CONTINUOUS UNIFORM DISTRIBUTION:

Q2. Suppose that the random variable X has the following uniform distribution:

$$f(x) = \begin{cases} 3 & , \quad \frac{2}{3} < x < 1 \\ 0 & , \quad other\ wise \end{cases}$$

1) p(0.33< x <0.5)=

(A) 0.49 (B) 0.51 (C) 0 (D) 3

2) p(x >1.25)=

(A) 0 (B) 1 (C) 0.5 (D) 0.33

(3) The variance of X is

(A) 0.00926 (B) 0.333 (C) 9 (D) 0.6944

HW: Q 1 , 3

EXPONENTIAL DISTRIBUTION

Q2. Suppose that the failure time (in hours) of a certain electrical device is distributed with a probability density function given by:

f(x)= (1/70) $e^{-x/70}$, x > 0 . [Hint: $\int e^{-ax}dx = -(1/a)\, e^{-ax} + c$]

1) the probability that a randomly selected device will fail within the first 50 hours is:

$P(X < 50) = \int_0^{50} \frac{1}{70} e^{-\frac{x}{70}}\, dx = \left[- e^{-\frac{x}{70}}\right]_0^{50} = -e^{-\frac{50}{70}} + e^0 = 0.5105$

(A) 0.4995 (B) 0.7001 (C) 0.5105 (D) 0.2999

2) the probability that a randomly selected device will last more than 150 hours is:

$P(X > 150) = \int_{150}^{\infty} \frac{1}{70} e^{-\frac{x}{70}}\, dx = \left[- e^{-\frac{x}{70}}\right]_{150}^{\infty} = -e^{-\infty} + e^{-\frac{150}{70}} =$

(A) 0.8827 (B) 0.2788 (C) 0.1173 (D) 0.8827

3) the average failure time of the electrical device is: $\mu = \beta$

(A) 1/70 (B) 70 (C) 140 (D) 35

4) the variance of the failure time of the electrical device is: $\sigma^2 = \beta^2$

(A) 4900 (B) 1/49000 (C) 70 (D) 1225

Q3. The lifetime of a specific battery is a random variable X with probability density function given by: f(x)=(1/200) $e^{-x/200}$, x>0

(1) The mean life time of the battery equals to
(A) 200 (B) 1/200 (C) 100 (D) 1/100 (E) Non of these

(2) P(X>100) =

$P(X > 100) = \int_{100}^{\infty} \frac{1}{200} e^{-\frac{x}{200}}\, dx = \left[- e^{-\frac{x}{200}}\right]_{100}^{\infty} = -e^{-\infty} + e^{-\frac{100}{200}} = e^{-\frac{1}{2}}$

(A) 0.5 (B) 0.6065 (C) 0.3945 (D) 0.3679 (E)0.6321

(3) P(X=200) =

(A) 0.5 (B) 0.0 (C) 0.3945 (D) 0.3679 (E) 1.0

HW: Q 1 , 6 ; Deleted: Q4

NORMAL DISTRIBUTION:

Q1. **(A)** Suppose that Z is distributed according to the standard normal distribution.

1) the area under the curve to the left of z =1.43 is: $p(Z < 1.43)$

(A) 0.0764 (B) 0.9236 (C) 0 (D) 0.8133

2) the area under the curve to the left of z = 1.39 is: $p(Z < 1.39)$

(A) 0.7268 (B) 0.9177 (C) 0.2732 (D) 0.0832

3) the area under the curve to the right of z = −0.89 is:

$p(Z > -0.89) = p(Z < 0.89) = 1 - p(Z < -0.89) = 1 - 0.1867$

(A) 0. 7815 (B) 0.8133 (C) 0.1867 (D) 0.0154

4) the area under the curve between z = −2.16 and z = −0.65 is:

$p(-2.16 < Z < -0.65) = p(Z < -0.65) - p(Z < -2.16) = 0.2578 - 0.0154$

(A) 0.7576 (B) 0.8665 (C) 0.0154 (D) 0.2424

(B) Suppose that Z is distributed according to the standard normal distribution. Find:

