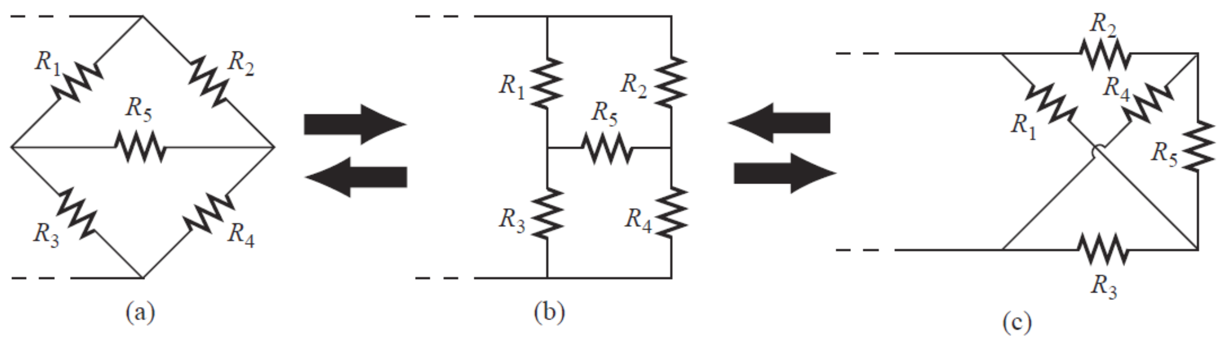
Methods of Analysis and Selected Topics (dc)

8.11 BRIDGE NETWORKS

The **bridge** network has many application:

- DC and AC meters
- In electronic rectifying circuit (convert AC to DC)

The bridge may appear in any of the three forms shown:

planar network can be made to appear nonplanar as in (c).	 **FIG. 8.63** *Various formats for a bridge network.*

The network is called symmetrical lattice network if:

$$R_2 = R_3 \quad \text{and} \quad R_1 = R_4$$

Let's examine the standard bridge configuration shown using mesh and nodal analysis:

Mesh analysis (Fig. 8.65) yields

$$(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 = 20\text{ V}$$
$$(4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$
$$(2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

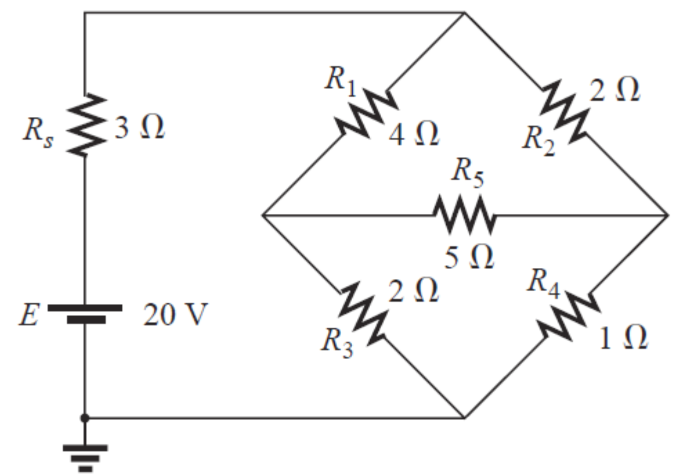
and

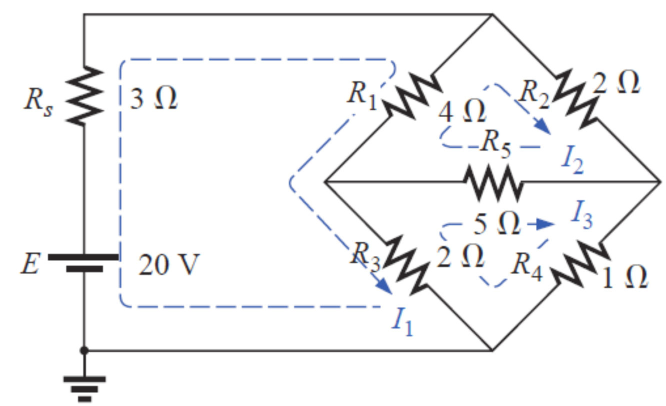
$$9I_1 - 4I_2 - 2I_3 = 20$$
$$-4I_1 + 11I_2 - 5I_3 = 0$$
$$-2I_1 - 5I_2 + 8I_3 = 0$$

with the result that

$$I_1 = \mathbf{4\ A}$$
$$I_2 = \mathbf{2.667\ A}$$
$$I_3 = \mathbf{2.667\ A}$$

The net current through the 5-Ω resistor is





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and

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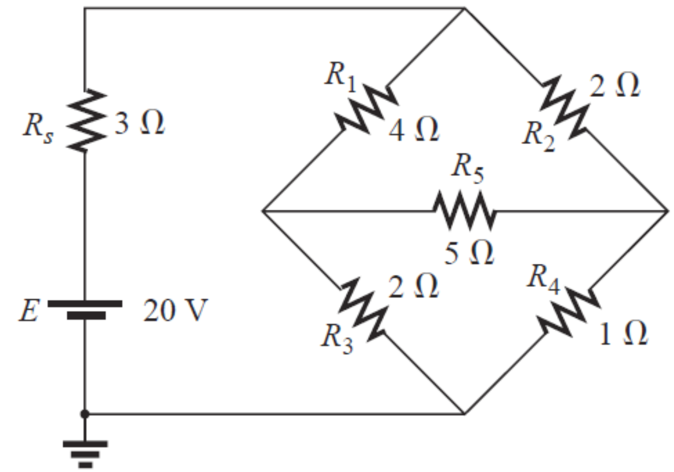
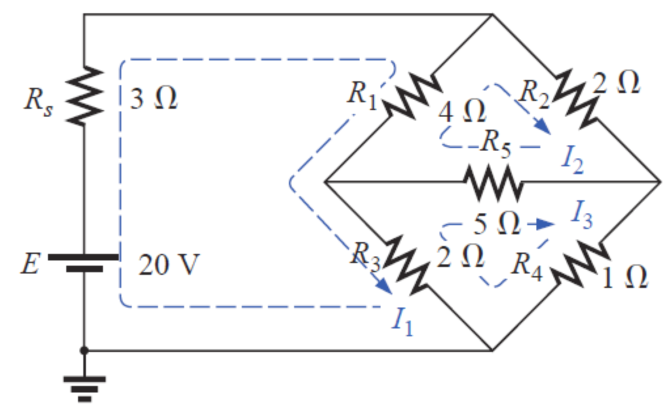
The net current through the 5-Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = \mathbf{2.667\ A - 2.667\ A = 0\ A}$$





