EXO 1

a/ Suppose we have $\mu_x = \mu$, $\forall x$

Find an expression for

$Pr(K_x = k)$ for, $k = 0, 1, 2, -$

in terms of μ and k.

Solution

we have ${}_tP_x = e^{-\mu t} \Rightarrow P_x = e^{-\mu}$, $q_x = 1 - e^{-\mu}$

then $P(K_x = k) = {}_{k|}q_x = {}_kP_x \, q_{x+k}$

$$= e^{-k\mu}(1 - e^{-\mu})$$

b/ Find an expression for

$Pr(K_x \leq k)$ for

$k = 0, 1, 2, --$ in terms of μ and k.

Solu.

$$Pr(K_x \leq k) = {}_{k+1}q_x = 1 - {}_{k+1}P_x$$

$$= 1 - e^{-(k+1)\mu}$$

c/ as we have $\mu = 0.01$ then

$$Pr(K_x = 10) = e^{-10(0,01)}(1 - e^{-0,01})$$

$$= 0.009$$

d/ and if $\mu = 0,01 \Rightarrow$

$$Pr(K_x \leq 10) = 1 - e^{-(10+1)0.01}$$

$$= 0.1042$$

Ex02 Suppose we have:

i/ $\mu_x = F + e^{2x}, \quad x \geq 0$

ii/ ${}_{0.4}P_0 = 0.5$

Calculate F?

Sol. We have

$${}_{0.4}P_0 = 0.5 = e^{-\int_0^{0.4} (F + e^{2s})\, ds}$$

$$= e^{-\left[F s + \frac{1}{2} e^{2s}\right]_0^{0.4}}$$

$$= \exp(-0.4F - 1.11277 + 0.5)$$

$$= \exp(-0.4F - 0.61277)$$

$$\Rightarrow \quad 0.5 = e^{-0.4F - 0.61277}$$

$$\Rightarrow \ln(0.5) = -0.4F - 0.6127$$

$$\Rightarrow \quad F \simeq 0.2.$$

Ex03 Suppose we have a population of individuals, where:

a/ Each individual has a constant force of mortality

b/ the forces of mortality are UD over the interval $(0,2)$.

calculate the probability that an individual drawn at random for this population dies within one year.

Sol.

Let us denote by M the force of Mortality for an individual drawn at random, We have M is UD over $(0,2)$ then the density function of M is given by:

$$f(\mu) = \begin{cases} \frac{1}{2} & \text{for } 0<\mu<2 \\ 0 & \text{for } \mu>2 \end{cases}$$

Let T be the future lifetime of the individual, then we have:

$$Pr(T \leq 1) = E[Pr(T \leq 1 | M)]$$

$$= \int_0^{\infty} Pr(T \leq 1 | M=\mu) f(\mu) d\mu$$

$${}_tp_x = e^{-\mu t}$$

$$p_x = e^{-\mu}$$

$$= \int_0^2 (1-e^{-\mu}) \frac{1}{2} d\mu$$

$$= \frac{1}{2}(2+e^{-2}-1)$$

$$= \frac{1}{2}(1+e^{-2})$$

$$= 0.5676$$

Exo 4 suppose we have

1/ $p_x = 0,97$.

2/ $p_{x+1} = 0.95$

3/ $e_{x+1.75} = 18.5$

4/ Deaths are UD between age x and $x+1$.

5/ the force of mortality is constant between $x+1$ and $x+2$

Find $e_{x+0,75}$

Sol. we have: $e_{x+0.75} = p_{x+0.75}(1+e_{x+1,75})$

then we need to calculate $p_{x+0,75}$

we have: $p_{x+0,75} = {}_{0,25}p_{x+0,75} \; {}_{0,75}p_{x+1}$

(4) $\Rightarrow$ using UDD we get: ${}_{0.25}p_{x+0.75} = \dfrac{p_x}{{}_{0,75}p_x}$

$$\boxed{p_x = {}_{0,75}p_x \cdot {}_{0,25}p_{x+0,75}} = \frac{0,97}{1-0,75(1-0,97)}$$

$${}_{0,75}p_x = 1 - {}_{0,75}q_x = 1 - 0,75\, q_x = 1 - 0,75(1-p_x) = 0,99227366$$

