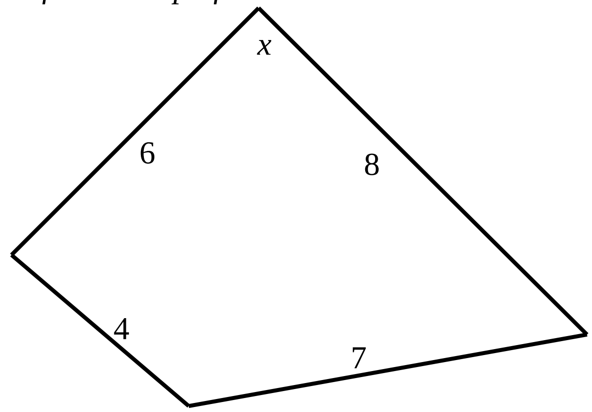
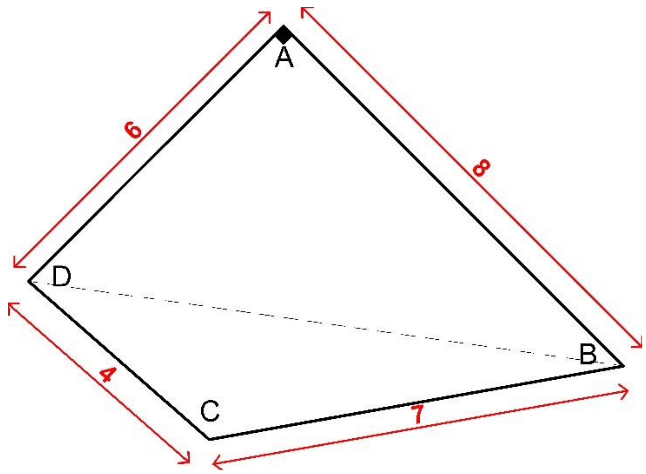
Compute the area for the boundary shown in figure below: angle x is right angle, dimensions shown are in mm, extracted from a map of scale 1:2000.





ABD is right angle triangle, so BD can be found $BD = \sqrt{6^2 + 8^2} = 10mm$

Area of triangle = $\sqrt{s \times (s-a) \times (s-b) \times (s-c)}$,   where $s = (a + b + c) \div 2$

$A_{ABD} = \sqrt{12 \times (12-6) \times (12-8) \times (12-10)} = 24\ mm^2$

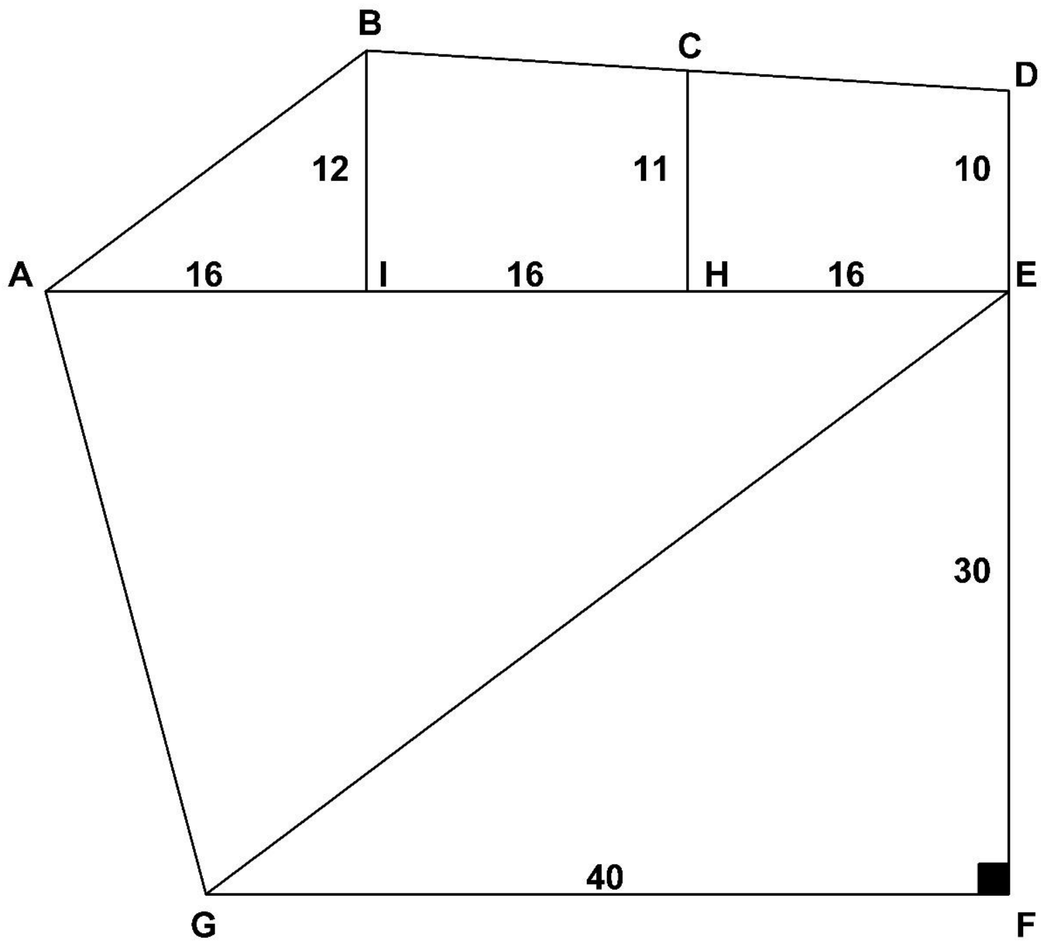
$A_{BCD} = \sqrt{10.5 \times (10.5-4) \times (10.5-7) \times (10.5-10)} = 10.929\ mm^2$

