PHYSICS 404
3rd HOMEWORK – FALL 2019
Prof. V. Lempesis

Hand in: Thursday 31st of October 2019

1. (i) Find the general solution of the following differential equation:

$$\frac{d^2u}{dx^2}+\frac{1}{x}\frac{du}{dx}+\left(1-\frac{9}{x^2}\right)u=0 .$$

(ii)Find the general solution of the following differential equation:

$$\frac{d^2u}{dx^2}+\frac{1}{x}\frac{du}{dx}+\left(1-\frac{1}{25x^2}\right)u=0 .$$

(ii) Find the general solution of the following differential equation:

$$x^2\frac{d^2u}{dx^2}+x\frac{du}{dx}+\left(4x^2-16\right)u=0$$

Hint: See the discussion we did in Q3 of Handout 6.

(3 marks)

2. Prove that $N_{\nu-1}-N_{\nu+1}=2N_\nu'$. (Hint: Q8 of Handout 6 could be very helpful)

(7 marks)

3. We have seen that a **function** $f(x)$ is expressed as a Bessel series: $f(x)=\sum_{n=1} b_n J_m\left(\alpha_{mn}x\right)$ with α_{mn} the nth root of J_m and $0<x<1$ where S

$$b_n=\frac{2}{J_{m+1}^2\left(\alpha_{mn}\right)}\int_0^1 f(x)J_m\left(\alpha_{mn}x\right)dx .$$

Assume that we have the function *f*(*x*)=x(1-x) and m =1. Find the first three coefficients b_1 , b_2 , b_3 of the expansion series $f(x)=\sum_{n=1} b_n J_1\left(\alpha_{1n}x\right)$

Hint:

a) To evaluate integrals use the Wolfram online integrator at:
http://www.wolframalpha.com/calculators/integral-calculator/

b) To find the zeros of the Bessel functions you can use any internet source. For example at
http://mathworld.wolfram.com/BesselFunctionZeros.html

c) To find values of Bessel functions you can use the follow link
http://keisan.casio.com/exec/system/1180573474

(10 marks)