PHYS 404
HANDOUT 10 – *Laguerre Polynomials*

1. Show that the Laguerre polynomials $L_n(x)$ are orthogonal in the interval $\left(0,\infty\right)$ with a weighting function e^{-x}.

(Sch. p. 160)

2. Show that $(n+1)L_{n+1}\left(x\right)=\left(2n+1-x\right)L_n\left(x\right)-nL_{n-1}\left(x\right)$.

(Sch. p. 160)

3. Expand the polynomial x^3+x^2-3x+2 in a series of Laguerre polynomials $\sum_{n=0}^{\infty}A_nL_n(x)$.

(Sch. p. 161)

4. Find the associated Laguerre polynomials a) $L_2^1(x)$, (b) $L_2^2(x)$, (c) $L_3^2(x)$ and (d) $L_3^4(x)$.

(Sch. p. 162)

5. Find the polynomial $L_4(x)$ and show that it satisfies the Hermite differential equation.

6. Use the series form of the Laguerre polynomials to show that (i) $L_n^{'}\left(0\right)=-n$ and (ii) $L_n^{''}\left(0\right)=n\left(n-1\right)$.

(Arf. 728)

7. A) Use the generating function to prove the recurrence relation $L_n^{'}(x)-L_{n-1}^{'}(x)+L_{n-1}(x)=0$.

B) Prove the recurrence relation $xL_n^{'}(x)=nL_n(x)-nL_{n-1}(x)$.

(Sch. p. 164)

8. Use the recurrence relation $L_{n+1}(x)=\left(2n+1-x\right)L_n(x)-n^2L_{n-1}(x)$ to find the polynomial $L_2(x)$. We are given that $L_{-1}(x)=0$ and that $L_0(x)=1$.

(Sch. p. 164)

9. Evaluate the integral $\int_0^{\infty}e^{-x}x^{k+1}L_n^k\left(x\right)L_n^k\left(x\right)dx=\frac{\left(n+k\right)!}{n!}\left(2n+k+1\right)$. Use the recurrence relation $xL_n^k=\left(2n+k+1\right)L_n^k-\left(n+k\right)L_{n-1}^k-(n+1)L_{n+1}^k$.

10. Starting from the generating function for the associated Laguerre polynomials prove the orthogonality relation $\int_0^\infty e^{-x} x^k L_n^k(x) L_m^k(x)\, dx = \frac{(n+k)!}{n!}\delta_{nm}$