Nodal analysis (Fig. 8.66) yields

$$\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{4\ \Omega}\right)V_2-\left(\frac{1}{2\ \Omega}\right)V_3=\frac{20}{3}\text{ A}$$

$$\left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2-\left(\frac{1}{4\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_3=0$$

$$\left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_2=0$$

and

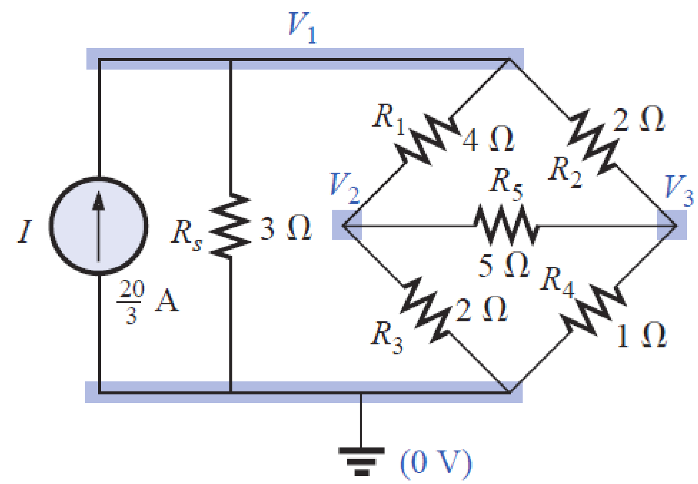
$$\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{4\ \Omega}\right)V_2-\left(\frac{1}{2\ \Omega}\right)V_3=\frac{20}{3}\text{ A}$$

$$-\left(\frac{1}{4\ \Omega}\right)V_1+\left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2-\left(\frac{1}{5\ \Omega}\right)V_3=0$$

$$-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_2+\left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3=0$$

$V_1 = \mathbf{8\ V}$

$V_2 = \mathbf{2.667\ V}$ and $V_3 = \mathbf{2.667\ V}$



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$$\left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2-\left(\frac{1}{4\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_3=0$$

$$\left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_2=0$$

and

$$\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{4\ \Omega}\right)V_2-\left(\frac{1}{2\ \Omega}\right)V_3=\frac{20}{3}\ \text{A}$$

$$-\left(\frac{1}{4\ \Omega}\right)V_1+\left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2-\left(\frac{1}{5\ \Omega}\right)V_3=0$$

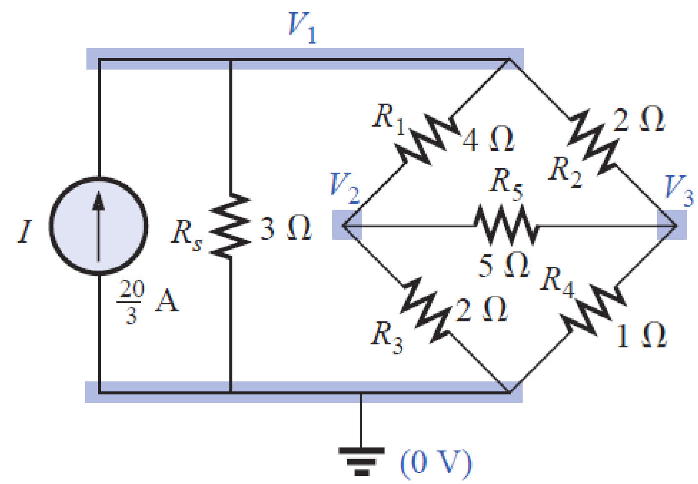
$$-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_2+\left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3=0$$

$V_1 = \mathbf{8\ V}$

$V_2 = \mathbf{2.667\ V}$ and $V_3 = \mathbf{2.667\ V}$

and the voltage across the 5-Ω resistor is

$$V_{5\Omega} = V_2 - V_3 = \mathbf{2.667\ V - 2.667\ V = 0\ V}$$



R_5 can be replaced by a short circuit

since $V_{R5} = 0$.

We obtain the same voltages at the nodes:

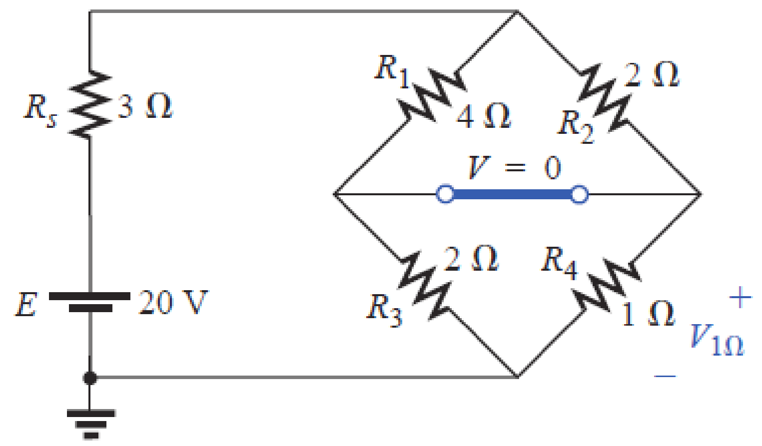
Using voltage divider rule:

