PHYS 404
HANDOUT 11 – *Hermite and Laguerre Polynomials in Physics*

1. The eigenstates of a simple harmonic oscillator are given by $\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n\left(x\sqrt{\frac{m\omega}{\hbar}}\right) e^{-m\omega x^2/2\hbar}$. Consider the operators $a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right), \quad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right)$ with $p = -i\hbar\frac{d}{dx}$. Find the action of these operators on the ground state of the SHO $\psi_0(x)$.

2. Repeat the above question for any of the states $\psi_n(x)$ of a SHO.

3. The transition probability between two oscillator states, *m* and *n*, depends on the integral $\int_{-\infty}^{+\infty} xe^{-x^2} H_n(x)H_m(x)dx$. Evaluate this integral.

(Arf. 719)

4. The calculation of the mean square-displacement in a quantum SHO involves the evaluation of the integral $\int_{-\infty}^{+\infty} x^2 e^{-x^2} H_n(x)H_n(x)dx$. Evaluate this integral.

(Arf. 719)

5. Evaluate this integral $\int_{-\infty}^{+\infty} x^2 e^{-x^2} H_n(x)H_m(x)dx$.

(Arf. 719)

6. Show that:

$$\int_{-\infty}^{+\infty} x^r e^{-x^2} H_n(x)H_{n+p}(x)dx = \begin{cases} 0, & p > r \\ 2^n\sqrt{\pi}\,(n+r)! & p = r \end{cases}.$$

(Arf. 719)

7. In the hydrogen atom after solving the Schroedinger equation in spherical coordinates with the method of separating variables we get a radial equation which has the following solutions:

$$R_{n\ell}(r) = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n\left[(n+\ell)!\right]}}\, e^{-r/na_0} \left(\frac{2r}{na_0}\right)^{\ell} L_{n+\ell}^{2\ell+1}(2r/a_0).$$

a) Find the expressions for the radial functions $R_{n=2,\ell=0}(r)$, $R_{n=2,\ell=1}(r)$.

b) Show that the functions $R_{n\ell}(r)$ are normalized.

8. Assume that solving the Schroedinger Equation for a quantum mechanical particle we get the solution:

$$\frac{d^2y}{dx^2} - \left[\frac{k^2-1}{4x^2} - \frac{(2n+k+1)}{2x} + \frac{1}{4}\right]y = 0 .$$

a) Find the solution $A(x)$ for large asymptotic values of x.

b) Find the solution $B(x)$ for small values of x such that $0 < x << 1$.

c) Create a solution of the form $y = A(x)B(x)C(x)$. Insert it in the differential equation and find the form of $C(x)$

(Arf. 729)