Phys 103

Chapter 9

Linear Momentum and Collisions

By

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LECTURE OUTLINE

- 9.1 Linear Momentum and Its
- Conservation
- 9.2 Impulse and Momentum
- <u>9.3 Collisions in One Dimension</u>
- <u>9.4 Two-Dimensional Collisions</u>

9.3 Collisions in One Dimension

The total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. In fact, whether or not kinetic energy is conserved is used to classify collisions as either ***elastic or inelastic.***

An elastic collision between two objects is one in which the total kinetic energy (as well as total momentum) of the system is the same before and after the collision.

An inelastic collision is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).

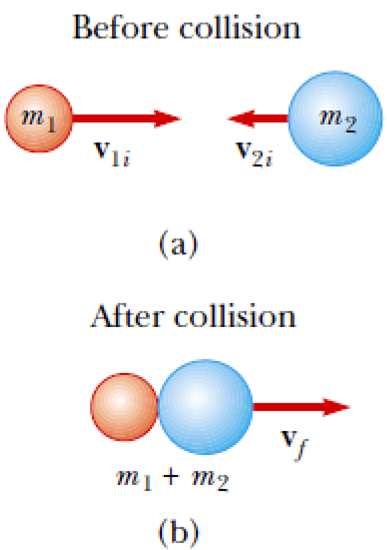
Inelastic collisions are of two types. When the colliding objects stick together after the collision, the collision is called **perfectly inelastic**, When the colliding objects do not stick together, but some kinetic energy is lost, the collision is called **inelastic**.

9.3 Collisions in One Dimension

- **Perfectly Inelastic Collisions**

Consider two particles of masses m1 and m2 moving with initial velocities v1i and v2i along the same straight line, as shown in Figure. The two particles collide head-on, stick together, and then move with some common velocity vf after the collision.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$



$$\Rightarrow v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

This is true only if the two objects

Stick together in one-object.

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

For this type of collisions: kinetic energy and liner momentum are conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1\, {v_{1i}}^2 + \frac{1}{2} m_2\, {v_{2i}}^2 = \frac{1}{2} m_1\, {v_{1f}}^2 + \frac{1}{2} m_2\, {v_{2f}}^2$$

We can use these equations directly to solve our problems to go directly to some special cases:

$$m_1\, {v_{1i}}^2 + m_2\, {v_{2i}}^2 = m_1\, {v_{1f}}^2 + m_2\, {v_{2f}}^2$$

$$m_1\, {v_{1i}}^2 - m_1\, {v_{1f}}^2 = m_2\, {v_{2f}}^2 - m_2\, {v_{2i}}^2$$

$$m_1({v_{1i}}^2 - {v_{1f}}^2) = m_2({v_{2f}}^2 - {v_{2i}}^2)$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \ldots.*$$

$$m_1({v_{1i}}^2 - {v_{1f}}^2) = m_2({v_{2f}}^2 - {v_{2i}}^2)$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \ldots.**$$

To obtain our final result, we divide Equation ** by Equation * and obtain:

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \ldots .*$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \ldots .**$$

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

Suppose that the masses and initial velocities of both particles are known:

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i} + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i}$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i} + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i}$$

Let us consider some special cases. If $m_1 = m_2$, then tow Equations show us that

$$v_{1f} = v_{2i}$$

And

$$v_{2f} = v_{1i}$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

If m_2 is initially at rest $v_{2i} = 0$

So the equations

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i} + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i}$$

becomes:

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} \ and \ \ v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}$$

9.3 Collisions in One Dimension

Perfectly Elastic Collisions

If m_2 is initially at rest $v_{2i} = 0$

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} \; and \;\; v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}$$

Now

If m1is much greater than m_2 and $v_{2i} = 0$, we see the tow Equations that $v_{1f} \approx v_{1i}\, and v_{2f} \approx v_{2i}$. That is, when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.