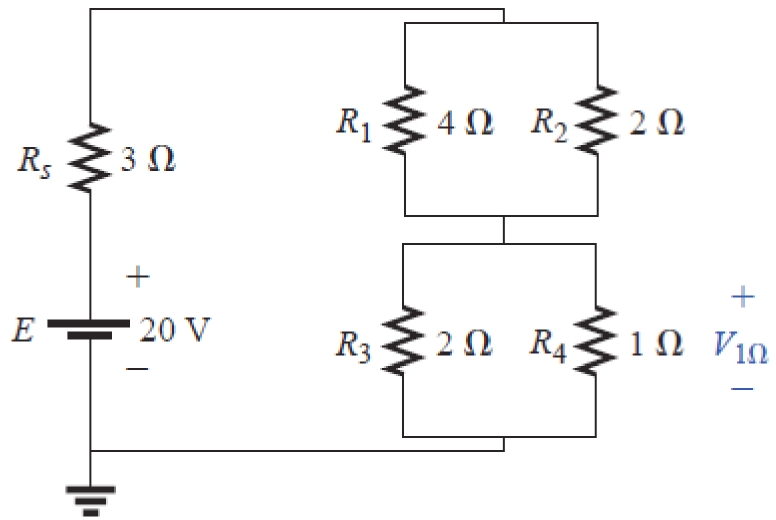
$$V_{1\Omega} = \frac{(2\ \Omega \parallel 1\ \Omega)20\ \text{V}}{(2\ \Omega \parallel 1\ \Omega) + (4\ \Omega \parallel 2\ \Omega) + 3\ \Omega}$$

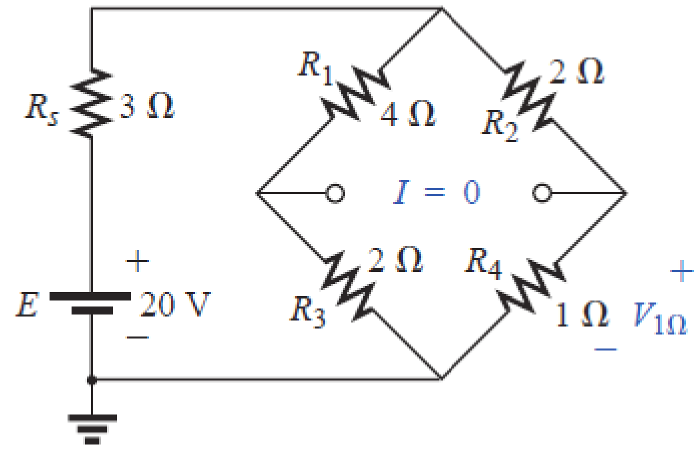
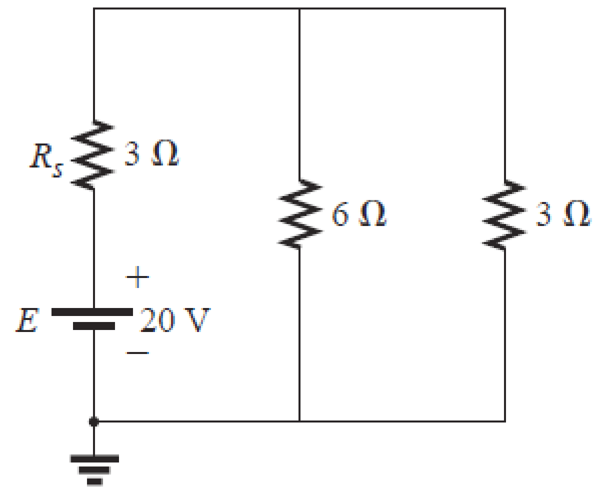
$$= \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}}$$

$$= \frac{2(20\ \text{V})}{2 + 4 + 9} = \frac{40\ \text{V}}{15} = \mathbf{2.667\ V}$$

VR3=VR4, they are in parallel!!!





R_5 can be replaced by a open circuit since **$I_{R5} = 0$**. We obtain the same voltages and currents: $$V_{3\Omega} = \frac{(6\ \Omega \parallel 3\ \Omega)(20\ V)}{6\ \Omega \parallel 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ V)}{2\ \Omega + 3\ \Omega} = 8\ V$$ $$V_{1\Omega} = \frac{1\ \Omega(8\ V)}{1\ \Omega + 2\ \Omega} = \frac{8\ V}{3} = \mathbf{2.667\ V}$$	 

The bridge network is balanced when the condition: **I = 0** or **V = 0** exists in the middle branch.

V = 0: short circuit between *a* and *b*:

$$V_1 = V_2 \Longrightarrow I_1 \cdot R_1 = I_2 \cdot R_2 \Longrightarrow \boldsymbol{I_1 = I_2 \cdot \frac{R_2}{R_1}}$$

In addition when V = 0:

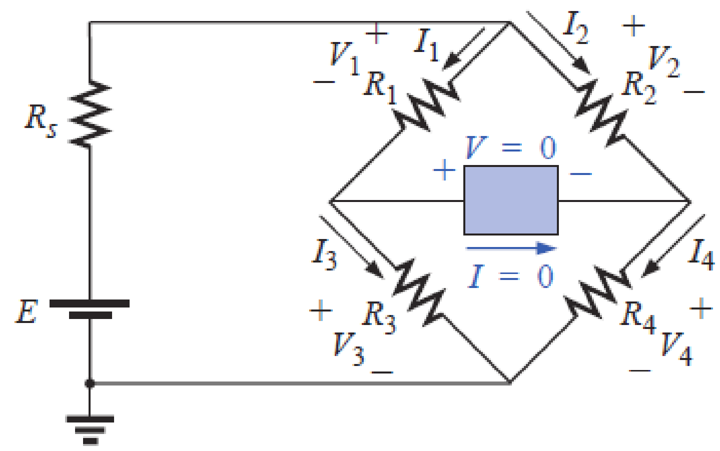
$$V_3 = V_4 \Longrightarrow I_3 \cdot R_3 = I_4 \cdot R_4 \Longrightarrow \boldsymbol{I_3 = I_4 \cdot \frac{R_4}{R_3}}$$

