzzle speed of 1 000 m/s at an angle of 37.0° with the horizontal. A second ball is fired at an angle of 90.0°. Use the conservation of energy principle to find (a) the maximum height reached by each ball and (b) the total mechanical energy at the maximum height for each ball. Let *y* =0 at the cannon.

SOLUTIONS TO PROBLEM:

(a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000 \sin 37.0°)^2}{2(9.80)} = \boxed{1.85 \times 10^4 \text{ m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10 \times 10^4 \text{ m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0 \text{ kg})(1\,000 \text{ m/s})^2 = \boxed{1.00 \times 10^7 \text{ J}}.$$

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

31. The coefficient of friction between the 3.00-kg block and the surface in Figure P8.31 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

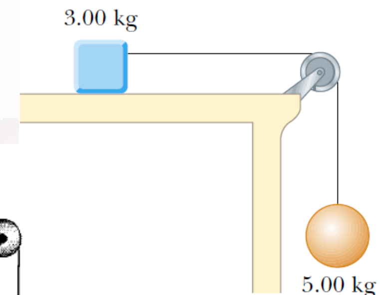

Figure P8.31

SOLUTIONS TO PROBLEM:

$U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$:

$$m_2 gh - fh = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$f = \mu n = \mu m_1 g$$

$$m_2 gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2) v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$

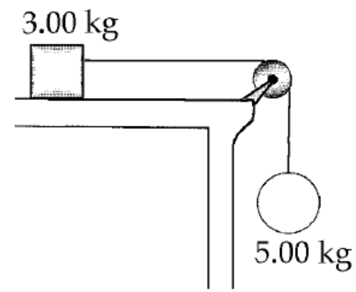

FIG. P8.31

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

33. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine

(a) the change in the block's kinetic energy, (b) the change in the potential energy of the block–Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

SOLUTIONS TO PROBLEM:

(a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$$

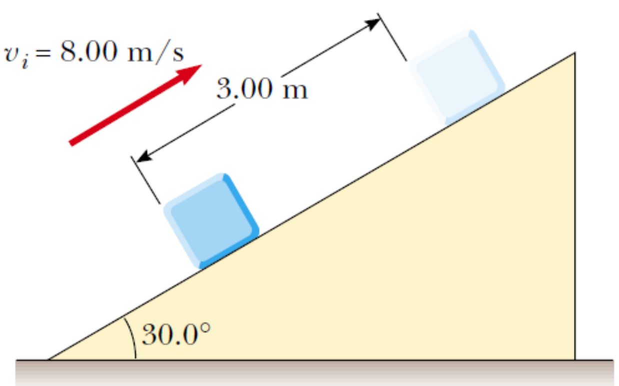

Figure P8.33

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

36. A 50.0-kg block and a 100-kg block are connected by a string as in Figure P8.36. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50.0 kg block and incline is 0.250. Determine the change in the kinetic energy of the 50.0-kg block as it moves from ! to ", a distance of 20.0 m.

SOLUTIONS TO PROBLEM:

$$\sum F_y = n - mg\cos 37.0° = 0$$

$$\therefore n = mg\cos 37.0° = 400\ \text{N}$$

$$f = \mu n = 0.250(400\ \text{N}) = 100\ \text{N}$$

$$-f\Delta x = \Delta E_{\text{mech}}$$

$$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$$

$$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90 \times 10$$

$$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96 \times 10^4$$

$$\Delta K_A = \frac{1}{2} m_A (v_f^2 - v_i^2)$$

$$\Delta K_B = \frac{1}{2} m_B (v_f^2 - v_i^2) = \frac{m_B}{m_A} \Delta K_A = 2\Delta K_A$$

Adding and solving, $\Delta K_A = \boxed{3.92\ \text{kJ}}$.

