

PHYS 404
HANDOUT 9 – Hermite Polynomials

1. Using the generating function of Hermite polynomials find a) $H_0(x)$, (b) $H_1(x)$, (c) $H_2(x)$ and (d) $H_3(x)$

(Sch. p. 157)

2. Show that $H'_n(x) = 2nH_{n-1}(x)$.

(Sch. p. 157)

3. Show that $H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} (e^{-x^2})$ (Rodrigues' formula).

(Sch. p. 157)

4. Show that $\int_{-\infty}^{+\infty} e^{-x^2} H_m(x) H_n(x) dx = \begin{cases} 0 & m \neq n \\ 2^n n! \sqrt{\pi} & m = n \end{cases}$.

(Sch. p. 157)

5. Show that the Hermite polynomials satisfy the differential equation: $y'' - 2xy' + 2ny = 0$.

(Sch. p. 158)

6. If $f(x) = \sum_{n=0}^{\infty} A_n H_n(x)$ show that $A_n = \frac{1}{2^n n! \sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2} f(x) H_n(x) dx$.

(Sch. p. 158)

7. Expand the function $f(x) = x^3$ in a series of Hermite polynomials.

(Sch. p. 158)

8. Write the Parseval's relation which corresponds to the series

$$f(x) = \sum_{n=0}^{\infty} A_n H_n(x).$$

9. Use the relation of question 3 above to find the polynomials a) $H_0(x)$, (b) $H_1(x)$, (c) $H_2(x)$ and (d) $H_3(x)$

10. From the generating function show that $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$. Find the polynomials $H_2(x)$ and $H_3(x)$ if you know that $H_0(x)=1$ and $H_1(x)=2x$.

11. Find the integral $\int_{-\infty}^{+\infty} e^{-x^2} x^2 H_n(x) dx$.

12. Show that $H_{2n}(0) = \frac{(-1)^n (2n)!}{n!}$.