If we set I = 0: $\Longrightarrow I_3 = I_1 \;\; and \;\; I_4 = I_2$

Thus:

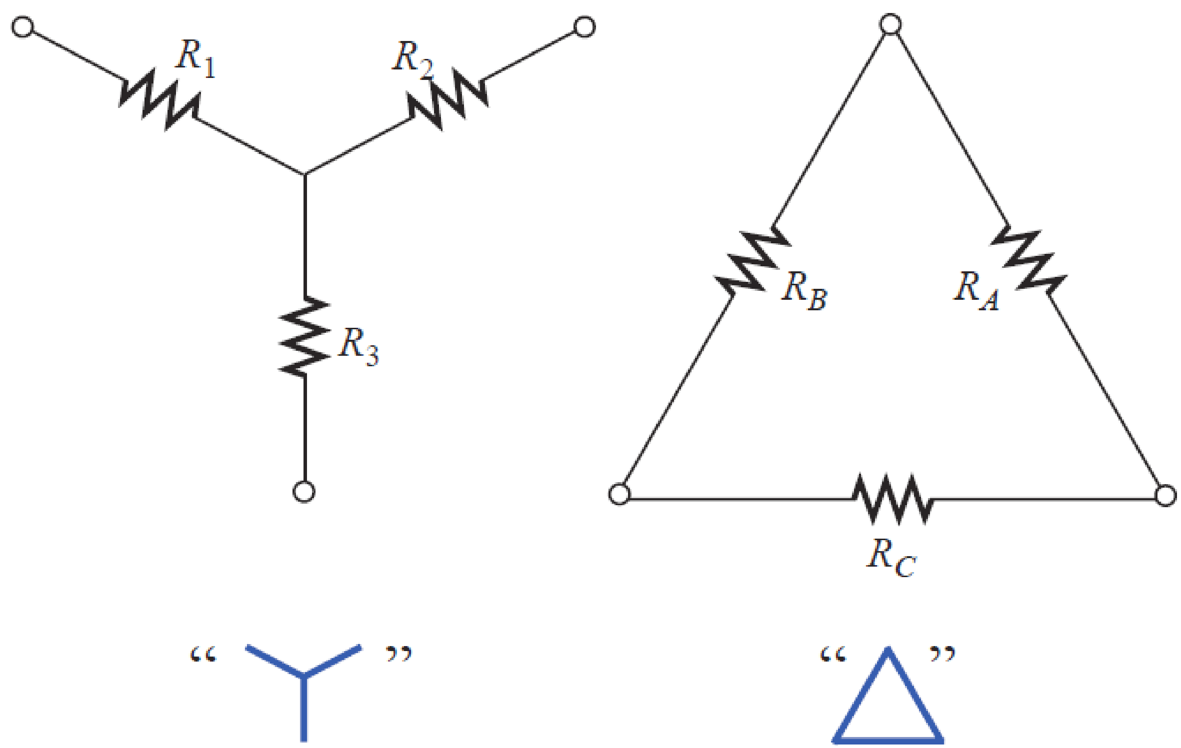
$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

This is the condition for the bridge to be balanced and therefore: V = 0 and I = 0.



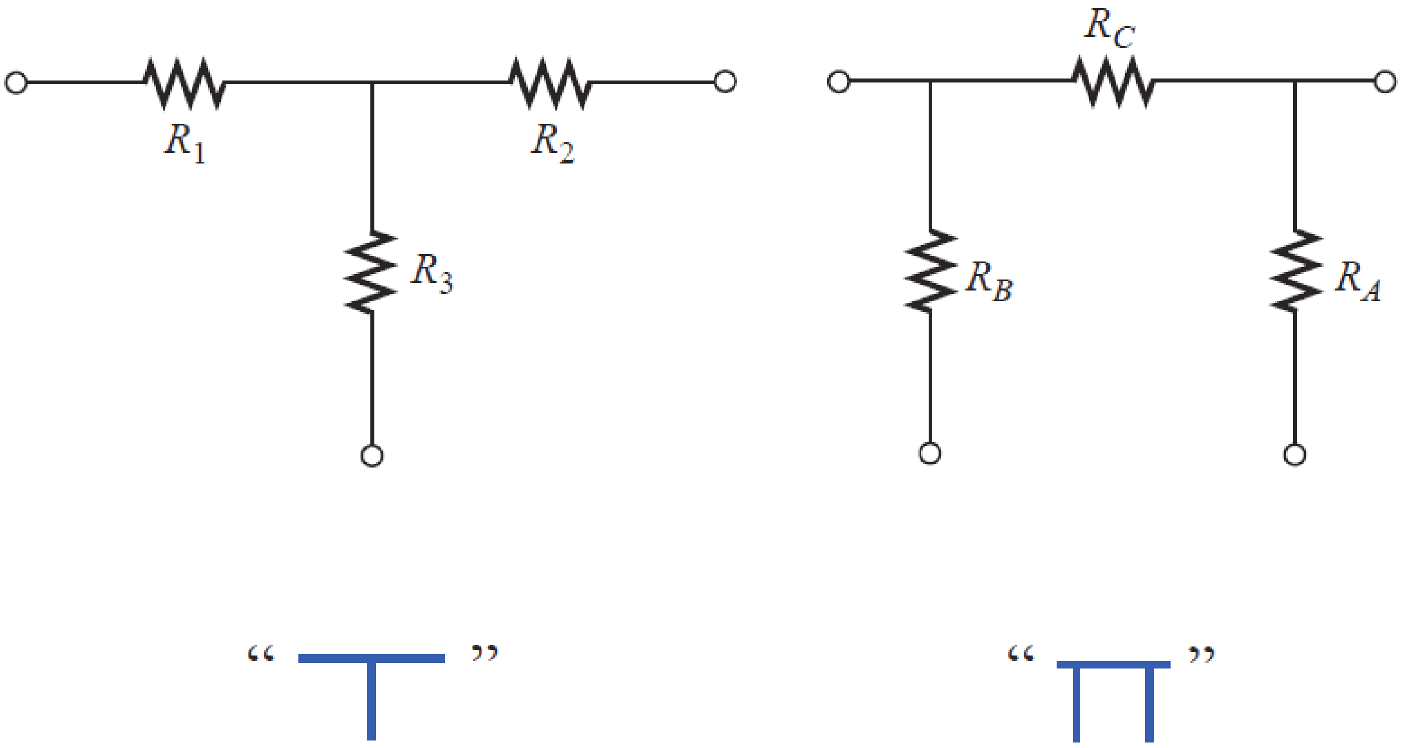
8.12 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

