Bose-Einstein Condensation

(For the 9th Edition of the McGraw-Hill Encyclopedia of Science & Technology)

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When a gas of bosonic particles is cooled below a critical temperature, it condenses into a Bose-Einstein condensate. The condensate consists of a macroscopic number of particles, which are all in the ground state of the system. Bose-Einstein condensation (BEC) is a phase-transition, which does not depend on the specific interactions between particles. It is based on the indistinguishability and wave nature of particles, both of which are at the heart of quantum mechanics.

Basic phenomenon in an ideal gas

In a simplified picture, particles in a gas may be regarded as quantum-mechanical wavepackets which have an extent on the order of a thermal de Broglie wavelength $\lambda_{dB} = (2\pi\hbar^2/mk_BT)^{1/2}$ where T is the temperature, m the mass of the particle, k_B Boltzmann's constant and $\hbar$ Planck's constant. λ_{dB} can be regarded as the position uncertainty associated with the thermal momentum distribution. At high temperature, λ_{dB} is small, and it is very improbable to find two particles within this distance. Therefore, the indistinguishability of particles is not important, and a classical description applies (Boltzmann statistics). When the gas is cooled to the point where λ_{dB} is comparable to the distance between particles, the individual wavepackets start to "overlap" and the indistinguishability of particles becomes crucial – an "identity crisis" occurs. For fermions, the Pauli exclusion principle prevents two particles from occupying the same quantum state, whereas for bosons, quantum statistics dramatically increases the probability of finding several particles in the same quantum state (Bose-Einstein statistics). The system undergoes a phase transition and forms a Bose-Einstein condensate, where a macroscopic number of particles occupy the lowest-energy quantum state (Fig. 1). The temperature and the density of particles n at the phase transition are related as $n\lambda_{dB}^3 = 2.612$. BEC is a phase-transition solely caused by quantum statistics, in contrast to other phase-transitions (like melting or crystallization) which depend on the inter-particle interactions.
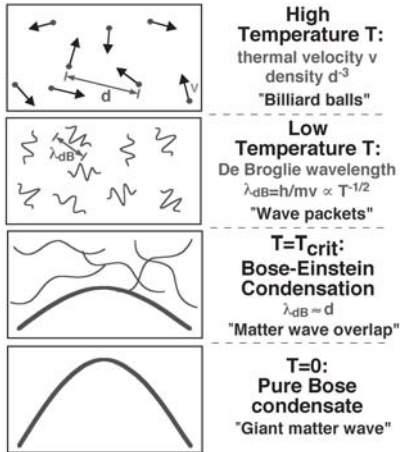
Fig. 1: Criterion for Bose-Einstein condensation. At high temperatures, a weakly interacting gas can be treated as a system of "billiard balls." In a simplified quantum description, the particles can be regarded as wavepackets with an extension λ_{dB}. At the BEC transition temperature, λ_{dB} becomes comparable to the distance d between particles, and a Bose-Einstein condensate forms (in the case of bosonic particles). As the temperature approaches zero, the thermal cloud disappears leaving a pure Bose condensate.

Bose-Einstein condensation can be described intuitively in the following way: when the quantum-mechanical wavefunctions of bosonic particles spatially "overlap", then the matter waves start to oscillate in concert - a coherent matter wave forms that comprises all particles in the ground state of the system. This transition from "disordered" to coherent matter waves can be compared to the step from incoherent light to laser light. Indeed, atom lasers based on Bose-Einstein condensation have been recently realized.

Bose-Einstein condensation occurs in thermal equilibrium. Although the condensate itself does not contribute to the entropy, its presence maximizes the entropy of the other particles, which are distributed among the higher energy states.

Atoms and molecules are composite particles. They are bosonic (fermionic) if they have integer (half-integer) spin, or equivalently, if the total number of electrons, protons, and neutrons they contain is even (odd). As long as the thermal energy is much less than internal excitation energies of the composite particles, we can regard them as pointlike particles obeying Bose-Einstein (Fermi-Dirac) statistics.

Experimental techniques

The phenomenon of Bose-Einstein condensation is responsible for the superfluidity of helium and for the superconductivity of an electron gas which involves "Bose condensed" electron pairs. However, these phenomena happen at high density and require a detailed treatment of the interactions.

The quest to realize BEC in a dilute weakly interacting gas was pursued in several different directions: liquid helium diluted in vycor, a sponge-like glass, excitons in semiconductors, which consist of weakly-bound electron-hole pairs, and in atomic gases. This article discusses the last case since most of the research activities have focused on these systems.

