PHYS 404
HANDOUT 9 – *Hermite Polynomials*

1. Using the generating function of Hermite polynomials find a) $H_0(x)$, (b) $H_1(x)$, (c) $H_2(x)$ and (d) $H_3(x)$

(Sch. p. 157)

2. Show that $H_n^{'}(x) = 2nH_{n-1}(x)$.

(Sch. p. 157)

3. Show that $H_n(x) = \left(-1\right)^n e^{x^2} \dfrac{d^n}{dx^n}\left(e^{-x^2}\right)$ (Rodrigues' formula).

(Sch. p. 157)

4. Show that $\int\limits_{-\infty}^{+\infty} e^{-x^2} H_m(x) H_n(x)\, dx = \begin{cases} 0 & m \neq n \\ 2^n n! \sqrt{\pi} & m = n \end{cases}$.

(Sch. p. 157)

5. Show that the Hermite polynomials satisfy the differential equation: $y^{''} - 2xy^{'} + 2ny = 0$.

(Sch. p. 158)

6. If $f(x) = \sum\limits_{n=0}^{\infty} A_n H_n(x)$ show that $A_n = \dfrac{1}{2^n n! \sqrt{\pi}} \int\limits_{-\infty}^{+\infty} e^{-x^2} f(x) H_n(x)\, dx$.

(Sch. p. 158)

7. Expand the function $f(x) = x^3$ in a series of Hermite polynomials.

(Sch. p. 158)

8. Write the Parseval's relation which corresponds to the series $f(x) = \sum\limits_{n=0}^{\infty} A_n H_n(x)$.

9. Use the relation of question 3 above to find the polynomials a) $H_0(x)$, (b) $H_1(x)$, (c) $H_2(x)$ and (d) $H_3(x)$

10. From the generating function show that $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$. Find the polynomials $H_2(x)$ and $H_3(x)$ if you know that $H_0(x)=1$ and $H_1(x)=2x$.

11. Find the integral $\int\limits_{-\infty}^{+\infty} e^{-x^2} x^2 H_n(x)\, dx$.

12. Show that $H_{2n}(0) = \dfrac{\left(-1\right)^n \left(2n\right)!}{n!}$.

13. Show the recurrence relation: $\left(2x-\frac{d}{dx}\right)^{n}1=H_{n}\left(x\right)$. (Hint: use induction)