9.3 Collisions in One Dimension

Example 9.6 Carry Collision Insurance

- An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, *what is the velocity of the entangled cars after the collision?*
- ***Solution:***

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

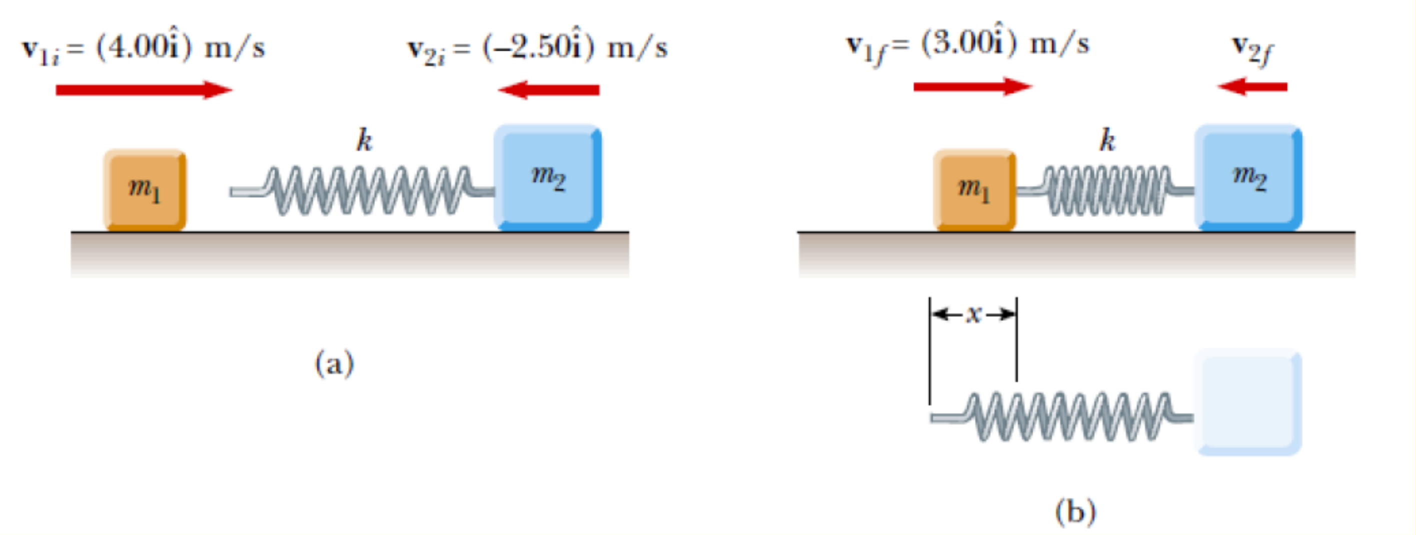
$$\therefore (1800)(0) + (900)(20) = (1800 + 900) v_f$$

$$\Rightarrow v_f = \frac{900 \times 20}{2700} = 6.67\, m/s$$

9.3 Collisions in One Dimension

Example 9.8 A Two-Body Collision with a Spring

- A block of mass $m_1 = 1.60$ kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass $m_2 = 2.10$ kg initially moving to the left with a speed of 2.50 m/s. The spring constant is 600 N/m.
- *(A) Find the velocities of the two blocks after the collision*



9.3 Collisions in One Dimension

Example 9.8 (Continued)

Solution:

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1.6)(4)+(2.10)(-2.5)=(1.6)v_{1f}+(2.10)v_{2f} \qquad (1)$$

$$(9.19) \rightarrow v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$\therefore (4)-(-2.5) = -v_{1f} + v_{2f}$$

$$\therefore 6.5 = -v_{1f} + v_{2f} \qquad (2)$$

$$(2)\times 1.6 \rightarrow 10.4 = (1.6)(-v_{1f}) + (1.6)(v_{2f}) \qquad (3)$$

$$(1)+(3): 11.55 = 3.7 v_{2f}$$

$$\Rightarrow v_{2f} = \frac{11.55}{3.7} = 3.12\, m/s \qquad (4)$$

$$(4) \text{ in } (2): v_{1f} = -3.38\, m/s \qquad (5)$$

9.3 Collisions in One Dimension

Example 9.8 (Continued)

- *(B) During the collision, at the instant block 1 is moving to the right with a velocity of +3.00 m/s, determine the velocity of block 2.*