At ultralow temperatures, all atomic gases liquefy or solidify in thermal equilibrium. Keeping the gas at sufficiently low density can prevent this. Elastic collisions, which ensure thermal equilibrium of the gas, are then much more frequent than three-body collisions, which are required for the formation of molecules and other aggregates. As a result the gaseous phase is metastable for seconds or even minutes and allows for the observation of BEC in a gas. Typical atomic densities between 10^{12} and 10^{15} particles/cm^3 imply BEC transition temperatures in the nanokelvin or microkelvin regime.

Table 1: Multi-stage cooling to BEC in the MIT sodium experiment. Through a combination of laser and evaporative cooling, the temperature of the gas is reduced by a factor of 10^9, while the density at the BEC transition is similar to the initial density in the atomic beam oven (all numbers are approximate). In each step shown, the ground state population (which is proportional to the phase-space density) increases by about 10^6, resulting in a total increase of phase-space density by 20 orders of magnitude.

	Temperature	Density [cm^{-3}]	Phase-space density
Oven	500 K	10^{14}	10^{-13}
Laser cooling	50 μK	10^{11}	10^{-6}
Evaporative cooling	500 nK	10^{14}	2.612
BEC		10^{15}	10^7

The realization of BEC in atomic gases required techniques to cool gases to such low temperatures, and atom traps to confine the gas at the required density and keep them away from the much warmer walls of the vacuum chamber. The experiments on alkali vapors (lithium, rubidium, sodium) use several laser-cooling techniques as precooling, then hold the atoms in a magnetic trap and cool them further by forced evaporative cooling (Fig. 2 and Table 1). For atomic hydrogen, the laser-cooling step is replaced by cryogenic cooling. Evaporative cooling is done by continuously removing the high-energy tail of the thermal distribution from the trap. The evaporated atoms carry away more than the average energy, which means that the temperature of the remaining atoms decreases. The high-energy tail must be constantly repopulated by collisions, thus maintaining thermal equilibrium and sustaining the cooling process. Evaporative cooling is a common phenomenon in daily life – it's how hot water cools down in a bathtub or in a cup of coffee.
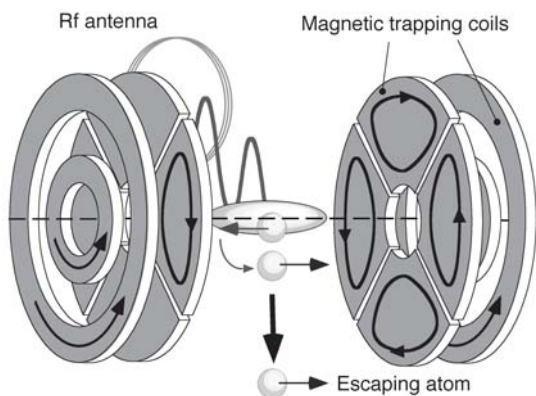

Fig 2: Experimental setup for cooling a gas of atoms to Bose-Einstein condensation. A cold cloud of atoms is confined in a magnetic trap and cooled by evaporative cooling. Flipping their spin with RF radiation evaporates the highest energy particles. Since this process reverses the magnetic force, these particles are expelled from the trapping region.

Observation of Bose-Einstein condensation

Bose-Einstein condensation was observed by suddenly switching off the atom trap releasing the atom cloud into a ballistic expansion. The cloud was illuminated with resonant laser light, and its shadow was imaged onto a CCD camera (Fig. 3). Such images showed the sudden appearance of a bimodal cloud consisting of a diffuse normal component and a dense core (the condensate) near the predicted transition temperature. The condensate expanded much faster than corresponding to the zero-point energy of the trapping potential – clear evidence for repulsive interactions between the atoms, which accelerate the ballistic expansion.
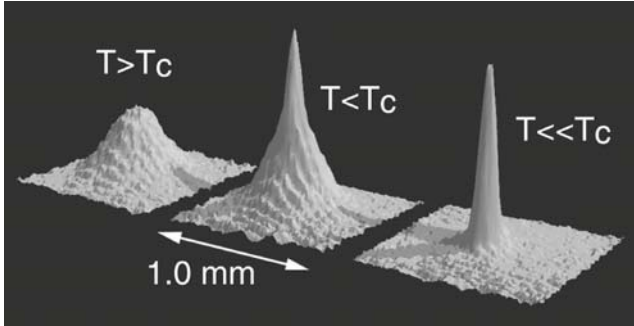
Fig. 3: Observation of Bose-Einstein condensation by absorption imaging. Shown is absorption vs. two spatial dimensions. The Bose-Einstein condensate is characterized by its slow expansion observed after 6 msec time of flight. The left picture shows an expanding cloud cooled to just above the transition point; middle: just after the condensate appeared; right: after further evaporative cooling has left an almost pure condensate. The total number of sodium atoms at the phase transition is about 7×10^5; the temperature T_c at the transition point is 2 μK.