- There are circuit configurations where resistors do not appear to be in series or parallel.
- May be necessary to convert the circuit from one form to another to easier find solutions.
- Two such configuration are: WYE (Y) and DELTA (Δ) configurations



(a)

- They are also referred as **TEE (T)** and **PI (π)**, the pi is an inverted delta



(b)

- Develop equations to convert from **Y** to **Δ** and vice versa
- Using these equations we can replace a Y to Δ (or Δ to Y) whichever is more convenient.
- Find R_1, R_2 and R_3 in terms of R_A, R_B and R_C and vice versa, which will ensure that the resistance between any two terminals is the same.

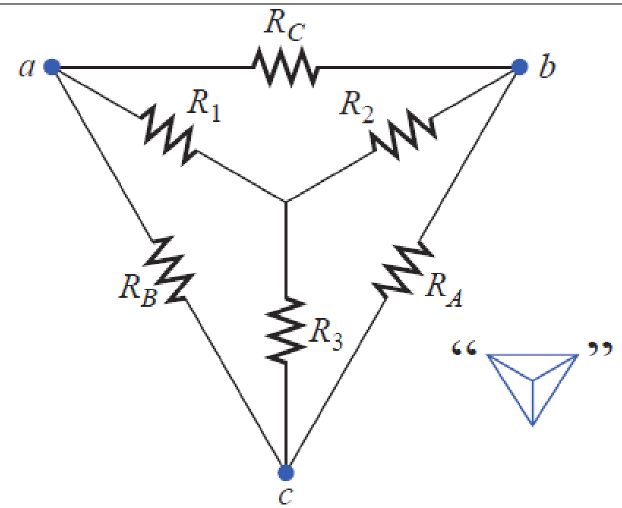


FIG. 8.73
Introducing the concept of Δ-Y or Y-Δ conversions.

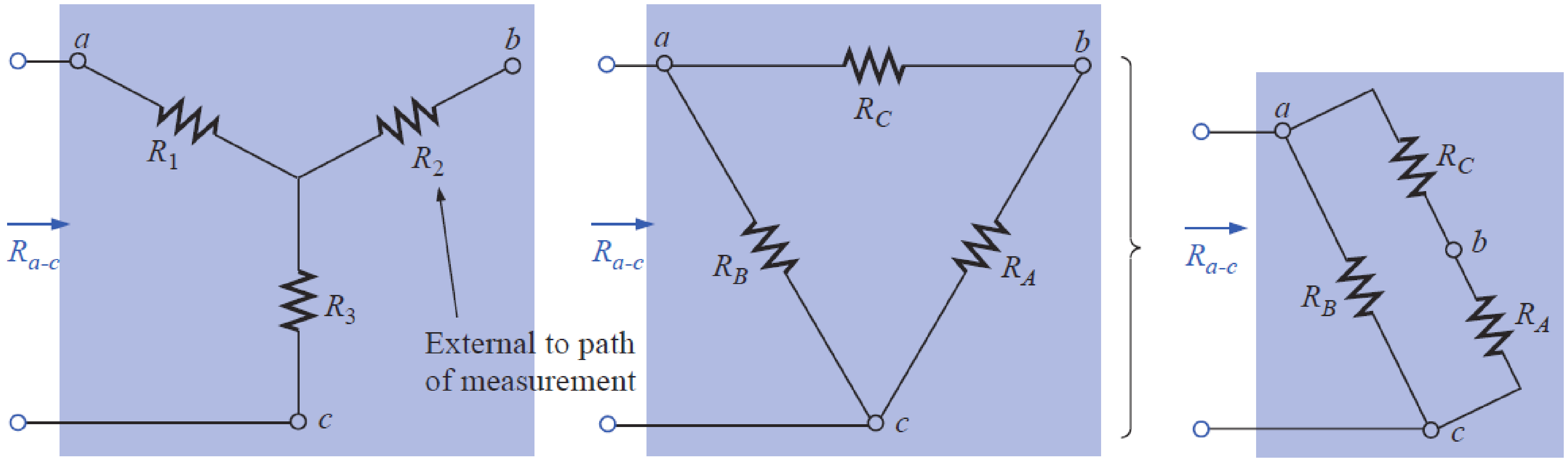


FIG. 8.74

Finding the resistance $R_{a\text{-}c}$ for the Y and Δ configurations.

