1) You are given ⓵

1) $P_x = 0{,}95, \quad x = 50, 51, 52$

ii) $i = 0{,}05$

Find $A^{1}_{50:\overline{3|}}$

$$A^{1}_{50:\overline{3|}} = v q_{50} + v^2 p_{50} q_{51} + v^3 p_{50} p_{51} q_{52}$$

$$= \frac{0{,}05}{1.05} + \frac{0{,}95(0.05)}{1.05^2} + \frac{0{,}95(0{,}95)(0{,}05)}{1.05^3}$$

$$= 0{,}1297$$

2) You are given

(i) $(DA)^{1}_{40:\overline{10|}} = 5{,}8$

(ii) $p_{40} = 0{,}9$

(iii) $i = 0{,}05$

Find $(DA)^{1}_{41:\overline{9|}}$

$$(DA)^{1}_{40:\overline{10|}} = 10 v q_{40} + v p_{40} (DA)^{1}_{41:\overline{9|}}$$

$$5{,}8 = \frac{10}{1{,}05} 0{,}1 + \frac{1}{1{,}05}(0{,}9)(DA)^{1}_{41:\overline{9|}}$$

$$(DA)^{1}_{41:\overline{9|}} = 5.6556$$

3/ For a continuous increasing whole life insurance on (x),

you are given (2)

(i) The force of mortality is constant

(ii) $\delta = 0,06$

(iii) ${}^2\bar{A}_x = 0.25$

calculate $(\bar{I}\bar{A})_x$

we have $\mu_x = \mu$, $\forall x$

$$\bar{A}_x = \frac{\mu}{\mu+\delta}, \quad {}^2\bar{A}_x = \frac{\mu}{\mu+2\delta}$$

$$0,25 = \frac{\mu}{\mu+2\delta} = \frac{\mu}{\mu+2(0,06)} \Rightarrow \mu = 0,04$$

We calculate

$(\bar{I}\bar{A})_x$ as follows

$$(\bar{I}\bar{A})_x = \int_0^\infty t \, v^t \, {}_tp_x \, \mu_{x+t} \, dt$$

$$= \int_0^\infty t \, e^{-0,06t} \, e^{-0,04t} \, 0,04 \, dt$$

$$= 0,04 \int_0^\infty t \, e^{-0,1t} \, dt$$

$$= -0,04\left(\left[t e^{-0,1t} \right]_0^\infty - \int_0^\infty e^{-0,1t} dt \right)$$

$$= \underline{4}$$

4/ You are given

(i) $i = 0.1$

(ii) $q_x = 0{,}04$, $q_{x+1} = 0{,}08$

(iii) Deaths are UD over each year of age.

Calculate $A^{1\,(12)}_{x:\overline{2}|}$

$$A^{1\,(12)}_{x:\overline{2}|} = \frac{i}{i^{(12)}} A^{1}_{x:\overline{2}|} \quad \text{under (UDD)}$$

For $i = 0.1$, we have:

$$A^{1}_{x:\overline{2}|} = v q_x + v^2 p_x\, q_{x+1}$$

$$= \frac{0{,}04}{1.1} + \frac{0{,}96\,(0{,}08)}{1.1^2}$$

$$= 0{,}0993347$$

$$i^{(12)} = 12\left(1.1^{1/12} - 1\right) = 0{,}0956816$$

$$A^{1\,(12)}_{x:\overline{2}|} = \frac{i}{i^{(12)}} A^{1}_{x:\overline{2}|}$$

$$= \frac{0{,}1}{0{,}09568967} (0.0993347)$$

$$= 0{,}10433$$

5/ Z is PV random variable for a 15-year pure endowment of 1 on (x): (4)

(i) the force of mortality is constant over the 15-year period.

(ii) $v = 0{,}9$

(iii) $\operatorname{var}(Z) = 0{,}065\, E(Z)$.

$$p_x = e^{-\mu}, \quad {}_n p_x = e^{-\mu n} = (p_x)^n$$

$$E(Z) = v^{15}\, {}_{15}p_x = (0{,}9)^{15} (p_x)^{15}$$

$$E(Z^2) = (v^{15})^2\, {}_{15}p_x = 0{,}9^{30}\, p_x^{15}$$

$$\operatorname{var}(Z) = E(Z^2) - (E(Z))^2$$

$$= (0{,}9)^{30} (p_x)^{15} - (0{,}9)^{30} (p_x)^{30}$$

$$= (0{,}9)^{30}\, p_x^{15} \left(1 - p_x^{15}\right)$$

$$(0{,}9)^{30}\, p_x^{15} \left[1 - p_x^{15}\right] = 0{,}065\, (0{,}9)^{15} (p_x)^{15}$$

$$\Rightarrow (0{,}9)^{15} \left(1 - p_x^{15}\right) = 0{,}065$$

$$p_x^{15} = 0{,}6843$$

$$\Rightarrow p_x = 0{,}975 \Rightarrow q_x = 0{,}025$$

6) For a whole life insurance of 10000 on (x) with benefits payable at the moment of death:

(i) $\delta_t = \begin{cases} 0{,}04 & ; 0 \leq t \leq 10 \\ 0{,}05 & ; t > 10 \end{cases}$

(ii) $\mu_{x+t} = \begin{cases} 0{,}06 & ; 0 \leq t \leq 10 \\ 0{,}07 & ; t > 10 \end{cases}$

Calculate the single benefit premium of this insurance;

$$10\,000\,\bar{A}_x = 10\,000\left(\bar{A}^1_{x:\overline{10|}} + {}_{10|}\bar{A}_x\right)$$

$$= 10\,000\left(\bar{A}^1_{x:\overline{10|}} + v^{10}\,{}_{10}p_x\,\bar{A}_{x+10}\right)$$

$\bar{A}^1_{x:\overline{10|}}$ and $v^{10}\,{}_{10}p_x$ $\left(\begin{matrix}\delta = 0{,}04 \\ \mu = 0{,}06\end{matrix}\right)$

$\bar{A}_{x+10}$ can be found $\delta = 0{,}05$
$\mu = 0{,}07$

$$\bar{A}^1_{x:\overline{10|}} = \int_0^{10} e^{-0.04t}\,e^{-0{,}06t}\,0{,}06\,dt$$

$$= 0{,}6\left(1 - e^{-1}\right) = 0{,}37927$$

$$v^{10}\,{}_{10}p_x = e^{-0.04(10)}\,e^{-0{,}06(10)} = e^{-1}$$

$$\bar{A}_{x+10} = \frac{0{,}07}{0{,}07 + 0{,}05} = 7/12$$

$\Rightarrow$

$$10000\left(0.37927 + e^{-7/2}\right) = 5930.87$$

7/ You are given

(I) $A_x = 0.4$

(II) $A_{x+2} = 0.4045$

(III) $i = 0.08$

(IV) $q_x = q_{x+1}$

(V) Death are UD between integral ages

a/ Calculate q_x.

b/ Calculate $\overline{A}_{x+1}$

$$A_x = v q_x + v^2 p_x q_{x+1} + v^3 p_x p_{x+1} q_{x+2} + v^4 p_x p_{x+1} p_{x+2} q_{x+2} + \dots$$

$$= v q_x + v^2 p_x q_{x+1} + v^2 p_x p_{x+1}\left(v q_{x+2} + v^2 p_{x+2} q_{x+3} + \dots\right)$$

$$= v q_x + v^2 p_x q_{x+1} + v^2 p_x p_{x+1} A_{x+2}$$

$$q_x = q_{x+1} \;,\; p_x = p_{x+1}$$

$$\Rightarrow A_x = v q_x + v^2 p_x q_x + v^2 (p_x)^2 A_{x+2}$$

$$= v q_x + v^2 (1-q_x) q_x + v^2 (1-q_x)^2 A_{x+2}$$

$$= v q_x + v^2 q_x - v^2 q_x^2 + v^2 (1 - 2 q_x + q_x^2) A_{x+2}$$

⑥

$$= q_x\left(v + v^2 - 2v^2 A_{x+2}\right) + q_x^2\left(v^2 A_{x+2} - v^2\right) + v^2 A_{x+2} \qquad (7)$$

$$0.4 = 1.089677641\, q_x - 0.510545267\, q_x^2 + 0.346793553$$

$$\Rightarrow q_x = \frac{1.089677461 \pm \sqrt{(1.084677461)^2 - 4(0.510545267)(0.053206447)}}{2(0.510545267)}$$

$$= 2.084341564 \text{ or } 0.04998977$$

$$0 \leq q_x \leq 1 \Rightarrow q_x = 0.04998977$$

b)

$$A_x = v q_x + v p_x A_{x+1}$$

$$A_{x+1} = 0.402105907$$

$$\overline{A}_{x+1} = \frac{i}{\delta} A_{x+1} = \frac{0.08}{\ln(1.08)} A_{x+1}$$

$$= 0.417993854$$