The sudden reduction of the speed of ballistic expansion has been called condensation in momentum space. However, BEC is a condensation into the lowest energy state; in an inhomogeneous trapping potential, the lowest state has the smallest dimension. Condensation into this states thus results in the formation of a dense core of Bose condensed atoms within a diffuse thermal cloud of atoms. This spatial structure of a Bose condensate was directly observed using dispersive imaging techniques.

Weak interactions

Bose-Einstein condensation in gases allows for a "first-principles" theoretical description because there is a clear hierarchy of length and energy scales. In a gas, the separation between atoms $n^{-1/3}$ is much larger than the effective range of the interatomic forces (characterized by the s-wave scattering length a, typically 1 to 5 nm for alkali atoms), i.e. the quantity $na^3 \ll 1$. This inequality expresses that binary collisions are much more frequent than three-body collisions. It is in this limit that the theory of the weakly interacting Bose gas applies. Interactions between the Bose condensed are described by a mean field energy $U_{int} = 4 \pi \hbar^2 n a /m$. The stability of large condensates requires repulsive interactions (positive a). For negative a, the interactions are attractive, and the condensate is unstable against collapse above a certain size.

Properties of the weakly interacting Bose gas have been studied both experimentally and theoretically. Of special interest were the collective excitations of the gas, different forms of sound (Bogoliubov, first and second sound) and the stability of vortices.

When condensates were realized in a mixture of atomic hyperfine states, multi-component condensates were observed. Their phase diagram, the spatial structure, possible metastable configurations and properties like miscibility and immiscibility are examples of new phenomena which have been studied. These examples show that gaseous Bose condensates are a new class of quantum fluids with properties quite different from the quantum liquids ^{3}He and ^{4}He. They provide a testing ground for aspects of many-body theories, which were developed many decades ago but never tested experimentally.

Macroscopic wavefunction

In an ideal gas, Bose condensed atoms all occupy the same single-particle ground-state wavefunction. The many-body ground-state wavefunction is then the product of N identical single-particle ground-state wavefunctions where N is the number of condensed atoms. This single-particle wavefunction is therefore called the condensate wavefunction or macroscopic wavefunction. This picture is valid even when weak interactions are included. The ground state

many-body wavefunction is still, to a very good approximation, a product of N single-particle wavefunctions, which are now obtained from the solution of a non-linear Schrödinger equation, which includes the interaction energy between atoms. The admixture of other configurations into the ground state is called quantum depletion. The theory of weakly interacting Bose gases shows that the fractional quantum depletion is $(8/3\pi^{1/2})\sqrt{na^3}$, typically 1 % or less for the alkali condensates. This means that even for the interacting gases, with 99 % accuracy, all the atoms can be regarded to have the same single-particle wavefunction. This is in contrast to liquid helium in which the quantum depletion is about 90 %.

The phase of the condensate and the coherence properties of its wavefunction are relevant for quantum fluids because the gradient of the phase is proportional to the superfluid velocity. It also determines the properties of atom lasers based on BEC. In superconductors and liquid helium, the existence of coherence and of a macroscopic wavefunction is impressively demonstrated through the Josephson effect. In the dilute atomic gases, the coherence has been demonstrated even more directly by interfering two Bose condensates (Fig. 4).
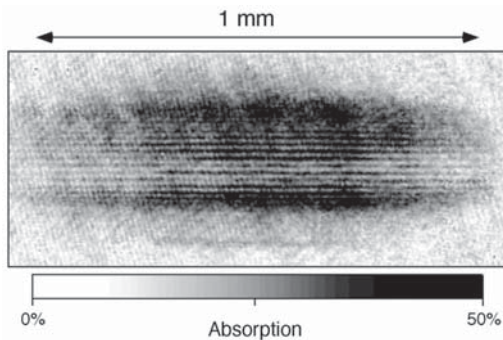

Fig. 4: Interference pattern of two expanding condensates demonstrating the coherence of Bose-Einstein condensates. This absorption image was observed after 40 ms time of flight. The interference fringes have a spacing of 15 μm, a huge length for matter waves. In contrast, the matter wavelength of atoms at room temperature is only 0.5 Å, less than the size of an atom.

For background information see Bose-Einstein statistics, Laser cooling, Particle trap, De Broglie wavelength, Atom laser, Coherence.

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