Total map area = 24 + 10.929 = 34.929 mm^2

Ground area = $34.929 \times 2000^2 = 139714986\ mm^2 = 139.715\ m^2$

■■■■

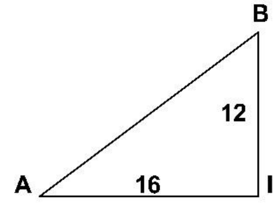
Compute the property ground area ABCDEFGA by dividing it into small geometric figures as shown. IB, HC and ED are offsets from AE, DEF is a straight line and angle EFG is right angle. All dimensions are in meters.



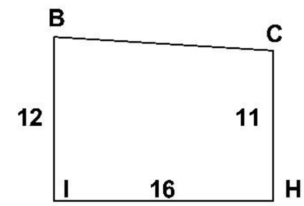
Assuming coordinates of point A: (x=0.0m, y=0.0m) with AE as the x-axis, use method of coordinates to compute the total area.

- By dividing into small geometric figures:

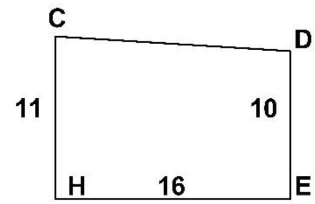
$$A_{ABI} = 12 \times \frac{16}{2} = 96\ m^2$$



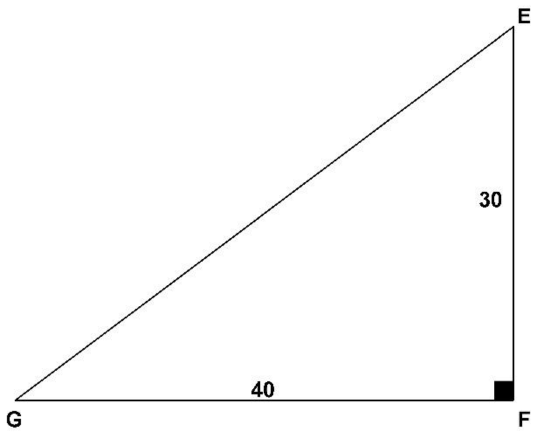
$$A_{BCHI} = (11 + 12) \times \frac{16}{2} = 184\ m^2$$



