1)

You are given:

i) $q_{x+1} = 0.1$

(ii) $p_{x+2} = 0.985$

(iii) ${}_3p_{x+2} = 0.95$

(iv) $q_{x+4} = 0.02$

Calculate:

a/ ${}_2p_{x+1} = p_{x+1} \; p_{x+2} = 0.99(0.985)$
$= 0.97515$

b/ $p_{x+4} = 1 - q_{x+4} = 0.98$

c/ ${}_2p_{x+2} = \dfrac{{}_3p_{x+2}}{p_{x+4}} = \dfrac{0.95}{0.98} \simeq 0.96939$

d/ ${}_3p_{x+1} = p_{x+1} \; {}_2p_{x+2} = 0.99(0.96939) = 0.959696$

e/ ${}_{1|2}q_{x+1} = p_{x+1} \; {}_2q_{x+2} = 0.0303$

2/ We call K_x the curtate future lifetime random variable.

You are given:

$q_{x+k} = 0.1(k+1)$,

$k = 0, 1, 2, \ldots, 9$

Calculate

a/ $Pr(K_x = 1) = {}_{1|}q_x = p_x \, q_{x+1} = (1-q_x)\, q_{x+1} = 0.9(0.2)$
$\boxed{= 0.18}$

b/ $Pr(K_x \leq 2) = Pr(K_x = 2) + Pr(K_x = 1) + Pr(K_x = 0)$

$$= q_x + {}_{1|}q_x + {}_{2|}q_x = 0.1 + p_x\, p_{x+1}\, q_{x+2} + p_x\, q_{x+1}$$

$$= 0.1 + 0.216 + 0.18$$

$$= 0.496$$

3°/ For mortality of certain population follow the De Moivre's law

$$\mu_x = \frac{1}{\omega - x}, \quad x \leq \omega$$

a°/ Show that the survival function for age-at-death random variable T_0 is:

$$S_0(x) = 1 - \frac{x}{\omega}, \quad 0 \leq x \leq \omega$$

b°/ Verify that the function in (a) is a valid survival function

c°/ Show that

$$_tp_x = 1 - \frac{t}{\omega - x}, \quad 0 \leq t \leq \omega - x, \quad x \leq \omega$$

Sln

a°/ $0 \leq x \leq \omega$

$$S_0(x) = \exp\left(-\int_0^x \mu_s\, ds\right) = e^{-\int_0^x \frac{1}{\omega - s} ds} = e^{\ln\left(1 - \frac{x}{\omega}\right)} = 1 - \frac{x}{\omega}.$$

b°/

(i) $S_0(0) = 1 - 0/\omega = 1$, (ii) $S_0(\omega) = 1 - \omega/\omega = 0$, $S_0'(x) = -1/\omega < 0$ $\forall\, 0 \leq x < \omega$

$S_0(x)$ is non-increasing

$\Rightarrow S_0$ is survival function

c°/

$$_tp_x = \frac{S_0(x+t)}{S_0(x)} = \frac{\omega - x - t}{\omega - x} = 1 - \frac{t}{\omega - x}, \quad 0 \leq t \leq \omega - x, \quad x < \omega$$

4% For a population which contains equal number of males and females at birth: ③

1) For males, $\mu_x^m = 1.01$, $\forall x \geq 0$

ii) For femals, $\mu_x^f = 0,06$, $\forall x \geq 0$

Calculate P_{20} for this population,

Solution

We have $S_0^m(t) = e^{-\int_0^t \mu_s^m ds} = e^{-\int_0^t 1.01 ds} = e^{-1.01 t}$

$S_0^f(t) = e^{-\int_0^t \mu_s^f ds} = e^{-0.06 t}$

$$\Rightarrow S_0(60) = \frac{e^{-1.01(60)} + e^{-0.06(60)}}{2} = 0.0137$$

$$S_0(61) = \frac{e^{-1.01(61)} + e^{-0,06(61)}}{2} = 0.0129$$

$$\Rightarrow P_{20} = \frac{S_0(61)}{S_0(60)} = \frac{0,0129}{0.0137} = 0.9416$$

5) you are given

$$\mu_x = \begin{cases} 0.07 & 50 \leq x \leq 60 \\ 0.03 & 60 \leq x \leq 70 \end{cases}$$

Calculate ${}_{4|14}q_{50}$

$${}_{4|14}q_{50} = {}_4P_{50} - {}_{18}P_{50} = e^{-0.07(4)} - ({}_{10}P_{50}\ {}_8P_{60})$$

$$= e^{-0.07(4)} - e^{-0.07(10)}\, e^{-0.03(8)}$$

$$= 0.3632$$

7/ You are given:

$$f_0(t) = \frac{20-t}{200} \qquad 0 \leq t \leq 20$$

Find $\mathring{e}_5$

$$\mathring{e}_5 = \int_0^{15} {}_tp_x\,dt = \int_0^{15} \frac{s_0(t+5)}{s_0(5)}\,dt = \int_0^{15} \left(1-\frac{t}{15}\right)^2 dt$$

$0 \leq t \leq 15$

where $S_0(t) = \int_t^{20} f_0(u)\,du = \frac{(20-t)^2}{400}$.

$$\Rightarrow \mathring{e}_5 = -\frac{15}{3}\left[\left(1-t/15\right)^3\right]_0^{15} = 5.$$

8) you are given

$$\mu_x = 0{,}02, \qquad x \geq 0$$

$$\mathring{e}_{20:\overline{20|}} = \int_0^{20} e^{-0{,}02t}\,dt = 16.484.$$

$$= \left[-\frac{e^{-0{,}02t}}{0{,}02}\right]_0^{20}$$

9/ you are given the following life table:

x	l_x	d_x	p_x
0		50	
1			0.98
2	890		

Find ${}_2p_0$

we have: ${}_2p_0 = p_0\,p_1$; $p_1 = 0.98$

$= p_0(0.98)$; $l_2 = 890$

$$l_1 = \frac{l_2}{p_1} = \frac{890}{0,98} = 908.1633$$

$$p_1 = \frac{l_2}{l_1} = \frac{890}{908.1683} = \underline{\underline{0,98}}$$

$$d_0 = l_0 - l_1 \Rightarrow l_0 = d_0 + l_1 = 50 + 908.1633$$

$$= 958.1630$$

$$p_0 = \frac{l_1}{l_0} = \frac{908.1633}{958.1633} = 0,9478$$

$$\Rightarrow {}_2p_0 = p_0 p_1 = 0.9478(0,98) = 0,9288$$