13.

Ernie makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest

$$\delta_t = \frac{t^2}{100}, t > 0.$$

The amount of interest earned from time 3 to time 6 is also X.

Calculate X.

(A)   385

(B)   485

(C)   585

(D)   685

(E)   785

16.

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40th payment is made.

(A)   6750

(B)   6890

(C)   6940

(D)   7030

(E)   7340

4.

John borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1,627.45 at the end of each year. Instead, John repays the 10,000 using a sinking fund that pays an annual effective interest rate of 14%. The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan and are made at the end of each year for 10 years.

Calculate the balance in the sinking fund immediately after repayment of the loan.

    (A)   2,130

    (B)   2,180

    (C)   2,230

    (D)   2,300

    (E)   2,370

15.

A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

    (i)   Equal annual payments at an annual effective interest rate of 8.07%.

    (ii)   Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Calculate i.

    (A)   8.75%

    (B)   9.00%

    (C)   9.25%

    (D)   9.50%

    (E)   9.75%

25.

A perpetuity-immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and a charity receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and the charity's share is K.

Calculate K.

    (A)   24%

    (B)   28%

    (C)   32%

    (D)   36%

    (E)   40%

27.

Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X.

Calculate X.

    (A)   28.00

    (B)   31.30

    (C)   34.60

    (D)   36.70

    (E)   38.90

29.

At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 3, is 32.

At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months, is X.

Calculate X.

      (A)   31.6

      (B)   32.6

      (C)   33.6

      (D)   34.6

      (E)   35.6

47.

Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The price assumes an annual nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of i.

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate i.

(A)   9.50%

(B)   9.75%

(C)   10.00%

(D)   10.25%

(E)   10.50%

48.

A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of X to a fund for 25 years. The fund earns an annual nominal interest rate of 8% compounded monthly.

On his 65th birthday, each 1000 of the fund will provide 9.65 of income at the beginning of each month starting immediately and continuing as long as he survives.

Calculate X.

(A)   324.70

(B)   326.90

(C)   328.10

(D)   355.50

(E)   450.70

54.

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

Calculate X.

- (A) 1400
- (B) 1420
- (C) 1440
- (D) 1460
- (E) 1480

55.

Toby purchased a 20-year par value bond with semiannual coupons of 40 and a redemption value of 1100. The bond can be called at 1200 on any coupon date prior to maturity, starting at the end of year 15.

Calculate the maximum price of the bond to guarantee that Toby will earn an annual nominal interest rate of at least 6% convertible semiannually.

- (A) 1251
- (B) 1262
- (C) 1278
- (D) 1286
- (E) 1295

74.

You are given the following information about two bonds, Bond A and Bond B:

  i) Each bond is a 10-year bond with semiannual coupons redeemable at its par value of 10,000, and is bought to yield an annual nominal interest rate of i, convertible semiannually.

  ii) Bond A has an annual coupon rate of $(i + 0.04)$, paid semiannually.

  iii) Bond B has an annual coupon rate of $(i - 0.04)$, paid semiannually.

  iv) The price of Bond A is 5,341.12 greater than the price of Bond B.

Calculate i.

  (A)   0.042
  (B)   0.043
  (C)   0.081
  (D)   0.084
  (E)   0.086

77.

Lucas opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible semiannually. Danielle also opens a bank account with 1000 at the same time as Lucas, but it grows at an annual nominal interest rate of 3% convertible monthly.

For each account, interest is credited only at the end of each interest conversion period.

Calculate the number of months required for the amount in Lucas's account to be at least double the amount in Danielle's account.

  (A)   276
  (B)   282
  (C)   285
  (D)   286
  (E)   288

81.

A borrower takes out a 50-year loan, to be repaid with payments at the end of each year. The loan payment is 2500 for each of the first 26 years. Thereafter, the payments decrease by 100 per year. Interest on the loan is charged at an annual effective rate of i ($0\% < i < 10\%$).

The principal repaid in year 26 is X.

Determine the amount of interest paid in the first year.

(A) Xv^{25}

(B) $2500v^{25} - Xv^{25}$

(C) $2500 - X$

(D) $2500 - XV^{25}$

(E) $25Xi$

88.

A borrower takes out a 15-year loan for 65,000, with level end-of-month payments. The annual nominal interest rate of the loan is 8%, convertible monthly.

Immediately after the 12th payment is made, the remaining loan balance is reamortized. The maturity date of the loan remains unchanged, but the annual nominal interest rate of the loan is changed to 6%, convertible monthly.

Calculate the new end-of-month payment.