$$A_{CDEH} = (10 + 11) \times \frac{16}{2} = 168\ m^2$$

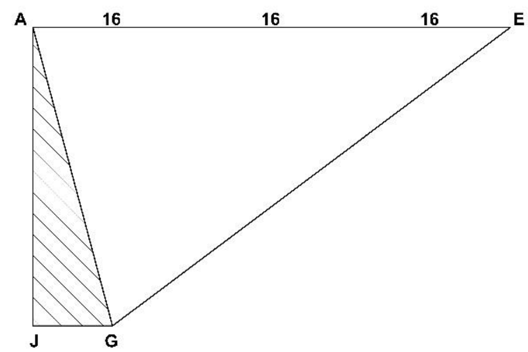


$$A_{EFG} = 30 \times \frac{40}{2} = 600\ m^2$$



To find A_{AEG}:

$$EG = \sqrt{30^2 + 40^2} = 50\ m$$
$$AG = \sqrt{30^2 + 8^2} = 31.0483\ m$$
$$AE = 16 + 16 + 16 = 48\ m$$
$$s = (50 + 31.0483 + 48)/2 = 64.52415\ m$$
$$A_{EGA} = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}$$
$$A_{EFG} = 720\ m^2$$



Total area = 720 + 600 + 168 + 184 + 96 = 1768 m^2

- By method of coordinates:

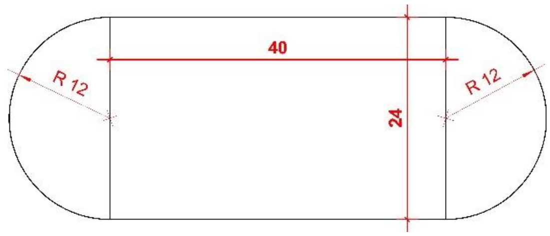
Point	Y	X
A	0	0
B	12	16
C	11	32
D	10	48
E	0	48
F	–30	48
G	–30	8
A	0	0

From right to left		From left to right	
0 × 16	= 0	0 × 12	= 0
12 × 32	= 384	16 × 11	= 176
11 × 48	= 528	32 × 10	= 320
10 × 48	= 480	48 × 0	= 0
0 × 48	= 0	48 × – 30	= – 1440
– 30 × 8	= – 240	48 × – 30	= – 1440
– 30 × 0	0	8 × 0	= 0
Sum	1152	Sum	– 2384

Area = [(1152) – (– 2384)]/2 = 1768 m^2

■■■■

A playground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 40m and 24m, find the area of the playground.



