King Saud University
College of Science
Department of Mathematics

First Midterm Exam

Monday, July 9, 2018	Math 106	Academic year 1438-39H
7:00 - 8:30 pm	**Integral Calculus**	Summer Semester

Student's Name	
ID number	
Section No.	
Classroom No.	
Teacher's Name	
Roll Number	

25

Instructions to Candidates

1. You may use both sides of the paper.
2. Please write your instructor name, student ID No. and section number.
3. Simple calculators are allowed.

Question	Total	Score
1	4	
2	3	
3	4	
4	5	
5	9	
Total	**25**	

Question 1 (4 Marks: 2 + 2)

a. Determine the function $f(x)$ given that $f'(x) = 8x^3 + 6x^2 + 4x + 5$, and $f(1) = 14$.

$f(x) = \int f'(x)\,dx = \int 8x^3 + 6x^2 + 4x + 5\,dx$ 1/2

$f(x) = 2x^4 + 2x^3 + 2x^2 + 5x + c$ 1/2

$f(1)=14$ $\quad 14 = 2+2+2+5+c \Rightarrow c = 3$ 1/2

$\therefore\ f(x) = 2x^4 + 2x^3 + 2x^2 + 5x + 3$ 1/2

b. Find the value of the constant c such that $\sum_{k=1}^{6}\left(k^2 + 2k + 7c\right) = 217$.

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{k=1}^{n} c = cn$

1

$\frac{6(7)(13)}{\cancel{6}} + \cancel{2}\,\frac{6(7)}{\cancel{2}} + 7c(6) = 217$ 1/2

$91 + 42 + 42c = 217$

$42c = 84$

$c = 2$ 1/2

Question 2 (3 Marks: 2 + 1)

Let R be the region under the graph of $f(x) = 2x + 3$ on the interval $[0,4]$. Let P be the regular partition of $[0,4]$ into n subintervals.

a. Find the Riemann sum if $c_i = \frac{4i}{n}$, $i = 1, 2, \cdots, n$.

1/2

Riemann Sum $= \sum_{i=1}^{n} f(c_i)\,\Delta x_i$, $\Delta x_i = \frac{b-a}{n} = \frac{4}{n}$

$= \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\frac{4}{n}$ 1/2

$= \frac{4}{n}\sum_{i=1}^{n}\left(2\left(\frac{4i}{n}\right)+3\right)$

$= \frac{4}{n}\sum_{i=1}^{n}\left(\frac{8i}{n}+3\right)$ 1/2

$= \frac{4}{n}\left[\frac{8}{\cancel{n}}\,\frac{\cancel{n}(n+1)}{2} + 3n\right]$

$= \frac{16(n+1)}{n} + 12$ 1/2

b. Compute the area of the region R by using Riemann sum in **part (a)**.

$$\text{Area} = \lim_{n\to\infty}\left[\frac{16(n+1)}{n} + 12\right] \quad \tfrac{1}{2}$$

$$= \lim_{n\to\infty}\left[16 + \frac{16}{n} + 12\right] = 28 \quad \tfrac{1}{2}$$

Question 3 **(4 Marks: 2 + 2)**

a. Find the value of c that satisfies the **Integral Mean Value Theorem** for the function $f(x) = 3x^2 - 2x + 3$ on $[-1, 3]$.

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx \quad \tfrac{1}{2}$$

$$f(c) = \frac{1}{4}\int_{-1}^{3}(3x^2 - 2x + 3)\,dx$$

$$3c^2 - 2c + 3 = \frac{1}{4}\left[x^3 - x^2 + 3x\right]_{-1}^{3} \quad \tfrac{1}{2}$$

$$3c^2 - 2c + 3 = \frac{1}{4}(27 - -5) \quad \tfrac{1}{2}$$

$$3c^2 - 2c + 3 = 8 \longrightarrow 3c^2 - 2c - 5 = 0$$

$$\rightarrow \boxed{c = -1}\,,\ \boxed{c = 5/3}$$

$(-1,3) \not\ni$ $\quad \in (1,3)$

$\tfrac{1}{2}$

b. If $F(x) = \displaystyle\int_{\cos x}^{1+\sin x} \sqrt{2+t}\,dt$, then find $F'(0)$.