The two configuration are equivalent if: $R_{a\text{-}c}\,(Y) = R_{a\text{-}c}\,(\Delta)$	$R_{a\text{-}c} = R_1 + R_3 = \dfrac{R_B(R_A + R_C)}{R_B + (R_A + R_C)}$
Similarly for $R_{a\text{-}b}$ and $R_{b\text{-}c}$:	$R_{a\text{-}b} = R_1 + R_2 = \dfrac{R_C(R_A + R_B)}{R_C + (R_A + R_B)}$ $R_{b\text{-}c} = R_2 + R_3 = \dfrac{R_A(R_B + R_C)}{R_A + (R_B + R_C)}$

$$(R_1 + R_2) - (R_1 + R_3) = \left(\frac{R_C R_B + R_C R_A}{R_A + R_B + R_C}\right) - \left(\frac{R_B R_A + R_B R_C}{R_A + R_B + R_C}\right)$$

$$R_2 - R_3 = \frac{R_A R_C - R_B R_A}{R_A + R_B + R_C}$$

$$(R_2 + R_3) - (R_2 - R_3) = \left(\frac{R_A R_B + R_A R_C}{R_A + R_B + R_C}\right) - \left(\frac{R_A R_C - R_B R_A}{R_A + R_B + R_C}\right)$$

$2R_3 = \dfrac{2R_B R_A}{R_A + R_B + R_C}$	$\Rightarrow$	$R_3 = \dfrac{R_A R_B}{R_A + R_B + R_C}$

Similarly we find:	$R_1 = \dfrac{R_B R_C}{R_A + R_B + R_C}$	$R_2 = \dfrac{R_A R_C}{R_A + R_B + R_C}$

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

To obtain the converse relations:

$\frac{R_3}{R_1} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_B R_C)/(R_A + R_B + R_C)} = \frac{R_A}{R_C}$	$\Rightarrow$	$R_A = \frac{R_C R_3}{R_1}$
$\frac{R_3}{R_2} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_A R_C)/(R_A + R_B + R_C)} = \frac{R_B}{R_C}$	$\Rightarrow$	$R_B = \frac{R_3 R_C}{R_2}$

Substituting for R_A and R_B in the equation of R_2 yields:

$$R_2 = \frac{(R_C R_3 / R_1) R_C}{(R_3 R_C / R_2) + (R_C R_3 / R_1) + R_C} = \frac{(R_3 / R_1) R_C}{(R_3 / R_2) + (R_3 / R_1) + 1}$$

Placing these over a common denominator, we obtain

$$R_2 = \frac{(R_3 R_C / R_1)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)/(R_1 R_2)} = \frac{R_2 R_3 R_C}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

We follow the same procedure for R_B and R_A:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

and

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

If all resistor are equal ($R_1 = R_2 = R_3 = R_Y$) and ($R_A = R_B = R_C = R_\Delta$) then

$R_Y = \frac{R_\Delta}{3}$	$R_\Delta = 3R_Y$

If only two elements of a Y (or a Δ) are the same, the corresponding Δ (or Y) of each will also have two equal elements.

The Y and the Δ will often appear as shown in Fig. 8.75. They are then referred to as a **tee (T)** and a **pi (π)** network, respectively.

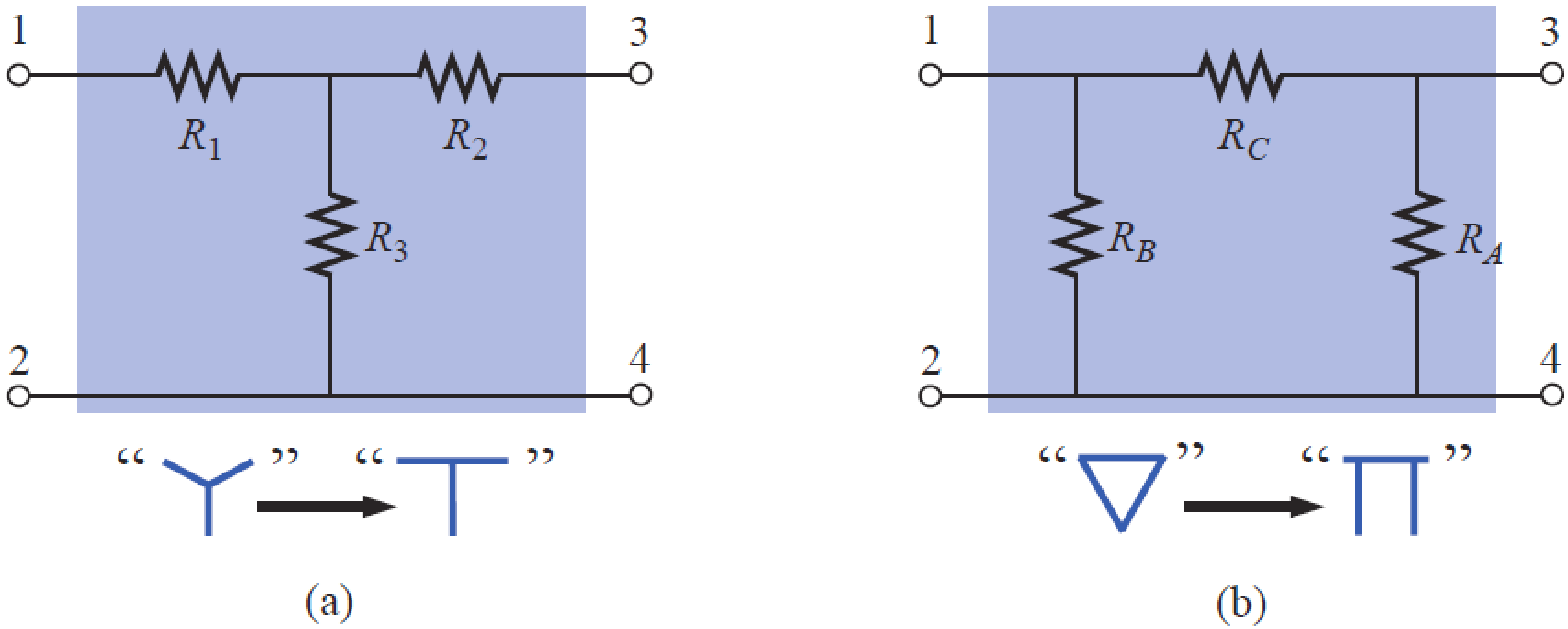


FIG. 8.75

The relationship between the Y and T configurations and the Δ and π configurations.

