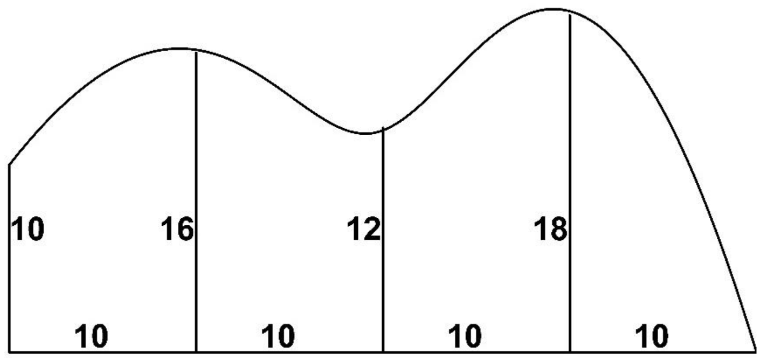
The figure below shows the boundary of a small farm as plotted on a map of scale 1:500, dimensions are in cms. Compute the actual ground area of the farm in sq. meters, using:

1- Average heights, 2- trapezoidal rule, 3- Simpson's rule.



- Average heights:

$$Average\ heights, H = \frac{\sum h_i}{n} = \frac{10 + 16 + 12 + 18 + 0}{5} = 11.2\ cm$$

$$Area_{map} = [(n-1)X] \times H = [(5-1)10] \times 11.2 = 448\ cm^2$$

$$Area_{ground} = 448 \times 500^2 = 112000000\ cm^2 = 11200\ m^2$$

- trapezoidal rule:

$$Area = \frac{X}{2}\left[h_1 + h_n + 2\sum(h_2 + h_3 + \cdots + h_{n-1})\right]$$

$$Area_{map} = \frac{10}{2}\left[10 + 0 + 2\sum(16 + 12 + 18)\right] = 510\ cm^2$$

$$Area_{ground} = 510 \times 500^2 = 127500000\ cm^2 = 12750\ m^2$$

- Simpson's rule:

$$Area = \frac{X}{3}\left[h_1 + h_n + 4\sum(h_2 + h_4 + h_6 + \cdots) + 2\sum(h_3 + h_5 + h_7 + \cdots)\right]$$

$$Area_{map} = \frac{10}{3}\left[10 + 0 + 4\sum(16 + 18) + 2\sum(12)\right] = 566.667\ cm^2$$

$$Area_{ground} = 566.667 \times 500^2 = 141666666.667\ cm^2 = 14166.667\ m^2$$

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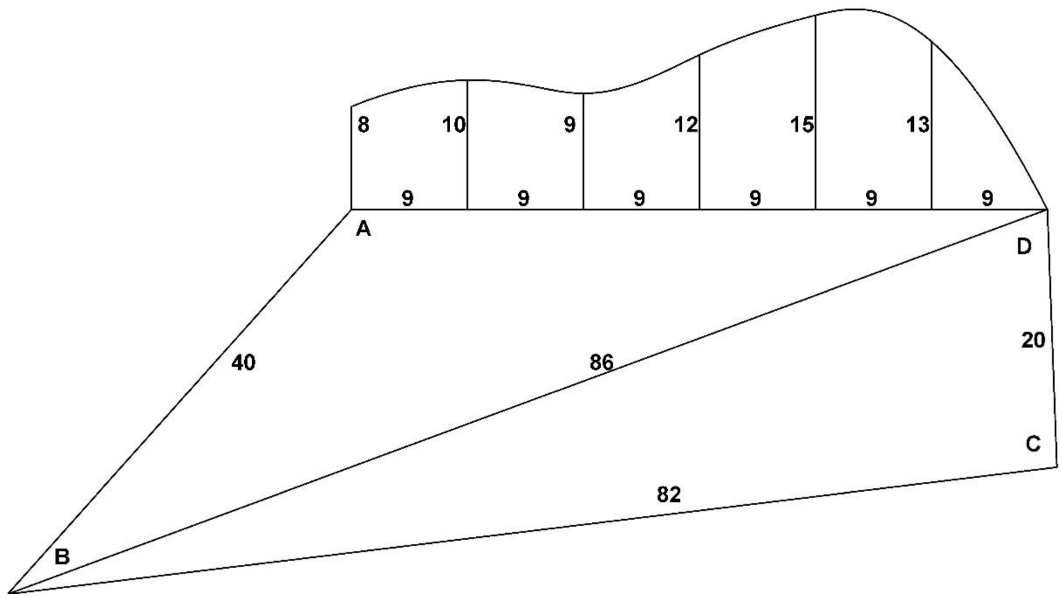
Exam Question: *The figure below shows a tract of land that has three straight boundaries: AB, BC, and CD. The fourth boundary DA is irregular. The measured lengths are as follows: AB = 40.00 m, BC = 82.00 m, CD = 20.00 m, BD = 86 m. offsets were measured from the boundary DA to the irregular boundary at a regular interval of 9.00 m and were recorded in meters as shown in the figure.*