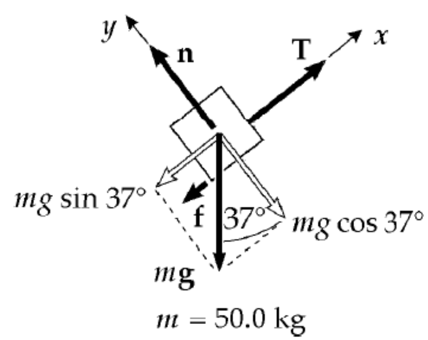

FIG. P8.36

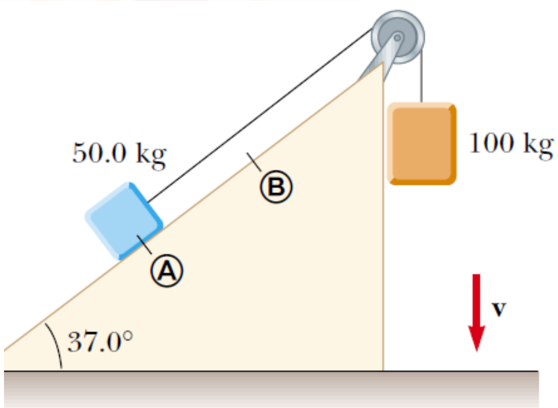

Figure P8.36

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

38. A 75.0-kg skysurfer is falling straight down with terminal speed 60.0 m/s. Determine the rate at which the skysurfer–Earth system is losing mechanical energy.

SOLUTIONS TO PROBLEM:

The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\ \text{kg})(9.80\ \text{m/s}^2)(60.0\ \text{m/s}) = \boxed{44.1\ \text{kW}}.$$

PROBLEMS

Section 8.5 Relationship Between Conservative Forces and Potential Energy

42. A potential-energy function for a two-dimensional force is of the form $U = 3x^3y - 7x$.

Find the force that acts at the point (x, y).

SOLUTIONS TO PROBLEM:

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial x} = -\left(9x^2y - 7\right) = 7 - 9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial y} = -\left(3x^3 - 0\right) = -3x^3$$

Thus, the force acting at the point (x, y) is $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

PROBLEMS

Additional Problems

55. Review problem. Suppose the incline is frictionless for the system described in Problem 54 (Fig. P8.54). The block is released from rest with the spring initially unstretched.

(a) How far does it move down the incline before coming to rest?

(b) (b) What is its acceleration at its lowest point? Is the acceleration constant? (c) Describe the energy transformations that occur during the descent.

SOLUTIONS TO PROBLEM:

(a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg\sin\theta)x$

and $U_f = \frac{1}{2}kx^2$

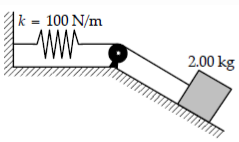

FIG. P8.55

Substituting values yields $(2.00)(9.80)\sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236\ \text{m}}$$

(b) $\sum F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg\sin\theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90\ \text{m/s}^2}$. The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.

U(spring) increases monotonically as the spring is stretched.

K initially increases, but then goes back to zero.

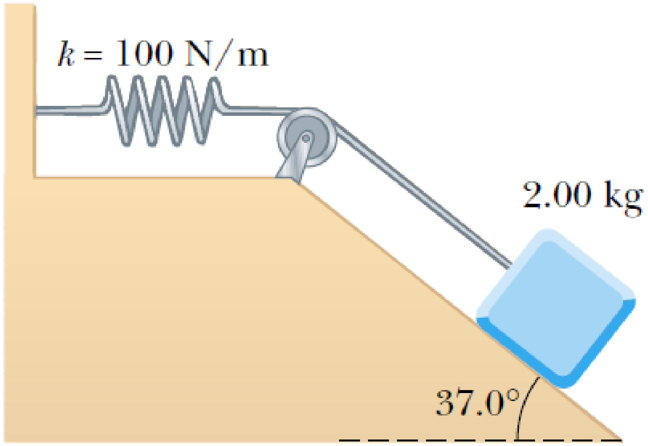

Figure P8.54 Problems 54 and 55.

PROBLEMS

Additional Problems

57. A 10.0-kg block is released from point A in Figure P8.57. The track is frictionless except for the portion between points B and C , which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily.

Determine the coefficient of kinetic friction between the block and the rough surface between B and C.

SOLUTIONS TO PROBLEM:

$$\Delta E_{mech} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

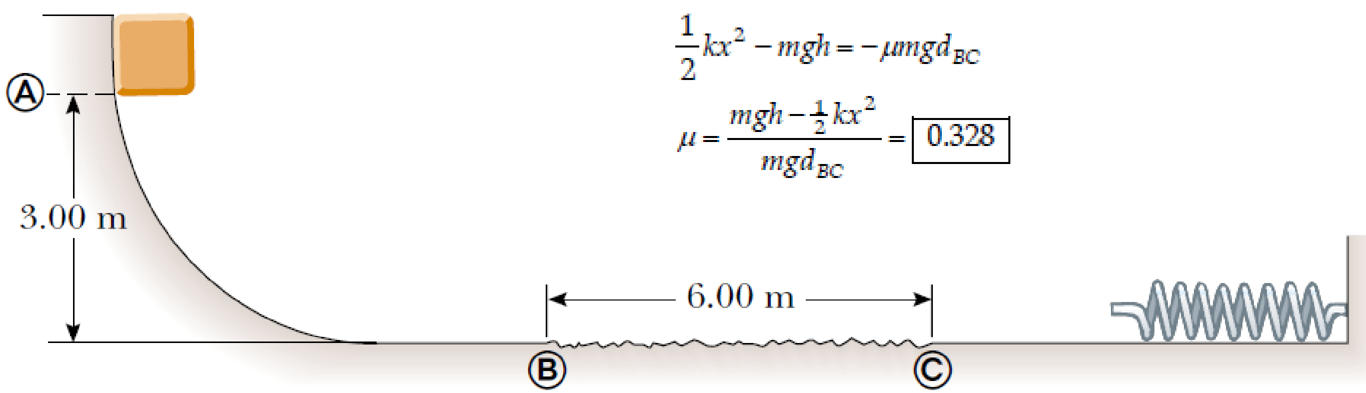

Figure P8.57

PROBLEMS

Additional Problems

59. A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.59. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and released from rest.

Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).

SOLUTIONS TO PROBLEM:

$$(K+U)_i = (K+U)_f$$

$$0+(30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})+\frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$=\frac{1}{2}(50.0\text{ kg})v^2+(20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J}+5.00\text{ J}=(25.0\text{ kg})v^2+25.2\text{ J}$$

$$\boxed{v=1.24\text{ m/s}}$$

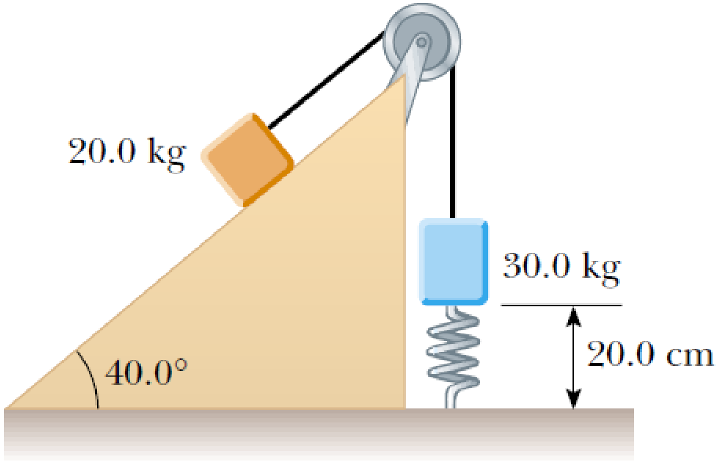

Figure P8.59

PROBLEMS

Additional Problems

60. A 1.00-kg object slides to the right on a surface having a coefficient of kinetic friction 0.250 (Fig. P8.60). The object has a speed of *vi*" 3.00 m/s when it makes contact with a light spring that has a force constant of 50.0 N/m. The object comes to rest after the spring has been compressed a distance *d.* The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally, the object comes to rest a distance *D* to the left of the unstretched spring. Find (a) the distance of compression *d*, (b) the speed *v* at the unstretched position when the object is moving to the left, and (c) the distance *D* where the object comes to rest.

SOLUTIONS TO PROBLEM:

(a) Between the second and the third picture, $\Delta E_{mech} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\ \text{N/m})d^2 + 0.250(1.00\ \text{kg})(9.80\ \text{m/s}^2)d - \frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s}^2) = 0$$

$$d = \frac{[-2.45 \pm 21.25]\ \text{N}}{50.0\ \text{N/m}} = \boxed{0.378\ \text{m}}$$

(b) Between picture two and picture four, $\Delta E_{mech} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\ \text{m/s})^2 - \frac{2}{(1.00\ \text{kg})}(2.45\ \text{N})(2)(0.378\ \text{m})}$$

$$= \boxed{2.30\ \text{m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{mech} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2$$

$$D = \frac{9.00\ \text{J}}{2(0.250)(1.00\ \text{kg})(9.80\ \text{m/s}^2)} - 2(0.378\ \text{m}) = \boxed{1.08\ \text{m}}$$

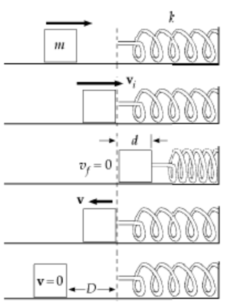

FIG. P8.60

Thank You


ACKNOWLEDGEMENTS