# Manual for SOA Exam FM/CAS Exam 2. Chapter 1. Basic Interest Theory. 

 Section 1.1. Amount and accumulation functions.(C)2009. Miguel A. Arcones. All rights reserved.

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- The effective rate of interest earned in the period $[s, t]$ is

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## Example 1

Simon invests $\$ 1000$ in a bank account. Six months later, the amount in his bank account is $\$ 1049.23$.
(i) Find the amount of interest earned by Simon in those 6 months.
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(ii) The (semiannual) effective rate of interest earned is

$$
\frac{1049.23-1000}{1000}=0.004923=0.4923 \% .
$$

## Amount function

Suppose that an amount $A(0)$ of money is invested at time 0 . $A(0)$ is the principal. Let $A(t)$ denote the value at time $t$ of the initial investment $A(0)$. The function $A(t), t \geq 0$, is called the amount function. Usually, we assume that the amount function satisfies the following properties:
(i) For each $t \geq 0, A(t)>0$.
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- The effective rate of interest earned in the period $[s, t]$ is

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\frac{A(t)-A(s)}{A(s)} .
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Jessica invests $\$ 5000$ on March 1, 2008, in a fund which follows the accumulation function $A(t)=(5000)\left(1+\frac{t}{40}\right)$, where $t$ is the number of years after March 1, 2008.
(i) Find the balance in Jessica's account on October 1, 2008.
(ii) Find the amount of interest earned in those 7 months.
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A(7 / 12)=(5000)\left(1+\frac{7 / 12}{40}\right)=5072.917
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(iii) The effective rate of interest earned in that period is

$$
\frac{A(7 / 12)-A(0)}{A(0)}=\frac{72.917}{5000}=0.0145834=1.45834 \%
$$

## Cashflows

Often, we consider the case when several deposits/withdrawals are made into an account following certain amount function. A series of (deposits/withdrawals) payments made at different times is called a cashflow. The payments can be either made by the individual or to the individual. An inflow is payment to the individual. An outflow is a payment by the individual. We represent inflows by positive numbers and outflows by negative numbers. In a cashflow, we have a contribution of $C_{j}$ at time $t_{j}$, for each $j=1, \ldots, n$. $C_{j}$ can be either positive or negative. We can represent a cashflow in a table:

| Investments | $C_{1}$ | $C_{2}$ | $\cdots$ | $C_{n}$ |
| :---: | :---: | :---: | :---: | :---: |
| Time (in years) | $t_{1}$ | $t_{2}$ | $\cdots$ | $t_{n}$ |

## Cashflow rules

Rule 1: Proportionality. If an investment strategy follows the amount function $A(t), t>0$, an investment of $\$ k$ made at time 0 with the previous investment strategy, has a value of $\$ \frac{k A(t)}{A(0)}$ at time $t$.

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- Investing 1 at time zero, we get $\frac{A(t)}{A(0)}$ at time $t$.
- Investing $k$ at time zero, we get $\frac{k A(t)}{A(0)}$ at time $t$.


## Present value

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Let $x$ be the amount which need to invest at time zero to get a balance of $k$ at time $t$. We have that $k=\frac{x A(t)}{A(0)}$. So, $x=\frac{k A(0)}{A(t)}$. Hence, the present value at time 0 of a balance of $k$ had at time $t$ is $\frac{k A(0)}{A(t)}$.

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- The present value at time $t$ of a deposit of $k$ made at time zero is $k a(t)\left(=\frac{k A(t)}{A(0)}\right)$.
- The present value at time 0 of a balance of $k$ had at time $t$ is $\frac{k}{a(t)}\left(=\frac{k A(0)}{A(t)}\right)$.


## Example 1

The accumulation function of a fund is $a(t)=(1.03)^{2 t}, t \geq 0$. (i) Amanda invests $\$ 5000$ at time zero in this fund. Find the balance into Amanda's fund at time 2.5 years.
(ii) How much money does Kevin need to invest into the fund at time 0 to accumulate $\$ 10000$ at time 3?

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(ii) How much money does Kevin need to invest into the fund at time 0 to accumulate $\$ 10000$ at time 3?
Solution: (i) The balance into Amanda's fund at time 2.5 years is

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k a(2.5)=(5000)(1.03)^{2(2.5)}=5796.370371 .
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Solution: (i) The balance into Amanda's fund at time 2.5 years is

$$
k a(2.5)=(5000)(1.03)^{2(2.5)}=5796.370371
$$

(ii) The amount which Kevin needs to invest at time 0 to accumulate $\$ 10000$ at time 3 is

$$
\frac{10000}{a(3)}=\frac{10000}{(1.03)^{2(3)}}=8374.842567 .
$$

## Cashflow rules

Rule 2. Grows-depends-on-balance rule. If an investment follows the amount function $A(t), t \geq 0$, the growth during certain period where no deposits/withdrawals are made depends on the balance on the account at the beginning of the period.

