لا يكتب في هذا الهامش

(5) $x_n = n e^{\sin(\frac{1}{n})} = e^{\frac{\sin n}{n}} \cdot \frac{\sin\frac{1}{n}}{\frac{1}{n}}$

$\lim_{n\to\infty} x_n = \lim_{n\to\infty} e^{\frac{\sin n}{n}} \cdot \lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$

$-\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$

$0 \qquad\qquad 0$

$\Rightarrow$ Sandwich Th: $\lim_{n\to\infty} \frac{\sin n}{n} = 0$

e^x is continuous function

$\Rightarrow \lim_{n\to\infty} e^{\frac{\sin n}{n}} = e^{\lim \frac{\sin n}{n}} = e^0 = 1$

$\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$ put $\frac{1}{n} = y$

$\Rightarrow \lim_{y\to 0} \frac{\sin y}{y} = 1 = \lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$

$\Rightarrow \lim_{n\to\infty} x_n = (1)(1) = 1$

Ex(2)(e): $x_n = \frac{n}{n^2} + \frac{(n+1)}{n^2} + \dots + \frac{2n}{n^2}$

(e) $0 < \frac{n}{n^2} + \frac{(n+1)}{n^2} + \dots + \frac{2n}{n^2} < \frac{2n}{n^2} + \frac{2n}{n^2} + \dots + \frac{2n}{n^2}$

$0 \leftarrow \neq \rightarrow 2 \leftarrow \frac{2n(n+1)}{n^2}$

Sandwich Th. does not work here!!

Try another method:

$\lim_{n\to\infty} \frac{n}{n^2} + \lim_{n\to\infty} \left(\frac{n+1}{n^2} + \frac{n+2}{n^2} + \dots + \frac{2n}{n^2} \right)$

$= \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n+k}{n^2}$

$= 0 + \lim_{n\to\infty} \left[\sum_{k=1}^{n} \frac{1}{n} + \frac{1}{n} \sum_{k=1}^{n} \left(\frac{k}{n}\right) \right]$

$f(x) = x$, on $[0,1]$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$x_k = 0 + k \cdot \Delta x = \frac{k}{n}$

$= \lim_{n\to\infty} \frac{n}{n} + \int_0^1 x\,dx$

$= \lim_{n\to\infty} 1 + \frac{1}{2}x^2 \Big]_0^1$

$= 1 + \frac{1}{2}(1) = \frac{3}{2}$

الممسوحة ضوئيا بـ CamScanner