# Physics 201

## Problem Set (7)

### Problem (1)

Provided that $\mathbf{u} = (-1, 0, 1)$ and $\mathbf{v} = (2, -1, 5)$ in $R^3$, find each vector.

(a) $\mathbf{u} + \mathbf{v}$ (b) $2\mathbf{u}$ (c) $\mathbf{v} - 2\mathbf{u}$

### Solution

(a) To add two vectors, add their corresponding components, as follows.

$$\mathbf{u} + \mathbf{v} = (-1, 0, 1) + (2, -1, 5) = (1, -1, 6)$$

(b) To multiply a vector by a scalar, multiply each component by the scalar, as follows.

$$2\mathbf{u} = 2(-1, 0, 1) = (-2, 0, 2)$$

(c) Using the result of part (b), you have

$$\mathbf{v} - 2\mathbf{u} = (2, -1, 5) - (-2, 0, 2) = (4, -1, 3).$$

## Problem (2)

Consider the vectors $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (6, 4, 2)$ in $R^3$. Show that $\mathbf{w} = (9, 2, 7)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ and that $w' = (4, -1, 8)$ is *not* a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

## Solution

***Solution*** In order for $\mathbf{w}$ to be a linear combination of $\mathbf{u}$ and $\mathbf{v}$, there must be scalars $k_1$ and $k_2$ such that $\mathbf{w} = k_1\mathbf{u} + k_2\mathbf{v}$; that is,

$$(9, 2, 7) = k_1(1, 2, -1) + k_2(6, 4, 2)$$

or

$$(9, 2, 7) = (k_1 + 6k_2, 2k_1 + 4k_2, -k_1 + 2k_2)$$

Equating corresponding components gives

$$\begin{aligned} k_1 + 6k_2 &= 9 \\ 2k_1 + 4k_2 &= 2 \\ -k_1 + 2k_2 &= 7 \end{aligned}$$

Solving this system using Gaussian elimination yields $k_1 = -3$, $k_2 = 2$, so

$$\mathbf{w} = -3\mathbf{u} + 2\mathbf{v}$$

Similarly, for $\mathbf{w}'$ to be a linear combination of $\mathbf{u}$ and $\mathbf{v}$, there must be scalars $k_1$ and $k_2$ such that $\mathbf{w}' = k_1\mathbf{u} + k_2\mathbf{v}$; that is,

$$(4, -1, 8) = k_1(1, 2, -l) + k_2(6, 4, 2)$$

or

$$(4, -1, 8) = (k_1 + 6k_2, 2k_1 + 4k_2, -k_1 + 2k_2)$$

Equating corresponding components gives

$$\begin{aligned} k_1 + 6k_2 &= 4 \\ 2k_1 + 4k_2 &= -1 \\ -k_1 + 2k_2 &= 8 \end{aligned}$$

This system of equations is inconsistent (verify), so no such scalars $k_1$ and $k_2$ exist. Consequently, $\mathbf{w}'$ is not a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

**<u>Problem (3)</u>**

**<u>Determine whether the set, together with the indicated operations, is a vector space. If it is not, identify at least one of the ten vector space axioms that fails.</u>**

1)

The set of all third-degree polynomials with the standard operations

2)

The set $\{(x, y): x \geq 0, y \text{ is a real number}\}$ with the standard operations in $R^2$

3)

The set $\{(x, x): x \text{ is a real number}\}$ with the standard operations

4)

The set of all $2 \times 2$ singular matrices with the standard operations

5)

The set of all $2 \times 2$ diagonal matrices with the standard operations

**Solution**

**1)**

The set is not a vector space. Axiom 1 fails because $x^3 + (-x^3 + 1) = 1$, which is not a third-degree polynomial. (Axioms 4, 5, and 6 also fail.)

**2)**

The set is not a vector space. Axiom 6 fails because $(-1)(x, y) = (-x, -y)$, which is not in the set when $x \neq 0$.

**3)**

The set is a vector space.

**4)**

The set is not a vector space. Axiom 1 fails because

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

which is not singular.

**5)**

The set is a vector space.