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If an account has a balance of $k$ at time $t$ and no deposits/withdrawals are made in the future, then the future balance in this account does not depend on how the balance of $k$ at time $t$ was attained.
In particular, the following two accounts have the same balance for times bigger than $t$ :

1. An account where a unique deposit of $k$ is made at time $t$.
2. An account where a unique deposit of $\frac{k}{A(t)}$ is made at time zero.

Theorem 1
If an investment follows the amount function $A(\cdot)$, the present value at time $t$ of a deposit of $\$ k$ made at time $s$ is $\$ \frac{k A(t)}{A(s)}=\frac{k a(t)}{a(s)}$.

## Theorem 1

If an investment follows the amount function $A(\cdot)$, the present value at time $t$ of a deposit of $\$ k$ made at time $s$ is $\$ \frac{k A(t)}{A(s)}=\frac{k a(t)}{a(s)}$.
Proof. We need to invest $\frac{k}{A(s)}$ at time 0 to get a balance of $k$ at time $s$. So, investing $k$ at time $s$ is equivalent to investing $\frac{k}{A(s)}$ at time 0 . The future value at time $t$ of an investment of $\frac{k}{A(s)}$ at time 0 is $\frac{k A(t)}{A(s)}$. Hence, investing $k$ at time $s$ is equivalent to investing $\frac{k A(t)}{A(s)}$ at time $t$.

Another way to see the previous theorem is as follows. The following three accounts have the same balance at any time bigger than $t$ :

1. An account where a unique deposit of $A(0)$ is made at time zero.
2. An account where a unique deposit of $A(s)$ is made at time $s$.
3. An account where a unique deposit of $A(t)$ is made at time $t$.

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The present value at time $t$ of an investment of $A(s)$ made at time $s$ is $A(t)$.

This means that:

- If $t>s$, an investment of $A(s)$ made at time $s$ has an accumulation value of $A(t)$ at time $t$.
- If $t<s$, to get an accumulation of $A(s)$ at time $s$, we need to invest $A(t)$ at time $t$.

We know that:
The present value at time $t$ of an investment of $A(s)$ made at time $s$ is $A(t)$, i.e.
$A(s)$ at time $s$ is equivalent to $A(t)$ at time $t$.

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By proportionality,

- The present value at time $t$ of an investment of 1 made at time $s$ is $\frac{A(t)}{A(s)}$, i.e.

1 at time $s$ is equivalent to $\frac{A(t)}{A(s)}$ at time $t$.

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- The present value at time $t$ of an investment of $k$ made at time $s$ is $\frac{k A(t)}{A(s)}$, i.e.
$k$ at time $s$ is equivalent to $\frac{k A(t)}{A(s)}$ at time $t$.


## Example 2

The accumulation function of a fund follows the function $a(t)=1+\frac{t}{20}, t>0$.
(i) Michael invests $\$ 3500$ into the fund at time 1. Find the value of Michael's fund account at time 4.
(ii) How much money needs Jason to invest at time 2 to accumulate $\$ 700$ at time 4.

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(i) Michael invests $\$ 3500$ into the fund at time 1. Find the value of Michael's fund account at time 4.
(ii) How much money needs Jason to invest at time 2 to accumulate $\$ 700$ at time 4.
Solution: (i) The value of Michael's account at time 4 is $3500 \frac{a(4)}{a(1)}=(3500) \frac{1+\frac{4}{20}}{1+\frac{1}{20}}=(3500) \frac{1.20}{1.05}=4000$.

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(i) Michael invests $\$ 3500$ into the fund at time 1. Find the value of Michael's fund account at time 4.
(ii) How much money needs Jason to invest at time 2 to accumulate $\$ 700$ at time 4.

Solution: (i) The value of Michael's account at time 4 is $3500 \frac{a(4)}{a(1)}=(3500) \frac{1+\frac{4}{20}}{1+\frac{1}{20}}=(3500) \frac{1.20}{1.05}=4000$.
(ii) To accumulate $\$ 700$ at time 4, Jason needs to invest at time 2, $700 \frac{a(2)}{a(4)}=700 \frac{1.1}{1.2}=641.67$.

