KING SAUD UNIVERSITY **FULL MARKS: 40**

M - 107

DEPARTMENT OF MATHEMATICS **TIME: 90min**

(SEMESTER I I, 1429-1430) SECOND MID-TERM

NOTE: Attempt all Questions.

Question: 1.(a) Find the area of triangle with vertices P(1, 4, 6), Q(-2, 5 ,-1) and R(1 , -1, 1). [5+5]

(b) Find the work done by a constant force *F = 3i +4 j+5 k*, if its point of application moves from point P(2, 1, 0) to Q(4, 6, 2).

Question: 2(a) Find the equation of the plane containing the point P (3, 0, -1) and perpendicular to planes x - 2y + z = 2 and 2x - z = 1. [6+6]

(b) Determine whether the lines

x = 4- 2t, y = 3t, z = -1 +2 t and

x = 4 + s, y =2s, z = -1 +3 s are parallel . If not find the angle between the lines.

Question:3. (**a**) A particle moves along the curve give by the vector valued function

[7+5+6] $r(t) = <2t^2, t^2 - 4t, 3t - 5>$, find the velocity and acceleration at t =1 also find the component of velocity and acceleration at t =1 in the direction of $a = <2, -4, 1>$.

(b) Find the curvature, center and radius of curvature of the curve $y = 2 - x^3$ at the point $(2, -6)$.

(c) Position of the moving point at time t is given by $r(t) = 4\cos t\ i + 9\sin t\ j + t\ k$, find the tangential and normal component of acceleration at any time t .

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(SEMESTER II, 1429-1430) SECOND MID-TERM

Question: 1. (a)

[5]

$\vec{PQ} = \langle -3, 1, -7 \rangle$, $\vec{PR} = \langle 0, -5, -5 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -3 & 1 & -7 \\ 0 & -5 & -5 \end{vmatrix} = \langle -40, -15, 15 \rangle$$

Area of $\Delta PQR = \frac{1}{2} \| \vec{PQ} \times \vec{PR} \| = \frac{1}{2}\sqrt{(-40)^2 + (-15)^2 + 15^2} = \frac{1}{2} 5\sqrt{82} = \frac{5}{2}\sqrt{82}$ unit^2

1.(b)

[5]

$F = 3i + 4j + 5k$

$d = \vec{PQ} = \langle 2, 5, 2 \rangle$ work done $= F \cdot d = \langle 3, 4, 5 \rangle \cdot \langle 2, 5, 2 \rangle$

$= 6 + 20 + 10$

$= 36$ unit

Question 2 (a)

[6]

$n_1 = \langle 1, -2, +1 \rangle$, $n_2 = \langle 2, 0, -1 \rangle$

A vector n normal to both n_1 and n_2 is $n = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 0 & -1 \end{vmatrix}$

$= \langle 2, 3, 4 \rangle$

Equation plane containing point $P(3, 0, -1)$ and having normal $n = \langle 2, 3, 4 \rangle$ is

$$2(x-3) + 3(y-0) + 4(z+1) = 0$$

$$2x + 3y - 4z = 2$$

2 (b)

[6]

vector parallel to line l_1 is $a = \langle -2, 3, 2 \rangle$

vector parallel to line l_2 is $b = \langle 1, 2, 3 \rangle$

$-\frac{2}{1} \neq \frac{3}{2} \neq \frac{2}{3} \Rightarrow$ a is not parallel to b $\Rightarrow$ lines are not parallel.

$$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{-2+6+6}{\sqrt{17}\sqrt{14}} = \frac{10}{\sqrt{17}\sqrt{14}} , \quad \theta = \cos^{-1}\left(\frac{10}{\sqrt{17}\sqrt{14}}\right)$$

Question 3 (a)

[7]

$r(t) = \langle 2t^2, t^2 - 4t, 3t - 5 \rangle$

velocity $r'(t) = \langle 4t, 2t - 4, 3 \rangle$

accel $= r''(t) = \langle 4, 2, 0 \rangle$

At $t = 1$ V = velocity $= \langle 4, -2, 3 \rangle$

A = accel $= \langle 4, 2, 0 \rangle$

$a = \langle 2, -4, 1 \rangle$

$\text{Comp}_a V = \frac{V \cdot a}{\|a\|} = \frac{19}{\sqrt{21}}$

$\text{Comp}_a A = \frac{A \cdot a}{\|a\|} = \frac{0}{\sqrt{21}} = 0$

3.(b)

[5]

$y = 2 - x^3$, $y' = -3x^2$, $y'' = -6x$

At $(2, -6)$ $y' = -12$, $y'' = -12$

Curvature

$$K = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{|-12|}{(1+(-12)^2)^{3/2}} = \frac{12}{145^{3/2}}$$

Radius of curvature $\rho = \frac{1}{K} = \frac{145^{3/2}}{12}$

center of curvature $h = 2 - \frac{(-12)(1+(-12)^2)}{(-12)} = -143$

$k = -6 + \frac{(1+(-12)^2)}{-12} = -\frac{217}{12}$

$(h, k) = \left(-143, -\frac{217}{12}\right)$

Q. 3(c)

[6]

$r(t) = 4\cos t\, i + 9\sin t\, j + t\, k$

$r'(t) = -4\sin t\, i + 9\cos t\, j + k$

$r''(t) = -4\cos t\, i - 9\sin t\, j$

$\|r'(t)\| = \sqrt{1 + 81\cos^2 t + 16\sin^2 t}$

$a_T = \frac{r' \cdot r''}{\|r'\|} = \frac{-65\sin t\cos t}{\sqrt{1 + 81\cos^2 t + 16\sin^2 t}}$

$a_N = \sqrt{\frac{16\cos^2 t + 81\sin^2 t + 1296}{81\cos^2 t + 16\sin^2 t + 1}}$