Compute the regular area ABCD and the irregular one using Simpson's rule. What is the total area?

$$s_{ABD} = (a + b + c) \div 2 = (40 + 86 + 54)/2 = 90\ m$$

$$A_{ABD} = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}$$

$$A_{ABD} = \sqrt{90 \times (90 - 40) \times (90 - 86) \times (90 - 54)} = 805\ m^2$$

$$s_{BCD} = (a + b + c) \div 2 = (82 + 86 + 20)/2 = 94\ m$$

$$A_{ABD} = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}$$

$$A_{BCD} = \sqrt{94 \times (94 - 82) \times (94 - 86) \times (94 - 20)} = 817\ m^2$$

$$Area_{irregular} = \frac{X}{3}\left[h_1 + h_n + 4\sum(h_2 + h_4 + h_6 + \cdots) + 2\sum(h_3 + h_5 + h_7 + \cdots)\right]$$

$$Area_{irregular} = \frac{9}{3}\left[8 + 0 + 4\sum(10 + 12 + 13) + 2\sum(9 + 15)\right] = 588\ m^2$$

Total area = 805 + 817 + 588 = 2210 m^2

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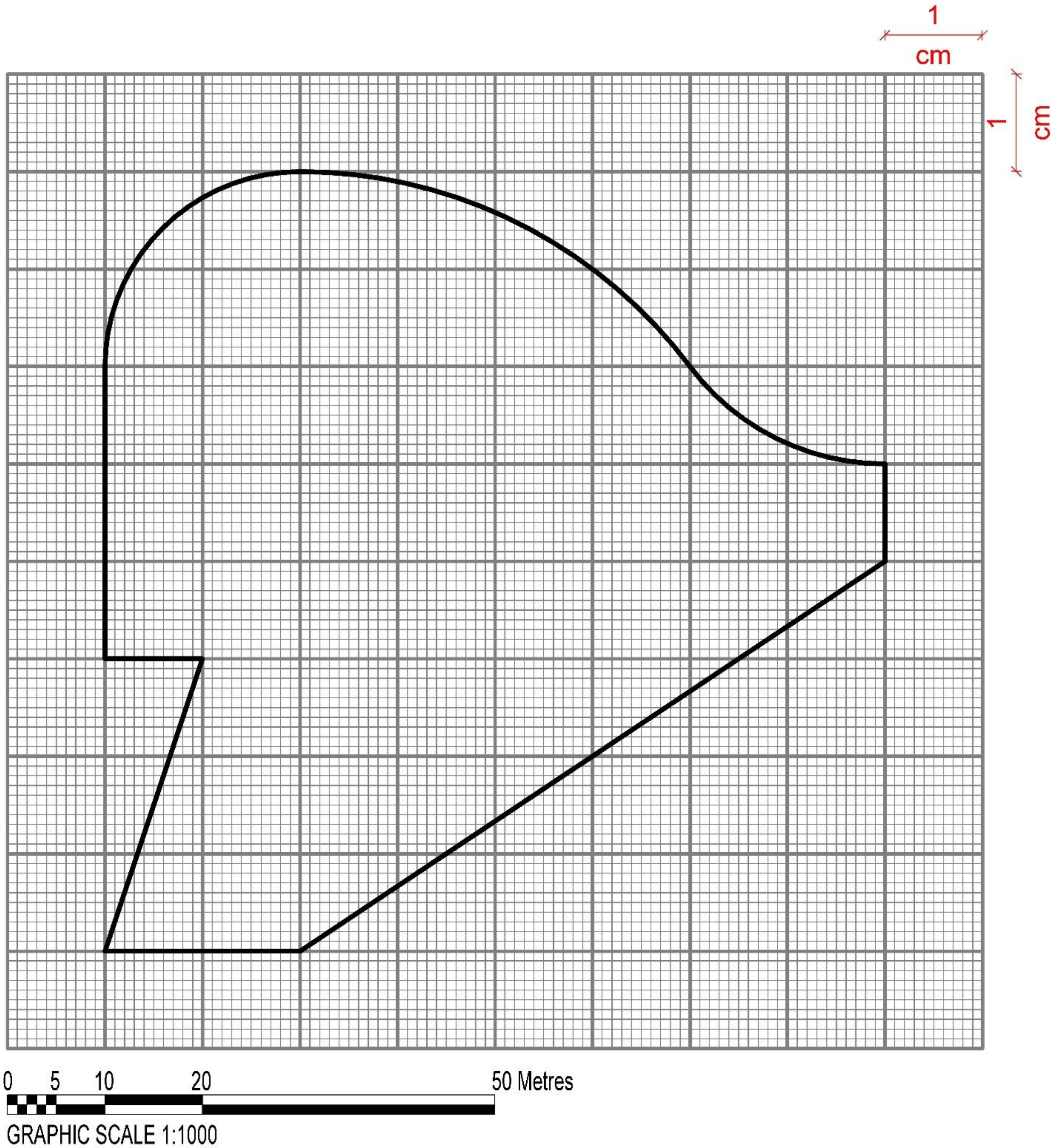
