

PHYS 404
HANDOUT 5 – Bessel Functions

1. Show that: $J_{-n}(x) = (-1)^n J_n(x)$ (n , integer).

(Arf. p. 576)

2. Show that: $J_n(x) + J_{n+2}(x) = 2(n+1)J_{n+1}(x)/x$

(Arf. p. 576)

3. Show that: $J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$.

4. Show that:

$$d/dx [x^n J_n(x)] = x^n J_{n-1}(x) \text{ and } d/dx [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x).$$

(Arf. p. 577)

5. Verify the following: $1 = J_0(x) + 2 \sum_{n=1}^{\infty} J_{2n}(x)$.

(Arf. p. 613)

6. Derive the Jacobi-Anger relation: $e^{iz \cos \theta} = \sum_{m=-\infty}^{\infty} i^m J_m(z) e^{im\theta}$. This is an expansion of a plane wave in a series of cylindrical waves.

(Arf. p. 585)

7. Show that the recurrence relation

$$J'_n(x) = \frac{1}{2} [J_{n-1}(x) - J_{n+1}(x)]$$

follows directly from differentiation of

$$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$$

(Arf. p. 589)

8. Show that $\frac{d}{dx} (x J_n(x) J_{n+1}(x)) = x (J_n^2(x) - J_{n+1}^2(x))$.

9. Evaluate $\int x^{-1} J_4(x) dx$.

10. From the product of the generating function $g(x, t) \cdot g(x, -t)$ show that $1 = [J_0(x)]^2 + 2[J_1(x)]^2 + 2[J_2(x)]^2 + \dots$

11. Using the generating function prove the addition theorem of Bessel equations: $J_n(u+v) = \sum_{m=-\infty}^{\infty} J_m(u)J_{n-m}(v)$.

12. Show that

$$(a) \cos x = J_0(x) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(x),$$

$$(b) \sin x = 2 \sum_{n=1}^{\infty} (-1)^{n+1} J_{2n+1}(x).$$

13. Using mathematical induction derive the relation:

$$J_n(x) = (-1)^n x^n \left(\frac{d}{x dx} \right)^n J_0(x).$$

Dr. Vasileios Lempesis