Risk Measurement

The Relation between the Law of Large Numbers & Risk

The value of Risk R = CV (CV is the coefficient of variation), then:

$R = \frac{\sigma}{\overline{x}} \times 100$ *where:* σ *is the standard deviation (SD) &* $\overline{x}$ *is the average.*

$R_1 = \frac{\sigma_1}{\overline{x_1}} \times 100$	$\overline{x_1}$ is the average of 1 unit	σ_1 is the Standard Deviation of 1 unit
$R_2 = \frac{\sigma_2}{\overline{x_2}} \times 100$	$\overline{x_2} = \overline{x_1} \times 2$	$\sigma_2 = \sigma_1 \times \sqrt{2}$
$R_3 = \frac{\sigma_3}{\overline{x_3}} \times 100$	$\overline{x_3} = \overline{x_1} \times 3$	$\sigma_3 = \sigma_1 \times \sqrt{3}$
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$R_{100} = \frac{\sigma_{100}}{\overline{x_{100}}} \times 100$	$\overline{x_{100}} = \overline{x_1} \times 100$	$\sigma_{100} = \sigma_1 \times \sqrt{100}$
$R_n = \frac{\sigma_n}{\overline{x_n}} \times 100$	$\overline{x_n} = \overline{x_1} \times n$	$\sigma_n = \sigma_1 \times \sqrt{n}$

If $\overline{x_1} = 1000$, $\sigma_1 = 1500$, *then:*

$$R_1 = \frac{\sigma_1}{\overline{x_1}} \times 100 = \frac{1500}{1000} \times 100 = 150\%$$

$$R_2 = \frac{\sigma_2}{\overline{x_2}} \times 100 = \frac{1500 \times \sqrt{2}}{1000 \times 2} \times 100 = \frac{1500 \times 1.4142}{1000 \times 2} \times 100 = \frac{2121.32}{2000} \times 100 = 106\%$$

$$R_3 = \frac{\sigma_3}{\overline{x_3}} \times 100 = \frac{1500 \times \sqrt{3}}{1000 \times 3} \times 100 = \frac{1500 \times 1.732051}{1000 \times 3} \times 100 = \frac{2598.076}{3000} \times 100 = 86.6\%$$

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$$R_{100} = \frac{\sigma_{100}}{\overline{x_{100}}} \times 100 = \frac{1500 \times \sqrt{100}}{1000 \times 100} \times 100 = \frac{1500 \times 10}{1000 \times 100} \times 100 = \frac{15000}{100000} \times 100 = 15\%$$

$$R_{10000} = \frac{\sigma_{10000}}{\overline{x_{10000}}} \times 100 = \frac{1500 \times \sqrt{10000}}{1000 \times 10000} \times 100$$

$$= \frac{1500 \times 100}{1000 \times 10000} \times 100 = \frac{150000}{10000000} \times 100 = 1.5\%$$

Remark:

$$R_2 = \frac{\sigma_1 \times \sqrt{2}}{\bar{x}_1 \times 2} \times 100 = \frac{\sigma_1 \times 1}{\bar{x}_1 \times \sqrt{2}} \times 100 = \frac{1500}{1000} \times \frac{1}{1.4142} \times 100 = 106\%$$

Or: $R_2 = R_1 \times \sqrt{\frac{1}{2}} = 150\% \times 0.7071 = 106\%$

$$R_3 = R_1 \times \sqrt{\frac{1}{3}} = 150\% \times 0.57745 = 86.6\%$$

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$$R_{100} = R_1 \times \sqrt{\frac{1}{100}} = 150\% \times 0.01 = 15\%$$

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$$R_{10000} = \frac{\sigma_{10000}}{\bar{\bar{x}}_{10000}} \times 100 = \frac{1500 \times \sqrt{10000}}{1000 \times 10000} \times 100$$

$$= \frac{1500 \times 100}{1000 \times 10000} \times 100 = \frac{150000}{10000000} \times 100 = 1.5\%$$