$$F'(x) = \sqrt{2 + 1 + \sin x}\,\cos x - \sqrt{2 + \cos x}\,(-\sin x) \quad 1$$

$$F'(0) = \sqrt{3} - 0 = \sqrt{3} \quad 1$$

Question 4

(5 Marks: 1 + 2 + 2)

Differentiate the following functions:

1. $y = 2^{\pi + \sin x}$

$y' = \cos x \cdot 2^{\pi + \sin x} \ln 2$ 1

2. $y = (1 + x^2)^{\tan x}$

$\ln y = \tan x \ln(1 + x^2)$ 1

$\frac{y'}{y} = \tan x \frac{2x}{1+x^2} + \sec^2 x \ln(1+x^2).$ 1/2

$y' = \left[\frac{2x \tan x}{1+x^2} + \sec^2 x \ln(1+x^2)\right](1+x^2)^{\tan x}$ 1/2

3. $y = \ln\left(\cosh^{-1} x\right)$

$y' = \frac{\frac{1}{\sqrt{x^2-1}}}{\cosh^{-1} x} = \frac{1}{\sqrt{x^2-1}\,\cosh^{-1} x}$ 1 1

Question 5

(9 Marks: 1 + 2 + 2 + 2+2)

Evaluate the following integrals:

1. $\int \left(x^{2/3} - 4x^{-1/5} + 4\right) dx$ $= \frac{3}{5} x^{5/3} - 4 \frac{5}{4} x^{4/5} + 4x + c$

$= \frac{3}{5} x^{5/3} - 5 x^{4/5} + 4x + c.$ 1

2. $\int_2^4 \frac{1}{9-2x}\,dx$

Let $u = 9-2x \rightarrow du = -2\,dx \rightarrow \frac{du}{-2} = dx$

1

$x=2 \rightarrow u=5$

$\cancel{x}=4 \rightarrow u=1$

$\therefore \int_5^1 \frac{1}{u}\,\frac{du}{-2} = -\frac{1}{2}\left(\ln|u|\Big|_5^1\right)$ 1

$= -\frac{1}{2}\left[\cancel{\ln 1}^{0} - \ln 5\right] = \frac{1}{2}\ln 5.$

3. $\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$

Let $u = \sqrt{y} \rightarrow du = \frac{1}{2\sqrt{y}}\,dy \rightarrow 2du = \frac{dy}{\sqrt{y}}$ 1

$\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy = \int 2e^{u}\,du = 2e^{u} + c$

1

$= 2e^{\sqrt{y}} + c.$

4. $\int 3^{\cos x^2} x \sin x^2\,dx$

Let $u = \cos x^2 \longrightarrow du = -2x\sin(x^2)\,dx$

$\rightarrow \frac{du}{-2} = x\sin(x^2)\,dx.$ 1

$I = \int 3^{u}\,\frac{du}{-2} = -\frac{1}{2}\,\frac{3^{u}}{\ln 3} + c$

$= \frac{-1}{2\ln 3}\,3^{\cos(x^2)} + c$ 1

5. $\displaystyle\int \frac{\sec(x+2)\tan(x+2)}{4+\sec^2(x+2)}\,dx$

~~Let u = 4 + sec(x+2)~~

$u = \sec(x+2) \rightarrow du = \sec(x+2)\tan(x+2)\,dx$ 1

$\therefore I = \displaystyle\int \frac{du}{4+u^2} = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + C$

$= \frac{1}{2}\tan^{-1}\left(\frac{\sec(x+2)}{2}\right) + C$ 1

GOOD LUCK