Theorem 3
Present value of a cashflow. If an investment account follows the amount function $A(t), t>0$, the (equation of value) present value at time $t$ of the cashflow

| Deposits | $C_{1}$ | $C_{2}$ | $\cdots$ | $C_{n}$ |
| :---: | :---: | :---: | :---: | :---: |
| Time | $t_{1}$ | $t_{2}$ | $\cdots$ | $t_{n}$ |

where $0 \leq t_{1}<t_{2}<\cdots<t_{n}$, is

$$
V(t)=\sum_{j=1}^{n} C_{j} \frac{A(t)}{A\left(t_{j}\right)}
$$

## Proof. Let $s>t_{n}$.

| Time | Balance before deposit | Balance after deposit |
| :--- | :--- | :--- |
| $t_{1}$ | 0 | $C_{1}$ |
| $t_{2}$ | $C_{1} \frac{a\left(t_{2}\right)}{a\left(t_{1}\right)}=\sum_{j=1}^{1} C_{j} \frac{a\left(t_{2}\right)}{a\left(t_{j}\right)}$ | $C_{1} \frac{a\left(t_{2}\right)}{a\left(t_{1}\right)}+C_{2}=\sum_{j=1}^{2} C_{j} \frac{a\left(t_{2}\right)}{a\left(t_{j}\right)}$ |
| $t_{3}$ | $\sum_{j=1}^{2} C_{j} \frac{a\left(t_{3}\right)}{a\left(j_{j}\right)}$ | $\sum_{j=1}^{3} C_{j} \frac{a\left(t_{3}\right)}{a\left(t_{j}\right)}$ |
| $t_{4}$ | $\sum_{j=1}^{3} C_{j} \frac{a\left(t_{4}\right)}{a\left(t_{j}\right)}$ | $\sum_{j=1}^{4} C_{j} \frac{a\left(t_{4}\right)}{a\left(t_{j}\right)}$ |
| $\ldots$ | $\cdots$ | $\cdots$ |
| $t_{n}$ | $\sum_{j=1}^{n-1} C_{j} \frac{a\left(t_{n}\right)}{a\left(t_{j}\right)}$ | $\sum_{j=1}^{n} C_{j} \frac{a\left(t_{n}\right)}{a\left(t_{j}\right)}$ |

Since the balance after deposit at time $t_{1}$ is $C_{1}$, the balance before deposit at time $t_{2}$ is $\frac{a\left(t_{2}\right)}{a\left(t_{1}\right)} C_{1}$.
Since the balance after deposit at time $t_{2}$ is $\sum_{j=1}^{2} C_{j} \frac{a\left(t_{2}\right)}{a\left(t_{j}\right)}$, the balance before deposit at time $t_{3}$ is $\frac{a\left(t_{3}\right)}{a\left(t_{2}\right)} \sum_{j=1}^{2} C_{j} \frac{a\left(t_{2}\right)}{a\left(t_{j}\right)}=\sum_{j=1}^{2} C_{j} \frac{a\left(t_{3}\right)}{a\left(t_{j}\right)}$.

Hence, the balance at time $s$ is

$$
\frac{a(s)}{a\left(t_{n}\right)} \sum_{j=1}^{n} C_{j} \frac{a\left(t_{n}\right)}{a\left(t_{j}\right)}=\sum_{j=1}^{n} C_{j} \frac{a(s)}{a\left(t_{j}\right)}
$$

The future value of the cashflow at time $t$ is

$$
\frac{a(t)}{a(s)} \sum_{j=1}^{n} C_{j} \frac{a(s)}{a\left(t_{j}\right)}=\sum_{j=1}^{n} C_{j} \frac{a(t)}{a\left(t_{j}\right)}
$$

end of proof.

Notice that the present value at time $t$ of the cashflow

| Deposits | $C_{1}$ | $C_{2}$ | $\cdots$ | $C_{n}$ |
| :---: | :---: | :---: | :---: | :---: |
| Time | $t_{1}$ | $t_{2}$ | $\cdots$ | $t_{n}$ |

is the same as the sum of the present values at time $t$ of $n$ separated investment accounts each following the amount function $A$, with the $j$-the account having a unique deposit of $C_{j}$ at time $t_{j}$.

## Example 4

The accumulation function of a fund follows the function $a(t)=1+\frac{t}{20}, t>0$. Jared invests $\$ 1000$ into the fund at time 1 and he withdraws $\$ 500$ at time 3. Find the value of Jared's fund account at time 5.

## Example 4

The accumulation function of a fund follows the function $a(t)=1+\frac{t}{20}, t>0$. Jared invests $\$ 1000$ into the fund at time 1 and he withdraws $\$ 500$ at time 3. Find the value of Jared's fund account at time 5.

Solution: The cashflow is

| deposit/withdrawal | 1000 | -500 |
| :---: | :---: | :---: |
| Time (in years) | 1 | 3 |.

The value of Jared's account at time 5 is

$$
\begin{aligned}
& 1000 \frac{a(5)}{a(1)}-500 \frac{a(5)}{a(3)}=1000 \frac{1+\frac{5}{20}}{1+\frac{1}{20}}-500 \frac{1+\frac{5}{20}}{1+\frac{3}{20}} \\
= & (1000) \frac{1.25}{1.05}-(500) \frac{1.25}{1.15}=1190.48-543.48=647.00 .
\end{aligned}
$$