(A) 528

(B) 534

(C) 540

(D) 546

(E) 552

89.

College tuition is 6000 for the current school year, payable in full at the beginning of the school year. College tuition will grow at an annual rate of 5%. A parent sets up a college savings fund earning interest at an annual effective rate of 7%. The parent deposits 750 at the beginning of each school year for 18 years, with the first deposit made at the beginning of the current school year. Immediately following the 18th deposit, the parent pays tuition for the 18th school year from the fund.

The amount of money needed, in addition to the balance in the fund, to pay tuition at the beginning of the 19th school year is X.

Calculate X.

      (A)   1439

      (B)   1545

      (C)   1664

      (D)   1785

      (E)   1870

91.

An investor purchases a 10-year callable bond with face amount of 1000 for price P. The bond has an annual nominal coupon rate of 10% paid semi-annually.

The bond may be called at par by the issuer on every other coupon payment date, beginning with the second coupon payment date.

The investor earns at least an annual nominal yield of 12% compounded semi-annually regardless of when the bond is redeemed.

Calculate the largest possible value of P.

      (A)   885

      (B)   892

      (C)   926

      (D)   965

      (E)   982

90.

A 1000 par value 20-year bond sells for P and yields a nominal interest rate of 10% convertible semiannually. The bond has 9% coupons payable semiannually and a redemption value of 1200.

Calculate P.

    (A)   914

    (B)   943

    (C)   1013

    (D)   1034

    (E)   1097

94.

A couple decides to save money for their child's first year college tuition.

The parents will deposit 1700 n months from today and another 3400 $2n$ months from today.

All deposits earn interest at a nominal annual rate of 7.2%, compounded monthly.

Calculate the maximum integral value of n such that the parents will have accumulated at least 6500 five years from today.

    (A)   11

    (B)   12

    (C)   18

    (D)   24

    (E)   25

95.

Let S be the accumulated value of 1000 invested for two years at a nominal annual rate of discount d convertible semiannually, which is equivalent to an annual effective interest rate of i.

Let T be the accumulated value of 1000 invested for one year at a nominal annual rate of discount d convertible quarterly.

$S/T = (39/38)^4$.

Calculate i.

(A)   10.0%
(B)   10.3%
(C)   10.8%
(D)   10.9%
(E)   11.1%

97.

Five deposits of 100 are made into a fund at two-year intervals with the first deposit at the beginning of the first year.

The fund earns interest at an annual effective rate of 4% during the first six years and at an annual effective rate of 5% thereafter.

Calculate the annual effective yield rate earned over the investment period ending at the end of the tenth year.

(A)   4.18%
(B)   4.40%
(C)   4.50%
(D)   4.58%
(E)   4.78%

99.

Jack inherited a perpetuity-due, with annual payments of 15,000. He immediately exchanged the perpetuity for a 25-year annuity-due having the same present value. The annuity-due has annual payments of X.

All the present values are based on an annual effective interest rate of 10% for the first 10 years and 8% thereafter.

Calculate X.

       (A)   16,942

       (B)   17,384

       (C)   17,434

       (D)   17,520

       (E)   18,989

101.

A 30-year annuity is arranged to pay off a loan taken out today at a 5% annual effective interest rate. The first payment of the annuity is due in ten years in the amount of 1,000. The subsequent payments increase by 500 each year.

Calculate the amount of the loan.

      (A)   58,283

      (B)   61,197

      (C)   64,021

      (D)   64,257

      (E)   69,211

105.

A bank agrees to lend 10,000 now and X three years from now in exchange for a single repayment of 75,000 at the end of 10 years. The bank charges interest at an annual effective rate of 6% for the first 5 years and at a force of interest $\delta_t = \dfrac{1}{t+1}$ for $t \geq 5$.

Calculate X.

      (A)   23,500

      (B)   24,000

      (C)   24,500

      (D)   25,000

      (E)   25,500

106.

A company takes out a loan of 15,000,000 at an annual effective discount rate of 5.5%. You are given:

    i)   The loan is to be repaid with n annual payments of 1,200,000 plus a drop payment one year after the nth payment.

    ii)   The first payment is due three years after the loan is taken out.

Calculate the amount of the drop payment.

    (A)   79,100

    (B)   176,000

    (C)   321,300

    (D)   959,500

    (E)   1,180,300

107.

Tim takes out an n-year loan with equal annual payments at the end of each year.

The interest portion of the payment at time $(n-1)$ is equal to 0.5250 of the interest portion of the payment at time $(n-3)$ and is also equal to 0.1427 of the interest portion of the first payment.

Calculate n.