1) $P(Z < -3.9) = 0$

2) $P(Z > 4.5) = 0$ >> $P(Z > 4.5) = P(Z < -4.5) = 1 - P(Z > 4.5) = 1 - 1 = 0$

3) $P(Z < 3.7) = 1$

4) $P(Z > -4.1) = 1$ >> $P(Z > -4.1) = P(Z < 4.1) = 1$

Q5. If the random variable X has a normal distribution with the mean μ and the variance σ^2, then $P(X<\mu+2\sigma)$ equals to

$$P(X < \mu + 2\sigma) = p\left(Z < \frac{\mu + 2\sigma - \mu}{\sigma}\right) = p(Z < 2) =$$

(A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q11. If the random variable X has a normal distribution with the mean 10 and the variance 36, then :

2. The probability that the value of X is greater than 16 is

$$P(X > 16) = P\left(\frac{x-\mu}{\sigma} > \frac{16-10}{6}\right) = P(Z > 1) = P(Z < -1) = 0.1587$$

(A) 0.9587 (B) 0.1587 (C) 0.7587 (D) 0.0587 (E) 0.5587

Q12. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is

$$P(X < 60) = P\left(\frac{x-\mu}{\sigma} < \frac{60-65}{4}\right) = P(Z > -1.25) = 0.1056 \rightarrow 0.1056 \ * 100\%$$

(A) 20.56% (B) 90.56% (C) 50.56% (D) 10.56% (E)40.56%

Q13. The average rainfall in a certain city for the month of March is 9.22 centimeters. Assuming a normal distribution with a standard deviation of 2.83 centimeters, then the probability that next March, this city will receive:

(1) less than 11.84 centimeters of rain is:

$$P(X < 11.84) = P\left(\frac{x-\mu}{\sigma} < \frac{11.84-9.22}{2.83}\right) = P(Z < 0.93) =$$

(A) 0.8238 (B) 0.1762 (C) 0.5 (D) 0.2018

(2) more than 5 centimeters but less than 7 centimeters of rain is:

$$P(5 < X < 7) = P\left(\frac{5-9.22}{2.83} < \frac{x-\mu}{\sigma} < \frac{7-9.22}{2.83}\right) = P(-1.49 < Z < -0.78)$$

$$= p(Z < -0.78) - P(Z < -1.49) = 0.2177 - 0.0681 =$$

(A) 0.8504 (B) 0.1496 (C) 0.6502 (D) 0.34221

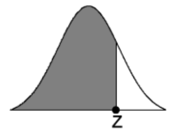
(3) more than 13.8 centimeters of rain is:

$$P(X > 13.8) = P\left(\frac{x-\mu}{\sigma} > \frac{13.8-9.22}{2.83}\right) = P(Z > 1.62) = P(Z < -1.62) =$$

(A) 0.0526 (B) 0.9474 (C) 0.3101 (D) 0.4053

HW: Q 2 , 4 ,6 ,7 ; Deleted:(Q1-5), Q 3 ,(Q4-2), Q5 ,Q9, Q10, (Q11-1)

Areas under the Standard Normal Curve



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

solution of some HW

CONTINUOUS UNIFORM DISTRIBUTION:

Q1. If the random variable X has a uniform distribution on the interval (0,10), then

1. P(X<6) equals to

   (A) 0.4 (B) 0.6 (C) 0.8
   (D) 0.2 (E) 0.1

2. The mean of X is

   (A) 5 (B) 10 (C) 2
   (D) 8 (E) 6

3. The variance X is

   (A) 33.33 (B) 28.33 (C) 8.33 (D) 25
   (E) None

1) $\int_0^6 \frac{1}{10} dx = 0.6$

2) $\int_0^{10} \frac{x}{10} dx = 5$

3) $\left(\int_0^{10} \frac{x^2}{10} dx - \left(\int_0^{10} \frac{x}{10} dx\right)^2\right) = 8.33$

Q3. Suppose that the continuous random variable X has the following probability density function (pdf): $f(x)=0.2$ for $0<x<5$. Then

(1) $P(X>1)$ equals to

(A) 0.4 (B) 0.2 (C) 0.1 (D) 0.8

(2) $P(X\geq 1)$ equals to

(A) 0.05 (B) 0.8 (C) 0.15 (D) 0.4

(3) The mean $\mu=E(X)$ equals to

(A) 2.0 (B) 2.5 (C) 3.0 (D) 3.5

(4) $E(X^2)$ equals to

(A) 8.3333 (B) 7.3333 (C) 9.3333
(D) 6.3333

(5) Var(X) equals to

(A) 8.3333 (B) 69.444 (C) 5.8333
(D) 2.0833

(6) If F(x) is the cumulative distribution function (CDF) of X, then F(1) equals to