(5) $\Rightarrow$ using the force of mortality is constant between $x+1$ and $x+2$ we get

$${}_{0,75}p_{x+1} = (p_{x+1})^{0,75} = 0,95^{0,75}$$
$$= 0.9622606$$

$$\Rightarrow \quad p_{x+0,75} = 0,99227366(0,9622606)$$
$$= 0,954878$$

and $e_{x+0,75} = 0,954878(1 + 18.5)$
$$= 18.620.$$

Ex 05 suppose we have:

1/ Mortality follows De Moivre's Law $\quad l_x = \omega - x, \; 0 \le x \le \omega$

2/ $\mathring{e}_{20} = 30$ $\quad S_0(x) = 1 - \frac{x}{\omega}$

Find q_{20} $\quad S_0(x) = \dfrac{\omega - x}{\omega} = \dfrac{l_x}{l_0}$

Solution: De Moivre's Law $\quad S_0(x) = \dfrac{l_x}{l_0} = \dfrac{\omega - x}{\omega} = 1 - \dfrac{x}{\omega}, \; S_x(t) = \dfrac{S_0(x+t)}{S_0(x)}$

$$\Rightarrow S_x(t) = \frac{l_{x+t}}{l_x} = \frac{\omega - x - t}{\omega - x} = 1 - t/\omega - x \Rightarrow \mu_{x+t} = -\frac{S'_x(t)}{S_x(t)} =$$

$$= - \frac{-\frac{1}{\omega - x}}{1 - \frac{t}{\omega - x}} = \frac{\frac{1}{\omega - x}}{\frac{\omega - x - t}{\omega - x}} = \frac{1}{\omega - x - t}$$

$$\Rightarrow \mu_x = \frac{1}{\omega - x}$$

De Moivre's Law

$$\mu_x = \frac{1}{\omega - x} \quad \text{or} \quad l_x = \omega - x, \quad 0 \leq x < \omega$$

$$2/\Rightarrow \quad \mathring{e}_{20} = 30 = \int_0^{\omega - 20} {}_tp_{20}\, dt = \int_0^{\omega - 20} \frac{l_{t+20}}{l_{20}}\, dt = \int_0^{\omega - 20} \frac{\omega - t - 20}{l_{20}}\, dt$$

$$= \int_0^{\omega - 20} \frac{\omega - t - 20}{\omega - 20}\, dt$$

$$= \int_0^{\omega - 20} \left(\frac{\omega - 20}{\omega - 20} - \frac{t}{\omega - 20} \right) dt$$

$$= \frac{\omega - 20}{2}.$$

De Moivre's law implies that the age at death random variable (T_0) is UD over $[0, \omega)$.
and the future lifetime random variable T_x is UD over $[0, \omega - x)$

$${}_tP_x = \frac{l_{x+t}}{l_x}$$

$$= \frac{\omega - x - t}{\omega - x} = 1 - \frac{t}{\omega - x}, \quad 0 \leq t < \omega - x$$

In general: $\int_0^{\omega - x} \left(1 - \frac{t}{\omega - x}\right) dt = (\omega - x) - \frac{(\omega - x)^2}{2(\omega - x)}$

$$= \omega - x - \frac{\omega - x}{2}$$

$$= \frac{\omega - x}{2}.$$

or

we have $l_x = \omega - x$
then the problem's uniform distribution
${}_tP_x$ is when $0 \leq t \leq \omega - x$

$$\Rightarrow \mathring{e}_x = \int_0^{\omega - x} {}_tP_x\, dt = \frac{\omega - x}{2},$$

as we have $\mathring{e}_x = \frac{\omega - x}{2}$ and $\mathring{e}_{20} = 30 \Rightarrow$

$$\mathring{e}_{20} = \frac{\omega - 20}{2} = 30$$

$$\Rightarrow \omega = 60 + 20 = 80$$

as we have the death is UD over $[0, 60[$

$$\Rightarrow q_{20} = \frac{1}{60}.$$