Then: $R_n = R_1 \times \sqrt{\frac{1}{n}}$

Also : $R_3 = R_2 \times \sqrt{\frac{2}{3}} = 106\% \times 0.8165 = 86.6\%$

$$R_{100} = R_3 \times \sqrt{\frac{3}{100}} = 86.6\% \times 0.173205 = 15\%$$

Then: $R_{new\,n} = R_{old\,n} \times \sqrt{\frac{old\ n}{new\ n}} =$

$$R_{100} = R_3 \times \sqrt{\frac{3}{100}}$$

& $R_3 = R_{100} \times \sqrt{\frac{100}{3}} = 15\% \times 5.7735 = 86.6\%$

Net or Pure Premium

*If the loss distribution follows the normal distribution, the pure premium **P** will be calculated as follow:*

$$P = \bar{x} + \sigma \times z_{99.9\%}$$

If the number of insured = 1 then:

$$P_1 = \bar{x}_1 + \sigma_1 \times z_{99.9\%}$$

where $z_{99.9\%}$ is the standard value for the confidence degree 99.9% & = 3.09 (we will approximate it to 3 to facilitate the calculation).

$$P_2 = \bar{x}_2 + \sigma_2 \times z_{99.9\%}$$

$$P_3 = \bar{x}_3 + \sigma_3 \times z_{99.9\%}$$

$$P_{100} = \bar{x}_{100} + \sigma_{100} \times z_{99.9\%}$$

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$$P_{10000} = \bar{x}_{10000} + \sigma_{10000} \times z_{99.9\%}$$

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$$P_{1000000} = \bar{x}_{1000000} + \sigma_{1000000} \times z_{99.9\%}$$

Based on the previous value of $\bar{x} = 1000 \; \& \; \sigma = 1500 \text{ then:}$

If n = 1 (i.e. if there is only one insured), then the premium for one insured would be:

$$P_1 = \bar{x}_1 + \sigma_1 \times z_{99.9\%}$$

$$P_1 = 1000 + 1500 \times 3 = 1000 + 4500 = 5500$$

If n = 2 (i.e. if there is only two insureds), then the premium for two insureds would be:

$$P_2 = \bar{x}_2 + \sigma_2 \times z_{99.9\%}$$

$$P_2 = (1000\times 2) + ((1500\times\sqrt{2})\times 3)$$
$$= 2000 + ((1500\times 1.4142)\times 3)$$
$$= 2000 + 6363.96 = 8363.96$$

Then the premium for each insured $= \dfrac{P_2}{2} = \dfrac{8363.96}{2} = 4141.98$

If n = 3 (i.e. if there is only three insureds) then the premium for three insureds would be:

$$P_3 = \bar{x}_3 + \sigma_3 \times z_{99.9\%}$$
$$P_3 = (1000\times 3) + ((1500\times\sqrt{3})\times 3)$$
$$= 3000 + ((1500\times 1.732051)\times 3)$$
$$= 3000 + 7794.23 = 10794.23$$

Then the premium for each insured $= \dfrac{P_3}{3} = \dfrac{10794.2286}{3} = 3598.08$

If n = 100 (i.e. if there is 100 insureds) then the premium for 100 insureds would be:

$$P_{100} = \bar{x}_{100} + \sigma_{100} \times z_{99.9\%}$$
$$P_{100} = (1000\times 100) + ((1500\times\sqrt{100})\times 3)$$
$$= 100000 + ((1500\times 10)\times 3)$$
$$= 100000 + 45000 = 145000$$

Then the premium for each insured $\dfrac{P_{100}}{100} = \dfrac{145000}{100} = 1450$

If n = 10000 (i.e. if there is 10000 insureds), then the premium for 10000 insureds would be:

$$P_{10000} = \bar{x}_{10000} + \sigma_{10000} \times z_{99.9\%}$$
$$P_{10000} = (1000\times 10000) + ((1500\times\sqrt{10000})\times 3)$$
$$= 10000000 + ((1500\times 100)\times 3)$$
$$= 10000000 + 450000 = 10450000$$

Then the premium for each insured $= \dfrac{P_{10000}}{10000} = \dfrac{10450000}{10000} = 1045$

If n = 1000000 (i.e. if there is 1000000 insureds), then the premium for 1000000 insureds would be:

$$P_{1000000} = \bar{x}_{1000000} + \sigma_{1000000} \times z_{99.9\%}$$

$$P_{10000} = (1000 \times 1000000) + (1500 \times \sqrt{1000000}) \times 3$$

$$= 1000000000 + (1500 \times 1000) \times 3$$

$$= 1000000000 + (1500000 \times 3)$$

$$= 1000000000 + 4500000 = 1004500000$$

Then the premium for each insured $= \frac{P_{1000000}}{1000000} = \frac{1004500000}{1000000} = 1004.5$