Physics 201

Problem set (3)

Problem (1)

Cosider the folloewing matrcies:

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix},$$

$$E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

Compute the following operations:

1) AB = $\begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$,

2) D-E= $\begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

3) 2B-C (undefined because B and C are not of the same size)

4) tr (D)= 5

5) tr (A) (Undefined)

6) $\underline{C}C^T = \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$

7) $(DA)^T = \begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$,

8) tr $(DD)^T$ = 61

Problem (2)

Find the inverse of the following matrcies (Use Gauss Jordan Elimination Method) :

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, C= $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$D = \begin{bmatrix} 1 & -2 & 4 \\ 1 & 0 & -2 \\ -3 & 12 & -32 \end{bmatrix}$

Solution:

$A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

B^{-1} (See solution below)

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \\ Row_3 - Row_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ Row_3 \\ Row_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ \\ Row_3 - 2Row_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & -3 & 2 & 1 & -2 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{1}{3}Row_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right]$$

$$\begin{array}{r} \\ Row_2 - 2Row_3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right]$$

C^{-1} (No inverse, See solution below)

Solution.

$$[A|I] = \left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} Row_2 \\ Row_1 \end{array} \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & -2 & 0 & 1 \end{array}\right]$$

$$Row_2 - 2Row_1 \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$$

The matrix has no inverse.

D^{-1} (See solution below)

Solution. (a) We use Gaussian elimination as follows:

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 4 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 \\ -3 & 12 & -32 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -2 & 4 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -2 & 6 & -3 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & 4 & -1 \\ 0 & 2 & 0 & -19 & 10 & -3 \\ 0 & 0 & -2 & 6 & -3 & 1 \end{array}\right).$$

Dividing the second row through by 2 and the third by -2, we conclude that the inverse of the given matrix is

$$\begin{pmatrix} -6 & 4 & -1 \\ -19/2 & 5 & -3/2 \\ -3 & 3/2 & -1/2 \end{pmatrix}.$$