(A) 0.75 (B) 0.25 (C) 0.8 (D) 0.2

1 & 2) $\int_1^5 0.2\,dx = 0.8$

3) $\int_0^5 0.2 * x\,dx = 2.5$

4) $\int_0^5 0.2 * x^2\,dx = 8.33$

5) $\int_0^5 0.2 * x^2\,dx - \left(\int_0^5 0.2 * x\,dx\right)^2 = 2.0833$

6) F(1)=0.2

EXPONENTIAL DISTRIBUTION

Q1. If the random variable X has an exponential distribution with the mean 4, then:

1. P(X<8) equals to

   (A) 0.2647 (B) 0.4647 (C) 0.8647
   (D) 0.6647 (E) 0.0647

2. The variance of X is

   (A) 4 (B) 16 (C) 2
   (D) 1/4 (E) 1/2

$*f(x) = \frac{1}{4}e^{-\frac{x}{4}};\ \ x > 0$

1) $\int_0^8 \frac{1}{4}\text{Exp}[-\frac{1}{4}x]\,dx = 0.8647$

2) V(X)=16

Q3. Assume the length (in minutes) of a particular type of a telephone conversation is a random variable with a probability density function of the form:

$$f(x) = \begin{cases} 0.2\ e^{-0.2x} & ;\ x \geq 0 \\ 0 & ;\ elsewhere \end{cases}$$

1. $P(3 < x < 10)$ is:

   (a) 0.587 (b) -0.413 (c) 0.413 (d) 0.758

1) $\int_3^{10} 0.2 * \text{Exp}[-0.2 * x]\, dx = 0.413$

Q4. The length of time for one customer to be served at a bank is a random variable X that follows the exponential distribution with a mean of 4 minutes.

(1) The probability that a customer will be served in less than 2 minutes is:

(A) 0.9534 (B) 0.2123 (C) 0.6065
(D) 0.3935

(2) The probability that a customer will be served in more than 4 minutes is:

(A) 0.6321 (B) 0.3679 (C) 0.4905
(D) 0.0012

(3) The probability that a customer will be served in more than 2 but less than 5 minutes is:

(A) 0.6799 (B) 0.32 (C) 0.4018
(D) 0.5523

(4) The variance of service time at this bank is

(A) 2 (B) 4 (C) 8
(D) 16

$*f(x) = \frac{1}{4}e^{-\frac{x}{4}};\ \ x > 0$

1) $\int_0^2 \frac{1}{4}\text{Exp}[-\frac{1}{4}x]\,dx = 0.3935$

2) $\int_4^\infty \frac{1}{4}\text{Exp}[-\frac{1}{4}x]\,dx = 0.3679$

3)

$$P(2 \le X \le 5) = F(5) - F(2) = \left(1 - e^{-\frac{5}{4}}\right) - \left(1 - e^{-\frac{2}{4}}\right)$$

$$= e^{-\frac{2}{4}} - e^{-\frac{5}{4}} = 0.32$$

NORMAL DISTRIBUTION

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

1) the proportion of rings that will have inside diameter less than 12.05 centimeters is:

   (A) 0.0475 (B) 0.9525 (C) 0.7257
   (D) 0.8413

2) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:

   (A) 0.0475 (B) 0.8413 (C) 0.1587
   (D) 0.4514

3) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:

   (A) 0.905 (B) –0.905 (C) 0.4514 (D) 0.7257

$$X \sim N(12, 0.03)$$

1) $P(X < 12.05) = P\left(\frac{X-\mu}{\sigma} < \frac{12.05-12}{0.03}\right) = P(Z < 1.666) = P(Z < 1.67) = 0.9525$

2) $P(X > 11.97) = P\left(\frac{X-\mu}{\sigma} > \frac{11.97-12}{0.03}\right) = P(Z > -1)$

$$= 1 - P(Z < -1) = 0.8413$$

3) $P(11.95 < X < 12.05) = P(-1.67 < Z < 1.67)$

$$= P(Z < 1.67) - P(Z < -1.67) = 0.905$$

Q3. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

(1) The percentage of fat persons with weights at most 110 kg is P(X<110)=P(Z<-2)

(A) 0.09 % (B) 90.3 % (C) 99.82% (D) 2.28%

(2) The percentage of fat persons with weights more than 149 kg is P(X>149)=P(Z>2.33)

(A) 0.09% (B) 0.99% (C) 9.7% (D) 99.82%