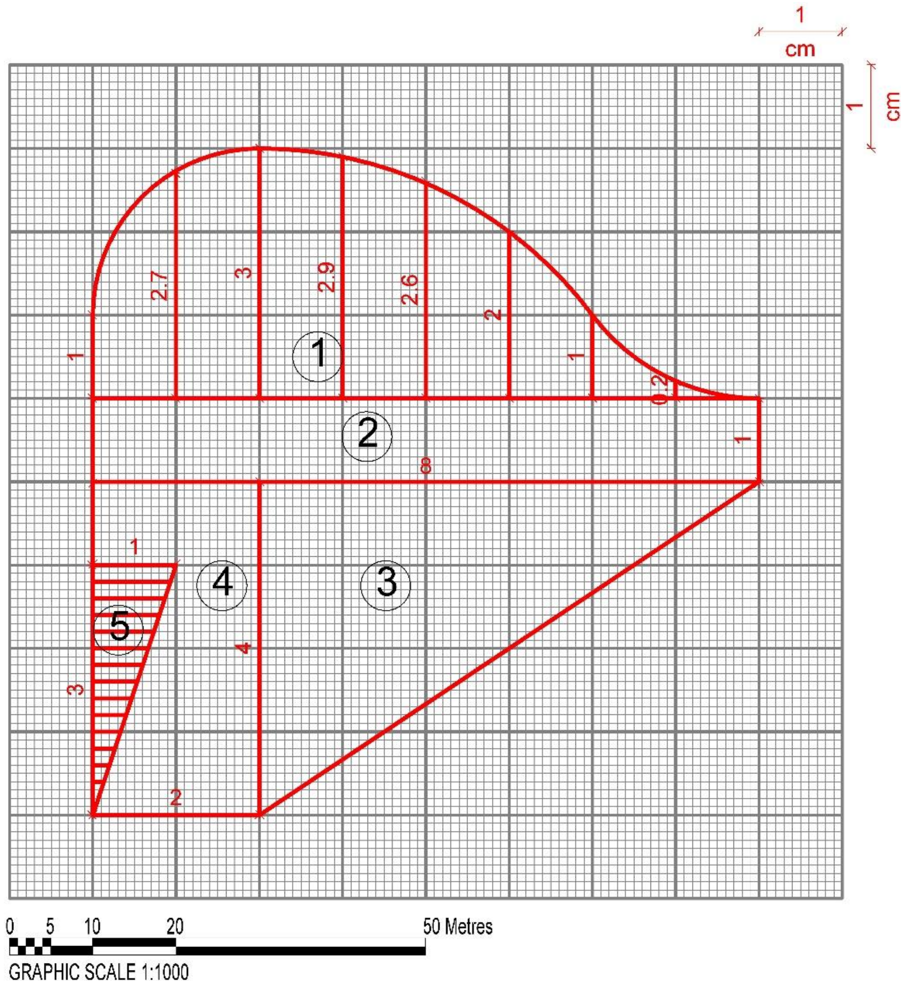
Exam Question: *Given the data in the plan below. Compute the ground area of the land tract in square meters. Use Simpson's rule for the area bounded by a curve.*





A1: Simpson's rule: $A_1 = \frac{1}{3}[1 + 0 + 4(2.7 + 2.9 + 2 + 0.2) + 2(3 + 2.6 + 1)] = 15.13\ cm^2$

A_2: Rectangular: $A_2 = 1 \times 8 = 8\ cm^2$

A_3: Right angle triangle: $A_3 = 0.5 \times 4 \times 6 = 12\ cm^2$

A_4: Rectangular: $A_2 = 2 \times 4 = 8\ cm^2$

A_5: Right angle triangle: $A_5 = 0.5 \times 1 \times 3 = 1.5\ cm^2$

Total plan area: $A_{total\ plan} = A_1 + A_2 + A_3 + A_4 - A_5$

$$A_{total\ plan} = 15.13 + 8 + 12 + 8 - 1.5 = 41.63\ cm^2$$

Total ground area:

$$A_{total\ ground} = 41.63 \times\ 1000^2 = 41630000\ cm^2 = 4163\ m^2$$

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