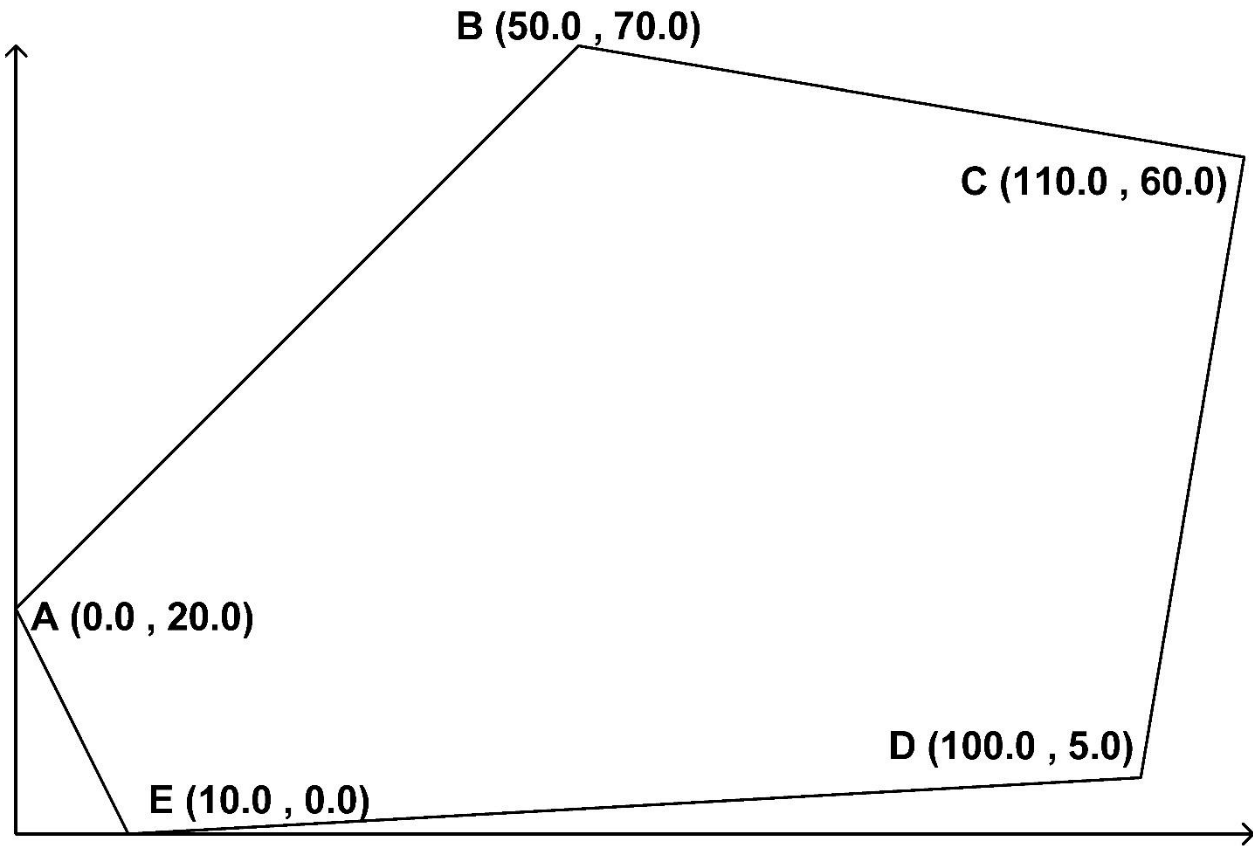
Area of rectangle = 40 × 24 = 960 m^2

Area of circle = $\pi r^2 = 3.14 \times 12^2 = 452.57\ m^2$

Total Area = 960 m^2 + 452.57 m^2 = 1412.57 m^2

■■■■

Compute the area of the land parcel shown in figure below, with the coordinates of vertices given in meters, using:



1- Method of Coordinates

2- Dividing the parcel in triangles: ABC, ACD and ADE.

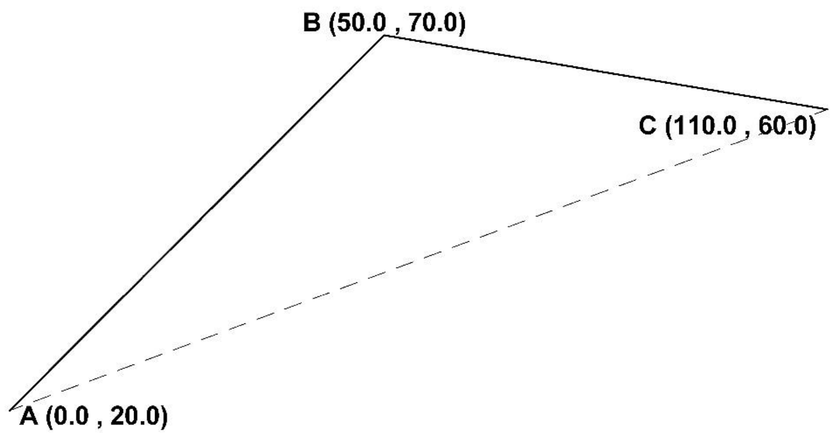
- By method of coordinates:

Point	Y	X
A	20	0
B	70	50
C	60	110
D	5	100
E	0	10
A	20	0

From right to left		From left to right	
20 × 50	= 1000	0 × 70	= 0
70 × 110	= 7700	50 × 60	= 3000
60 × 100	= 6000	110 × 5	= 550
5 × 10	= 50	100 × 0	= 0
0 × 0	= 0	10 × 20	= 200
Sum	14750	Sum	3750