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1.6)(4)+(2.10)(-2.5)=(1.6)(3)+(2.10)v_{2f} \Rightarrow v_{2f} = -1.74\, m/s$$

- *(C) Determine the distance the spring is compressed at that instant.*

$$\because K_i + U_i = K_f + U_f$$

$$\therefore \frac{1}{2} m_1 {v_{1i}}^2 + \frac{1}{2} m_2 {v_{2i}}^2 + 0 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2 + \frac{1}{2} k x^2$$

$$\Rightarrow \frac{1}{2}(1.6)(4)^2 + \frac{1}{2}(2.1)(-2.5)^2 = \frac{1}{2}(1.6)(3)^2 + \frac{1}{2}(2.1)(-1.74)^2 + \frac{1}{2}(600)x^2$$

$$\therefore x = \sqrt{\frac{8.98 \times 2}{600}} = 0.173\, m$$

9.3 Collisions in One Dimension

PROBLE M-SOLV ING HI NTS

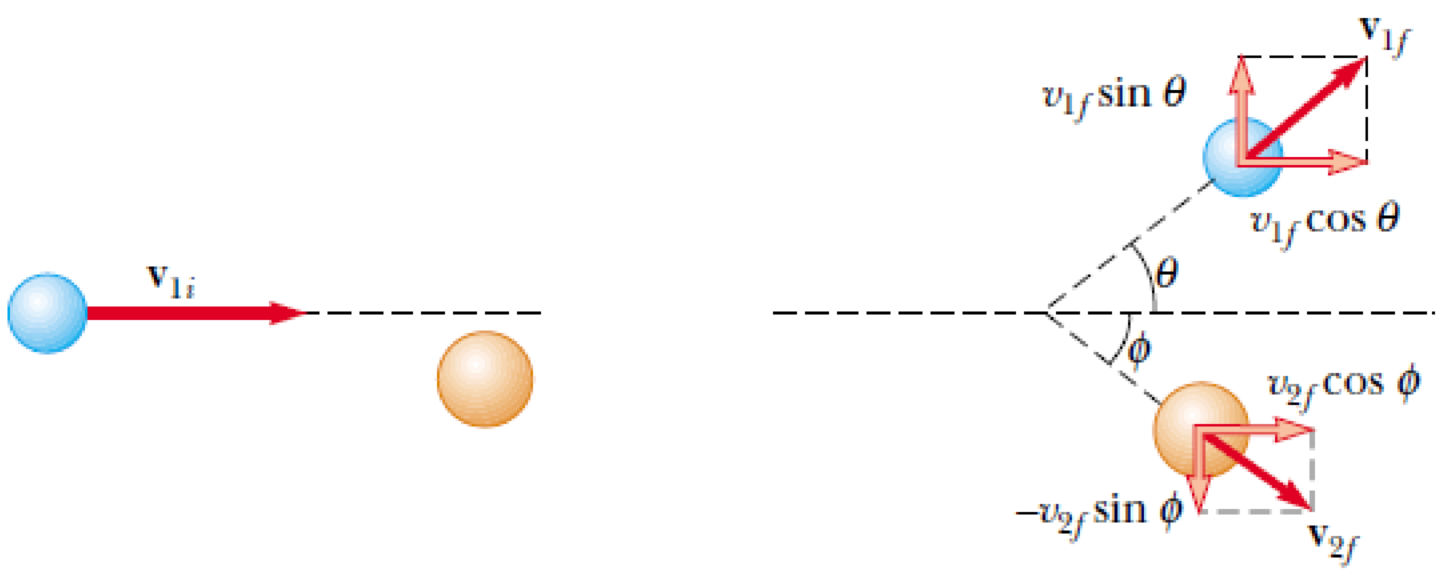
- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the *x and y components of the momentum of each* object before and after the collision.
- Write expressions for the total momentum of the system in the *x direction* before and after the collision and equate the two.
- If the collision is inelastic, kinetic energy of the system is *not conserved, and* additional information is probably required.
- If the collision is *perfectly* inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is *elastic, kinetic energy of the system is conserved, and you can* equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

