You are given

(i) $p_x = 0.90$, for $x = 60, 61, 62$

(ii) $i = 0.06$

Find $A^{\,1}_{60:\overline{3}|}$

solution

$$A^{\,1}_{60:\overline{3}|} = v\, q_{60} + v^2 p_{60} q_{61} + v^3 p_{60} p_{61} q_{62}$$

$$= \frac{0.1}{1.06} + \frac{0.90(0.1)}{(1.06)^2} + \frac{(0.9)(0.9)(0.1)}{(1.06)^3}$$

$$= 0.246529013883944$$

You are given

i) $\mu_x = \mu$ for all x

ii) The force of interest is constant

(iii) $\mathring{e}_x = 25$

(iv) $\bar{A}_x = 0.4$

Find ${}_{10|}\bar{A}_x$

as we have $\mu_x = \mu, \forall x$, then $\mathring{e}_x = \int_0^\infty {}_tp_x \, dt$

$$= \int_0^\infty e^{-\mu t} dt$$

$$= \frac{1}{\mu} = 25$$

$$\Rightarrow \mu = \frac{1}{25} = 0.04$$

$$\boxed{\text{Note} \quad \mathring{e}_{x:\overline{n}|} = \int_0^n {}_tp_x\, dt = \frac{1 - e^{-\mu n}}{\mu}}$$

and we have

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = \frac{0.04}{0.04 + \delta} = 0.4$$

$$\Rightarrow \delta = 0.06$$

we have $\,_{10|}\bar{A}_x = v^{10}\,_{10}p_x\,\bar{A}_{x+10}$ where $\,_{10}E_x = v^{10}\,_{10}p_x = e^{-\delta(10)}e^{-\mu 10}$

(2) $= e^{-0.4}e^{-0.6}$

$= 0.3678$

$\bar{A}_{x+10} = 0.4$

$= 0.3678\,(0.4)$

$= 0.1472$

and we have $\bar{A}_{x:\overline{10|}} = \bar{A}^{1}_{x:\overline{10|}} + \,_{10}E_x$

then $\bar{A}^{1}_{x:10} = \bar{A}_x - \,_{10|}\bar{A}_x = 0.4 - 0.1472$
$= 0.2528$

$\Rightarrow \bar{A}_{x:10} = 0.2520 + 0.3678$
$= 0.62073$.

For a special deferred term insurance on (40) with death benefit payable at the end of the year of death, you are given:

1) the death benefit is 0 in year 1-10; 1000 in years 11-20; 2000 in years 21-30, 0 otherwise.

(II) Mortality follows the ILT (Illustrative life Table)

(III) $i = 0.06$

(IV) the random variable Z is the present value, at age 40 of the death benefits.

a/ Write an expression for Z in terms of K_{40}, the curtate time until death random variable.

Solution

- the benefit is 0, if (40) dies in the first 10 years & $[K_{40} < 10]$
- " " is 1000, if (40) dies in year 11 to 20 $[10 \leq K_{40} < 20]$
- " " is 2000; if (40) dies in year 21 to 30 $[20 \leq K_{40} < 30]$
- " " , 0, if (40) dies after year 30 $[K_{40} \geq 30]$

$$\Rightarrow \quad z = \begin{cases} 0 & \text{if } \quad K_{40} < 10 \quad \text{or} \quad K_{40} \geq 30 \\ \dfrac{1000}{(1.06)^{K_{40}+1}} & \text{if } \quad 10 \leq K_{40} < 20 \\ \dfrac{2000}{(1.06)^{K_{40}+1}} & \text{if } \quad 20 \leq K_{40} < 30 \end{cases}$$

b/ Calculate $\Pr(z=0)$

$$\Pr(z=0) = \Pr(K_{40} < 10) + \underbrace{\Pr(K_{40} \geq 30)}_{\boxed{1-\Pr(K_{40}<30)}}$$