Area = (14750 – 3750)/2 = 5500 m^2

- Dividing the parcel in triangles: ABC, ACD and ADE



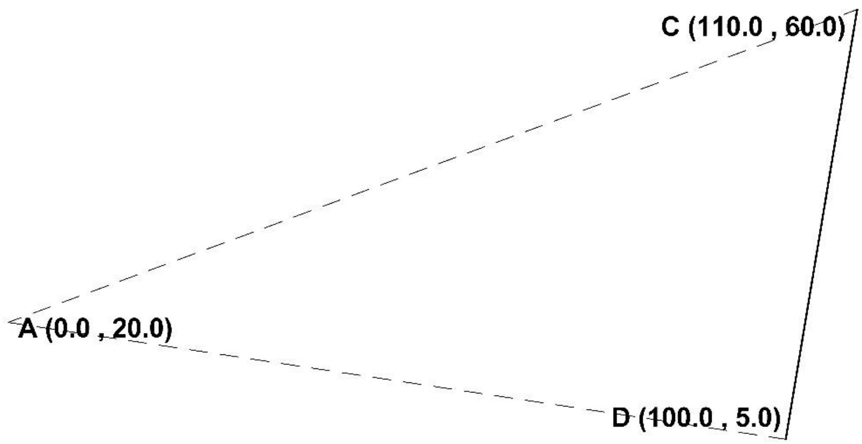
$$AB = \sqrt{50^2 + 50^2} = 70.711\ m$$

$$BA = \sqrt{60^2 + 10^2} = 60.828\ m$$

$$AC = \sqrt{40^2 + 110^2} = 117.047\ m$$

$$s = (a + b + c)/2 = (70.711 + 60.828 + 117.047)/2 = 124.293\ m$$

$$A_{ABC} = \sqrt{s \times (s - a) \times (s - b) \times (s - c)} = 1750\ m^2$$



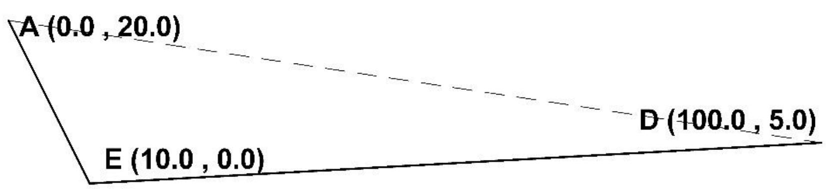
$$AC = 117.047\ m$$

$$DC = \sqrt{55^2 + 10^2} = 55.902\ m$$

$$AD = \sqrt{15^2 + 100^2} = 101.119\ m$$

$$s = (a + b + c)/2 = (117.047 + 55.902 + 101.119)/2 = 137.034\ m$$

$$A_{ACD} = \sqrt{s \times (s - a) \times (s - b) \times (s - c)} = 2825\ m^2$$



$$AD = 101.119\ m$$

$$AE = \sqrt{10^2 + 20^2} = 22.361\ m$$

$$ED = \sqrt{5^2 + 90^2} = 90.139\ m$$

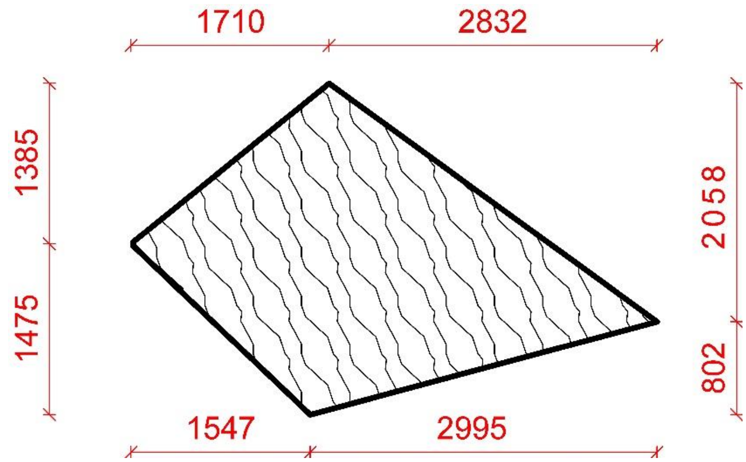
$$s = (a + b + c)/2 = (22.361 + 90.139 + 101.119)/2 = 106.810\ m$$

