HW-II assignment date: 12 April 2020

Homework - II
KSU / Math-244 / Semester-II (1440-1441H)

Note 1: The students must submit the PDF file of completed homework through email to the respective class teachers within 2 days from its assignment date; in their own handwritings with signatures.

Problem 1: *Throughout this problem* **1**, *let* $x_1x_2x_3x_4x_5x_6x_7x_8x_9$ *be your present university* ID.
[*for example, if your* ID *is* 423897615 $x_1 = 4$, $x_2 = 2$, $x_3 = 3$, $x_4 = 8$, $x_5 = 9$, $x_6 = 7$, $x_7 = 6$, $x_8 = 1$, $x_9 = 5$.]

(a) *If* $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}$, *then find*: (i). *rank* $(\mathbf{A})$ (ii). *nullity* $(\mathbf{A}^T)$.

(b) *If* ${}_{\mathbf{S}}\mathbf{P}_{\mathbf{B}} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & x_5 & 5 \end{bmatrix}$ *is the transition matrix from the basis* $\mathbf{B} = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ *of* $\mathbb{R}^3$ *to its standard basis* $\mathbf{S}=\{\boldsymbol{e}_1 = (1, 0, 0), \boldsymbol{e}_2 = (0, 1, 0), \boldsymbol{e}_3 = (0, 0, 1)\}$, *then find* $[\boldsymbol{v_2}]_{\mathbf{S}}$.

(c) *For the Euclidean inner product space* $\mathbb{R}^3$:

(i). *Find* $\cos\theta$, *where* θ *is the angle between the vectors* (x_1, x_2, x_3) *and* (x_2, x_4, x_5).

(ii). *Use the Gram-Schmidt process to obtain an orthonormal basis from the given basis* $\{(x_3, 0, 0), (1, 1, 0), (1, x_9, 1)\}$ *for* $\mathbb{R}^3$.

(d) *Let* $\mathbf{T}: \mathbb{R}^3 \to \mathbb{R}$ *be the transformation defined by*: $\mathbf{T}(\boldsymbol{v}) = \langle \boldsymbol{v}, (x_2, x_4, x_6) \rangle$ *for all* $\boldsymbol{v} \in \mathbb{R}^3$, *where* "$\langle\, , \rangle$" *denotes the Euclidean inner product on* $\mathbb{R}^3$. Then:

(i) *Show that* **T** *is a linear transformation with* $Im(\mathbf{T}) = \mathbb{R}$.

(ii) *Find a basis of* $Ker(\mathbf{T})$.

Problem 2: (*a*) *Determine a linear transformation* $T: \mathbb{R}^4 \to \mathbb{R}^3$ *such that*:

$$Ker(T) = span\{(1, 1, -2, 3), (-1, 2, 0, 1)\}.$$

(*b*) *Determine a linear transformation* $T: \mathbb{R}^5 \to \mathbb{R}^5$ *such that*:

$$Im(T) = span\{(1, 1, -1, 4, 3), (-2, 1, 5, 1, 0)\}.$$

Problem 3: *Let* V *be a* real *inner product space and* $B = \{v_1, v_2, v_3, v_4\}$ *be an orthogonal basis for* V *such that* $\|v_1\| = 2$, $\|v_2\| = 3$, $\|v_3\| = 5$ *and* $\|v_4\| = 2$. Suppose $T: V \to V$ *is* a *linear transformation such that*:

$T(v_1) = v_1 - v_2 + v_3 - v_4$;

$T(v_2) = v_2 + 2v_3 + 2v_4$;

$T(v_2) + T(v_3) = -v_1 + 2v_2 + 4v_3 + 2v_4$;

$T(v_4) = v_1 - v_2 + 4v_3 - 2v_4$.

Then:

(a) *Calculate*:

(*i*) $\langle u, v_4 \rangle$

(*ii*) $\langle u, T(v_3) \rangle$

where $u \in V$ *and* $[u]_B = (2, -2, -1, 3)$.

(b) *Find a basis for* $Ker(T)$.

(*c*) *Show that* $\{T(v_1), T(v_2), T(v_3)\}$ *is a basis for* $Im(\mathrm{T})$.

(*d*) *Show that* $v_2 \notin Im(T)$.

(*e*) *Find* $w \in V$ *such that* $T(w) = v_2 + 5v_3 + v_4$.

===...===