EXAMPLE 8.27 Convert the Δ of Fig. 8.76 to a Y.

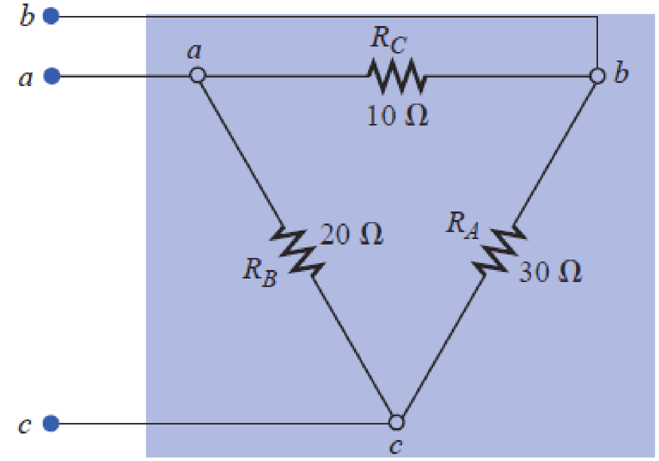
Solution:

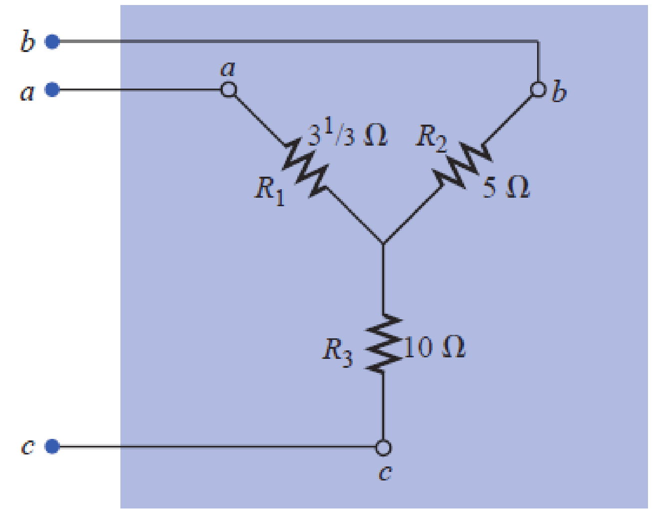
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 8.77 (page 298).





EXAMPLE 8.28 Convert the Y of Fig. 8.78 to a Δ.

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

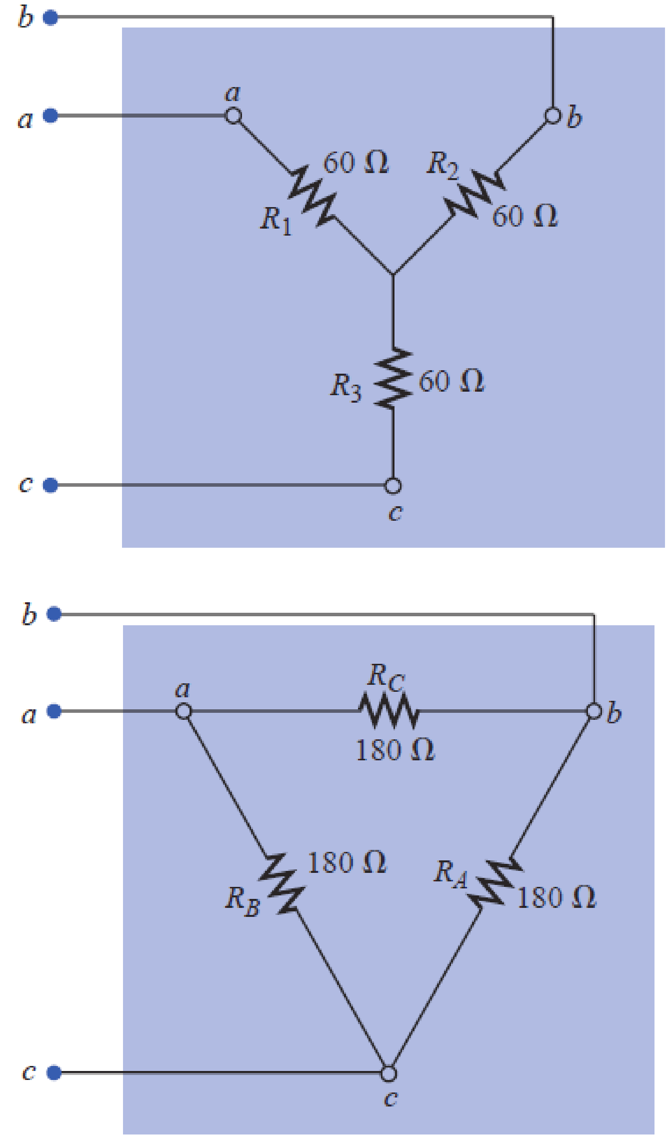
$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. (8.8) and yielding

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and $$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig. 8.79.