$$= \frac{l_{40} - l_{50}}{l_{40}} + \frac{l_{70}}{l_{40}}$$

$$\simeq 0.75$$

c/ Calculate $\Pr(z > 400)$

we have only $\underline{2}$ possibly $10 \leq K_{40} < 20$ or

$20 \leq K_{40} < 30$

- Fist case $10 \leq K_{40} < 20$

$$z > 400 \Leftrightarrow \frac{1000}{(1.06)^{K_{40}+1}} > 400$$

$$\Leftrightarrow \quad K_{40} \leq 14$$

then we need look on the case

$$10 \leq K_{40} \leq 14$$

- 2" case $20 \leq K_{40} < 30$

$$z > 400 \Leftrightarrow \frac{2000}{(1.06)^{K_{40}+1}} > 400 \Leftrightarrow K_{40} \leq 26$$

$(K_{40} < 26{,}62)$

$$\Rightarrow \Pr(Z > 400) = \Pr(10 \leq K_{40} \leq 14) + \Pr(20 \leq K_{40} < 26)$$

$$= \frac{l_{50} - l_{55}}{l_{40}} + \frac{l_{60} - l_{67}}{l_{40}}$$

$$\simeq 0.1392.$$

You are given

i) $i = 0.1$

ii) $q_x = 0.04$, $q_{x+1} = 0.08$

iii) Deaths are UD over each year of age

Calculate $A^{1\,(12)}_{x:\overline{2}|}$

$$A^{1\,(12)}_{x:\overline{2}|} = \frac{i}{i^{(12)}} A^{1}_{x:\overline{2}|} \quad \text{under (UDD)}$$

$$A^{1}_{x:\overline{2}|} = v q_x + v^2 p_x q_{x+1} = \frac{0.04}{1.1} + \frac{0.96(0.08)}{1.1^2}$$

$$= 0.0998347$$

and $i^{(12)} = 12(1.1^{\frac{1}{12}} - 1) = 0.09568969$

$$\Rightarrow A^{1\,(12)}_{x:\overline{2}|} = 0.10433.$$

You are given an excerpt from a select and ultimate life:

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	$l_{[x]+4}$	$x+4$
40	100 000	99899	99724	99520	99288	44
41	99802	99689	99502	99283	99033	45
42	99597	99471	99268	99030	98752	46
43	99365	99225	99007	98747	98435	47
44	99120	98964	98796	98429	98067	48

Assuming an interest rate per year 8%.

calculate:

$$\textcircled{3}\quad v\, p_{[40]+1}\left(v\, p_{[40]+2}\right)$$

a/

$$A_{[40]+1:\overline{2}|} = \sum_{k=0}^{1} v^{k+1}\, {}_{k|}q_{[40]+1} + v^2\, {}_2p_{[40]+1}$$

$$= v\, {}_{0|}q_{[40]+1} + v^2\, {}_{1|}q_{[40]+1} + v^2\, {}_2p_{[40]+1}$$

$$= v\, d_{[40]+1} + v^2\, d_{[40]+2} + v^2\, l_{[40]+3}$$

$$\frac{\frac{175}{1.08} + \frac{204}{(1.08)^2} + \frac{99520}{(1.08)^2}}{99899} = \frac{v\, d_{[40]+1} + v^2\, d_{[40]+2} + v^2\, l_{[40]+3}}{l_{[40]+1}}$$

$$d_{[40]+1} = l_{[40]+1} - l_{[40]+2} = 175$$

$$d_{(40)+2} = l_{(40)+2} - l_{(40)+3} = 204$$

$$\Rightarrow \quad A_{[40]+1:\overline{2}|} = 0,85745896$$

b/ the standard deviation of the present value of a two-year term insurance, deferred one year, issued to newly selected life aged 40, with sum insured 100000 SR, payable at the end of the year of death

$$E(Z) = 100\,000 \sum_{k=1}^{2} v^{(k+1)}\, {}_{k|}q_{[40]}$$

$$= 100000\left[v^2\, {}_{1|}q_{40} + v^3\, {}_{2|}q_{40}\right]$$

$$= 100000\left[\frac{v^2\, d_{(40)+1} + v^3\, d_{(40)+2}}{l_{[40]}}\right]$$

$$= 100000\left[\frac{175/(1.08)^2 + 204/(1.08)^3}{100\,000}\right] = 311.976070720$$