    (A)   18

    (B)   20

    (C)   22

    (D)   24

    (E)   26

108.

You are given the following information about an eleven-year loan of L to be repaid by the sinking fund method:

i)   The sinking fund earns an annual effective interest rate of 4.70%.

ii)   Immediately following the seventh payment and deposit, the difference between what is owed to the lender on the loan and the accumulated value of the sinking fund is 6241.

Calculate the sinking fund deposit.

(A)   1019

(B)   1055

(C)   1067

(D)   1084

(E)   1104

109.

On January 1, 2003 Mike took out a 30-year mortgage loan in the amount of 200,000 at an annual nominal interest rate of 6% compounded monthly. The loan was to be repaid by level end-of-month payments with the first payment on January 31, 2003.

Mike repaid an extra 10,000 in addition to the regular monthly payment on each December 31 in the years 2003 through 2007.

Determine the date on which Mike will make his last payment (which is a drop payment).

(A)   July 31, 2013

(B)   November 30, 2020

(C)   December 31, 2020

(D)   December 31, 2021

(E)   January 31, 2022

110.

A 5-year loan of 500,000 with an annual effective discount rate of 8% is to be repaid by level end-of-year payments.

If the first four payments had been rounded up to the next multiple of 1,000, the final payment would be X.

Calculate X.

    (A)   103,500

    (B)   111,700

    (C)   115,200

    (D)   125,200

    (E)   127,500

111.

A company plans to invest X at the beginning of each month in a zero-coupon bond in order to accumulate 100,000 at the end of six months. The price of each bond as a percentage of redemption value is given in the following chart:

Maturity (months)	1	2	3	4	5	6
Price	99%	98%	97%	96%	95%	94%

Calculate X given that the bond prices will not change during the six-month period.

    (A)   15,667

    (B)   16,078

    (C)   16,245

    (D)   16,667

    (E)   17,271

112.

A loan of X is repaid with level annual payments at the end of each year for 10 years.
You are given:

i)   The interest paid in the first year is 3600; and

ii)   The principal repaid in the 6th year is 4871.

Calculate X.

(A)   44,000

(B)   45,250

(C)   46,500

(D)   48,000

(E)   50,000

113.

An investor purchased a 25-year bond with semiannual coupons, redeemable at par, for a price of 10,000. The annual effective yield rate is 7.05%, and the annual coupon rate is 7%.

Calculate the redemption value of the bond.

(A)   9,918

(B)   9,942

(C)   9,981

(D)   10,059

(E)   10,083

115.

A bank issues three annual coupon bonds redeemable at par, all with the same term, price, and annual effective yield rate.

The first bond has face value 1000 and annual coupon rate 5.28%.

The second bond has face value 1100 and annual coupon rate 4.40%.

The third bond has face value 1320 and annual coupon rate r.

Calculate r.

  (A) 2.46%
  (B) 2.93%
  (C) 3.52%
  (D) 3.67%
  (E) 4.00%

116.

An investor owns a bond that is redeemable for 250 in 6 years from now. The investor has just received a coupon of c and each subsequent semiannual coupon will be 2% larger than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 582.53 assuming an annual effective yield rate of 4%.

Calculate c.

  (A) 32.04
  (B) 32.68
  (C) 40.22
  (D) 48.48
  (E) 49.45

117.

An n-year bond with annual coupons has the following characteristics:

i) The redemption value at maturity is 1890;

ii) The annual effective yield rate is 6%;

iii) The book value immediately after the third coupon is 1254.87; and

iv) The book value immediately after the fourth coupon is 1277.38.

Calculate n.

(A) 16
(B) 17
(C) 18
(D) 19
(E) 20

118.

An n-year bond with semiannual coupons has the following characteristics:

i) The par value and redemption value are 2500;

ii) The annual coupon rate is 7% payable semi-annually;

iii) The annual nominal yield to maturity is 8% convertible semiannually; and

iv) The book value immediately after the fourth coupon is 8.44 greater than the book value immediately after the third coupon.

Calculate n.

(A) 6.5
B) 7.0
(C) 9.5
(D) 12.0
(E) 14.0

133.

An insurance company wants to match liabilities of 25,000 payable in one year and 20,000 payable in two years with specific assets. The following assets are currently available:

i) One-year bond with an annual coupon of 6.75% at par

ii) Two-year bond with annual coupons of 4.50% at par

iii) Two-year zero-coupon bond yielding 5.00% annual effective

Calculate the smallest amount the company needs to disburse today to purchase assets that will exactly match these liabilities.

(A) 41,220

(B) 41,390

(C) 41,560

(D) 41,660

(E) 41,750