$$\lim_{n\to\infty} \frac{n}{n^2+3n+1} = 0 \text{ (by definition)}$$

$\forall \varepsilon > 0 \ \exists N \in \mathbb{N}$ s.t :

$\forall n \geq N \Rightarrow \left| \frac{n}{n^2+3n+1} - 0 \right| < \epsilon$

$$\frac{n}{n^2+3n+1} < \frac{n}{n^2} = \frac{1}{n} < \epsilon$$

let $\varepsilon > 0 \xrightarrow{\text{Ar property}} \exists N \in \mathbb{N}$ s.t : $\varepsilon > \frac{1}{N}$

$\forall n \geq N \Rightarrow \epsilon > \frac{1}{N} \geq \frac{1}{n} = \frac{n}{n^2} > \frac{n}{n^2+3n+1}$

i.e $\varepsilon > \left| \frac{n}{n^2+3n+1} - 0 \right|$

بقية فقرات definition ، حاول فيها وأرسل لي
الكل (لمحاكاة حل السؤال أعلاه)

(3) $x_n = (-1)^n \frac{n+1}{n^2+1} \sin(2n+1)$

$|x_n| = \frac{n+1}{n^2+1} |\sin(2n+1)|$

$0 \le |\sin(2n+1)| \le 1$

$\underset{\downarrow \atop 0}{0} \le \frac{n+1}{n^2+1} |\sin(2n+1)| \le \underset{\downarrow \atop 0}{\frac{n+1}{n^2+1}}$

$\Rightarrow$ By Sandwich Theorem:

$\lim_{n \to \infty} \frac{n+1}{n^2+1} |\sin(2n+1)| = 0$

$\Rightarrow \lim_{n \to \infty} |x_n| = 0$

$\Rightarrow \lim_{n \to \infty} x_n = 0$ (conv)

(6) $x_n = (-1)^n n^2 \frac{n+1}{n^2+1} \sin(\frac{1}{n})$

$x_n = (-1)^n \frac{n(n+1)}{n^2+1} \frac{\sin \frac{1}{n}}{\frac{1}{n}}$

$x_n = (-1)^n \frac{n^2+n}{n^2+1} \cdot \frac{\sin \frac{1}{n}}{\frac{1}{n}}$

$\frac{n^2+n}{n^2+1} \to 1$

$\lim_{n \to \infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}}$

$\frac{1}{n} = y$

$\lim_{y \to 0} \frac{\sin y}{y} = 1$

Take subsequences:

$x_{2n} = \underset{1}{(-1)^{2n}} \frac{(2n)^2+2n}{(2n)^2+1} \frac{\sin \frac{1}{2n}}{\frac{1}{2n}} \to 1$

$x_{2n+1} = \underset{-1}{(-1)^{2n+1}} \frac{(2n+1)^2+(2n+1)}{(2n+1)^2+1} \frac{\sin(\frac{1}{2n+1})}{\frac{1}{2n+1}} \to -1$

$\lim x_{2n} \neq \lim x_{2n+1}$

$\Rightarrow \lim_{n \to \infty} x_n$ D.N.E