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$$E(Z^2) = 100\,000^2 \sum_{k=1}^{2} (v^{k+1})^2 \; {}_{k|}q_{40}$$

$$= 100\,000^2 \; \frac{v^2 \, {}_{1|}q[40] + v^4 \, {}_{2|}q[40]}{l(40)}$$

$$= 100000\left[\frac{125}{(1.08)^2} + \frac{204}{(1.08)^4}\right]$$

$$= 29998038.35232886$$

$$\sigma = \sqrt{E(Z^2) - (E(Z))^2} = 5468.1541020$$

For a whole life insurance of 10 000 on (x) with benefit payable at the moment of death;

(i) $\mu_{x+t} = \begin{cases} 0,05 & ; \; 0 \leq t \leq 20 \\ 0.07 & ; \quad t > 20 \end{cases}$

(ii) $\delta_t = \begin{cases} 0.03 & ; \; 0 \leq t \leq 20 \\ 0.06 & ; \quad t > 20 \end{cases}$

Calculate the single benefit premium for this insurance

$$\bar{A}_x = \bar{A}^{1}_{x:\overline{20}|} + {}_{20|}\bar{A}_x = \bar{A}^{1}_{x:\overline{20}|} + v^{20} \, {}_{20}p_x \, \bar{A}_{x+20}$$

if $t > 20 \Rightarrow \mu_{x+t} = 0,07, \; \delta_t = 0,06$

$$\Rightarrow \bar{A}_{x+20} = \frac{0.07}{0.06+0.07} = 7/13$$

if $0 \leq t \leq 20, \; \mu_{x+t} = 0,05, \; \delta_t = 0,03$

$$v^{20} \, {}_{20}p_x = e^{-0,03(20)} \, e^{-0,05(20)} = e^{-0,8(2)} = 0.2018$$

$$\Rightarrow \bar{A}^{1}_{x:\overline{20|}} = \int_0^{20} e^{-0.03t} e^{-0.05t} \, 0.05 \, dt$$

$$= \frac{0.05}{0.08}\left(1 - e^{-0.08(20)}\right) = 0.49881467$$

$$\Rightarrow 10000 \bar{A}_x = 10000\left(0.49881467 + 0.2018 \left(\tfrac{7}{13}\right)\right)$$

$$= 6074.762084$$

For a continuous whole life annuity of 1 on (x), you are given:

(i) T_x is the future lifetime random variable for (x)

(ii) the force of interest and force of mortality are equal and constant.

(iii) $\bar{a}_x = 12.50$

Calculate the standard deviation of $\bar{a}_{\overline{T_x|}}$

$Y = \bar{a}_{\overline{T_x|}}$ we have $\mu = \delta \Rightarrow \bar{a}_x = \frac{1}{\mu+\delta} = \frac{1}{2\delta} = 12.5$

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = \frac{\mu}{2\mu} = 1/2 \quad \Rightarrow \mu = \delta = 0.04$$

$${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{\mu}{3\mu} = 1/3 \Rightarrow \operatorname{Var}(Y) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2}$$

$$\sigma = \sqrt{\operatorname{Var}(Y)} = 7.217 \qquad = 52.0833$$

2 method

or we can use $\quad {}^2\bar{a}_x = \frac{1}{\mu+2\delta} = \frac{1}{3\mu} = 8.3333$

$$\operatorname{Var}(Y) = 2/\delta\left(\bar{a}_x - {}^2\bar{a}_x\right) - (\bar{a}_x)^2 = 52.0833 \Rightarrow \sigma = \sqrt{\operatorname{Var}(Y)} = 7.217$$