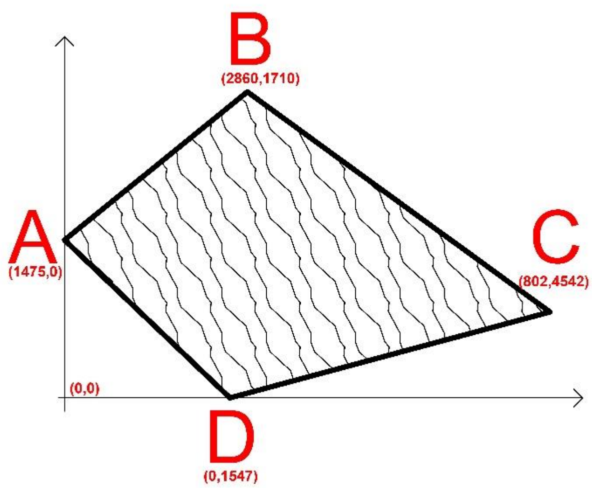
$$A_{ACD} = \sqrt{s \times (s - a) \times (s - b) \times (s - c)} = 925\ m^2$$

Total area = 1750 + 2825 + 925 = 5500 m^2

■■■■

Exam Question*: given the data in the in meters. Compute the ground area of the land tract.*





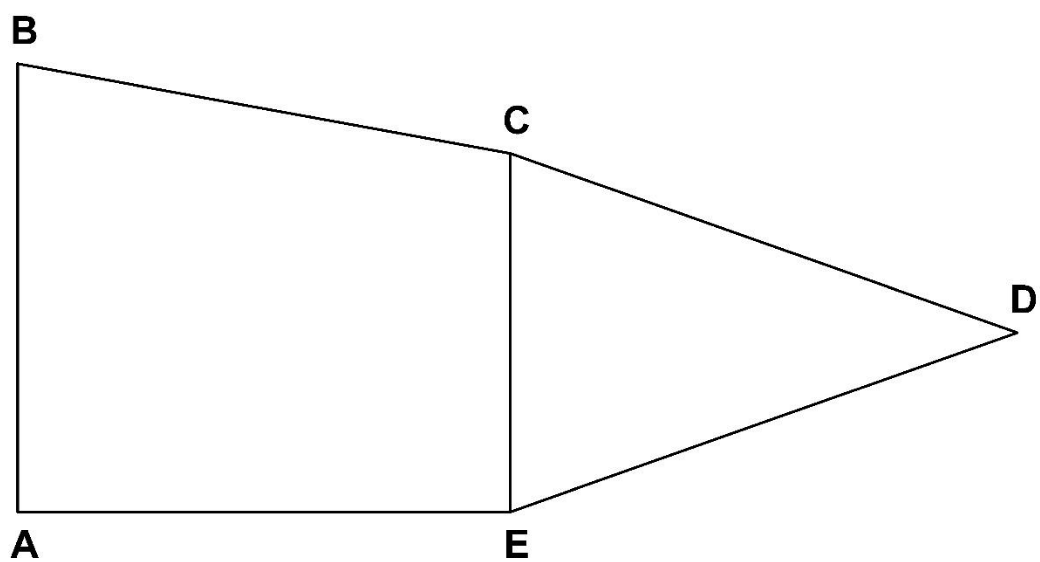
Point	Y	X
A	1475	0
B	2860	1710
C	802	4542
D	0	1547
A	1475	0

From right to left		From left to right	
1475 × 1710	= 2522250	0 × 2860	= 0
2860 × 4542	= 12990120	1710 × 802	= 1371420
802 × 1547	= 1240694	4542 × 0	= 0
0 × 0	= 0	1547 × 1475	= 2281825
Sum	16753064	Sum	3653245

Area = (16753064 – 3653245)/2 = 6549909.5 m^2

■■■■

Exam Question: *The figure below shows the plan view of a house. AB = 20.0m, AE = 22.0m, CD=ED = 24.0m and CE=16.0m; angles at A and E (EAB and CEA) are right angles, Assuming A as origin of a 2D coordinates system (AE is the x-axis). Use method of coordinates to calculate the area of the house with boundary ABCDEA.*



- Coordinates of corners of figure:

  A (0.00, 0.00); B (0.00, 20.00); C (22.00, 16.00);

  D (44.627, 8.00); Since DF = $[24.00^2 – 8.00^2]^{1/2}$ = 22.627 m

  E (22.00, 0.00)

- Area of figure = (1/2) {[20.00×22.00 + 16×44.627 + 8.00×22.00] – [22.00×8.00]}
  = 577.016 m^2

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