9.4 Two-Dimensional Collisions

- For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

consider a 2-D problem in which particle 1 of mass *m_1 collides* with particle 2 of mass *m_2, where particle 2 is initially at rest, as in Figure*



(a) Before the collision (b) After the collision

9.4 Two-Dimensional Collisions

Applying the law of conservation of momentum in component form and noting that the initial y component of the momentum of the two-particle system is zero, we obtain:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$
$$0 = m_1 v_{1f} \, sin\,\theta - m_2 v_{2f} \, sin\,\varphi$$

- where the minus sign in last Equation comes from the fact that after the collision, particle 2 has a y component of velocity that is downward.
- If the collision is elastic, we can also use Equation 9.16 (conservation of kinetic energy) with v_{2i}= 0 to give:

$$\frac{1}{2} m_1 \, {v_{1i}}^2 = \frac{1}{2} m_1 \, {v_{1f}}^2 + \frac{1}{2} m_2 \, {v_{2f}}^2$$

9.4 Two-Dimensional Collisions

PROBLE M-SOLV ING HI NTS

- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the x and y components of the momentum of each object before and after the collision.
- Write expressions for the total momentum of the system in the x and y directions before and after the collision and equate the two..
- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably required.
- If the collision is perfectly inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is elastic, kinetic energy is conserved, and you can equate the total kinetic energy before and after the collision.

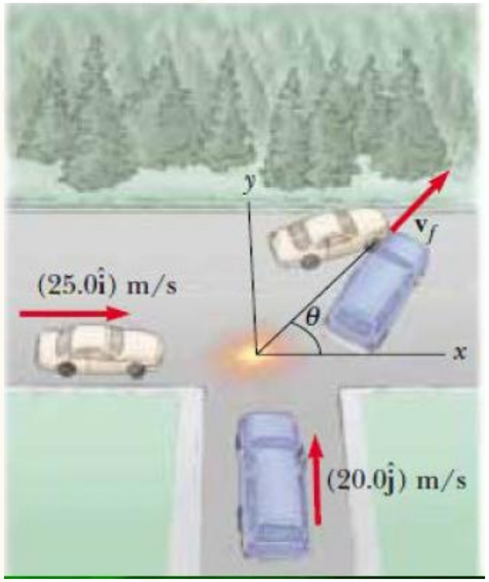
9.4 Two-Dimensional Collisions

Example 9.10 Collision at an Intersection

- A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.14. *Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).*
- ***Solution***:
- We shall apply the conservation of momentum in each direction.

$$x:\ m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \qquad (1)$$

$$y:\ m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy} \qquad (2)$$



9.4 Two-Dimensional Collisions

Example 9.10 (continued)

- Solving to find final velocity and direction:

$$(1) \rightarrow: \ (1500)(25) + (2500)(0) = (1500 + 2500) v_{fx} \qquad (3)$$

$$\therefore v_{fx} = \frac{37500}{4000} = 9.37\ m/s$$

$$(2) \rightarrow: \ (1500)(0) + (2500)(20) = (1500 + 2500) v_{fy} \qquad (4)$$

$$\therefore v_{fy} = \frac{50000}{4000} = 12.5\ m/s$$

$$(1) + (2) \rightarrow: v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{9.37^2 + 12.5^2} = 15.6\ m/s$$

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{12.5}{9.37}\right) = 53.1^o$$

Lecture Summary

Perfectly Inelastic Collisions

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Perfectly Elastic Collisions:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 \, {v_{1i}}^2 + \frac{1}{2} m_2 \, {v_{2i}}^2 = \frac{1}{2} m_1 \, {v_{1f}}^2 + \frac{1}{2} m_2 \, {v_{2f}}^2$$

When two particles collide, the total momentum of the isolated system before the collision always equals the total momentum after the collision, regardless of the nature of the collision. An inelastic collision is one for which the total kinetic energy of the system is not conserved. A perfectly inelastic collision is one in which the colliding bodies stick together after the collision. An elastic collision is one in which the kinetic energy of the system is conserved.

Lecture Summary

Two dimensional collisions

For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Thank You

ACKNOWLEDGEMENTS