Physics 201

Problem Set (4)

Problem (1)

Decide whether each matrix below is an elementary matrix

(a) $\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Solutions:

A) Elementary
B) Not Elementary
C) Not Elementary
D) Not Elementary

Problem (2)

In each part, an elementary matrix *E* and a matrix *A* are given. Write down the row operation corresponding to *E* and show that the product EAresults from applying the row operation to *A*

(a) $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

(b) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

Solution:

(a) Swap rows 1 and 2: $EA = \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix}$

(b) Add -3 times row 2 to row 3: $EA = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ -1 & 9 & 4 & -12 & -10 \end{bmatrix}$

Problem (3)

Use an inverse matrix to solve the following system (Hint: use Ax=b) .

$$\begin{aligned} 2x + 3y + z &= 4 \\ 3x + 3y + z &= 8 \\ 2x + 4y + z &= 5 \end{aligned}$$

Solution:

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -7 \end{bmatrix}$$

The solution is $x = 4$, $y = 1$, and $z = -7$.

Problem (4)

Find all the minors and cofactors of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

Solution:

$$M_{11} = 29,\ C_{11} = 29$$
$$M_{12} = 21,\ C_{12} = -21$$
$$M_{13} = 27,\ C_{13} = 27$$
$$M_{21} = -11,\ C_{21} = 11$$
$$M_{22} = 13,\ C_{22} = 13$$
$$M_{23} = -5,\ C_{23} = 5$$
$$M_{31} = -19,\ C_{31} = -19$$
$$M_{32} = -19,\ C_{32} = 19$$
$$M_{33} = 19,\ C_{33} = 19$$

Problem (5)

Find the determinant of the matrix by cofactor expansion

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

Solution

$$\det(A) = \begin{vmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{vmatrix} = 3\begin{vmatrix} -4 & 3 \\ 4 & -2 \end{vmatrix} - 1\begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} + 0\begin{vmatrix} -2 & -4 \\ 5 & 4 \end{vmatrix}$$
$$= 3(-4) - (1)(-11) + 0 = -1$$

Problem (6)

Use the arrow technique to evaluate the determinant of the given matrix

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{vmatrix}:$$

Solution

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 1 & 2 \\ -4 & 5 & 6 & -4 & 5 \\ 7 & -8 & 9 & 7 & -8 \end{vmatrix}$$

$$= [45 + 84 + 96] - [105 - 48 - 72] = 240$$