EXAMPLE 8.29 Find the total resistance of the network of Fig. 8.80, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

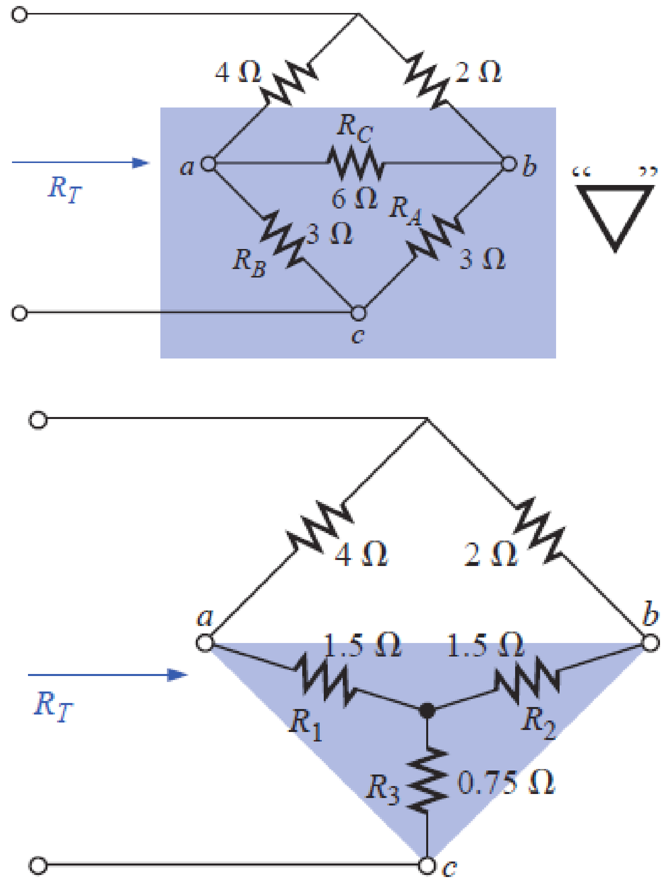
Replacing the Δ by the Y, as shown in Fig. 8.81, yields

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$

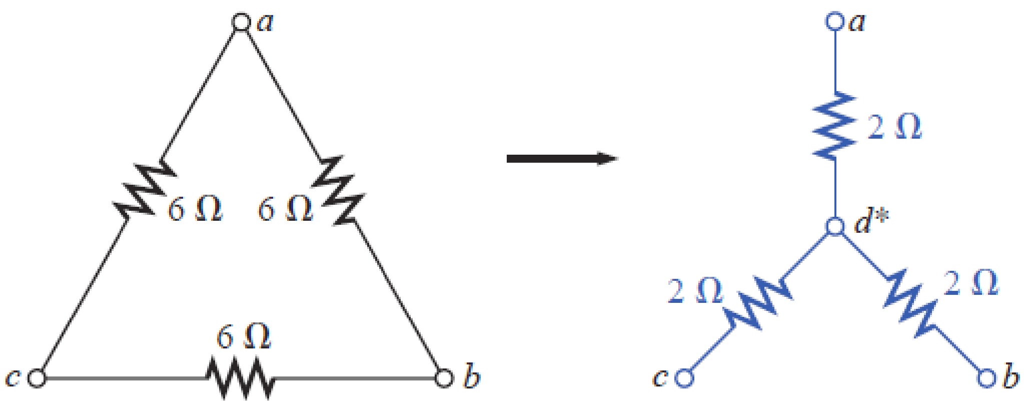


EXAMPLE 8.30 Find the total resistance of the network of Fig. 8.82.

Solutions: Since all the resistors of the Δ or Y are the same, Equations (8.8a) and (8.8b) can be used to convert either form to the other.

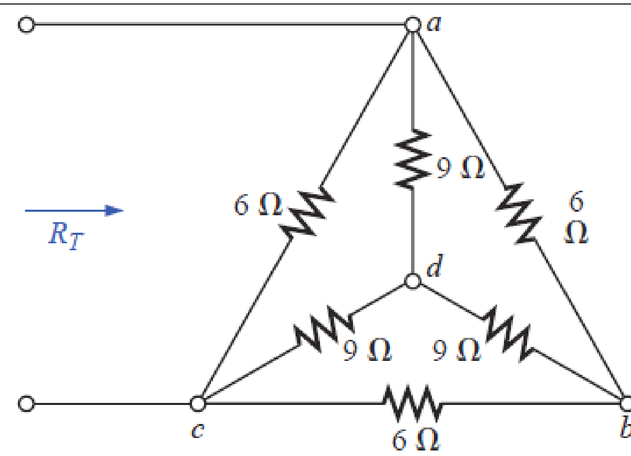
a. *Converting the Δ to a Y.* Note: When this is done, the resulting d' of the new Y will be the same as the point d shown in the original figure, only because both systems are "balanced." That is, the resistance in each branch of each system has the same value:

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega \qquad \text{(Fig. 8.83)}$$



The network then appears as shown in Fig. 8.84.

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.2727\ \Omega}$$



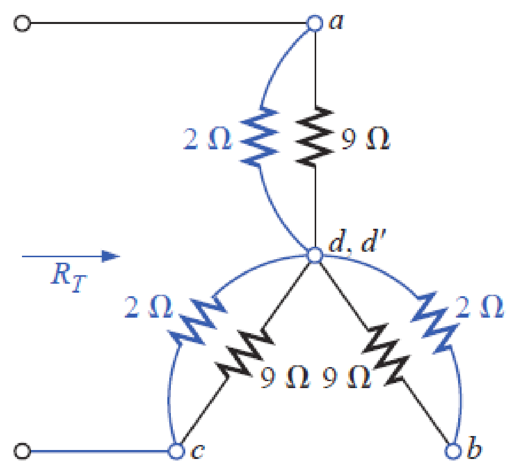


FIG. 8.84

Substituting the Y configuration for the converted Δ into the network of Fig. 8.82.

b. *Converting the Y to a Δ:*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega \qquad \text{(Fig. 8.85)}$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.9091\ \Omega$$

$$R_T = \frac{R'_T\,(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.9091\ \Omega)}{3} = \mathbf{3.2727\ \Omega}$$

which checks with the previous solution.

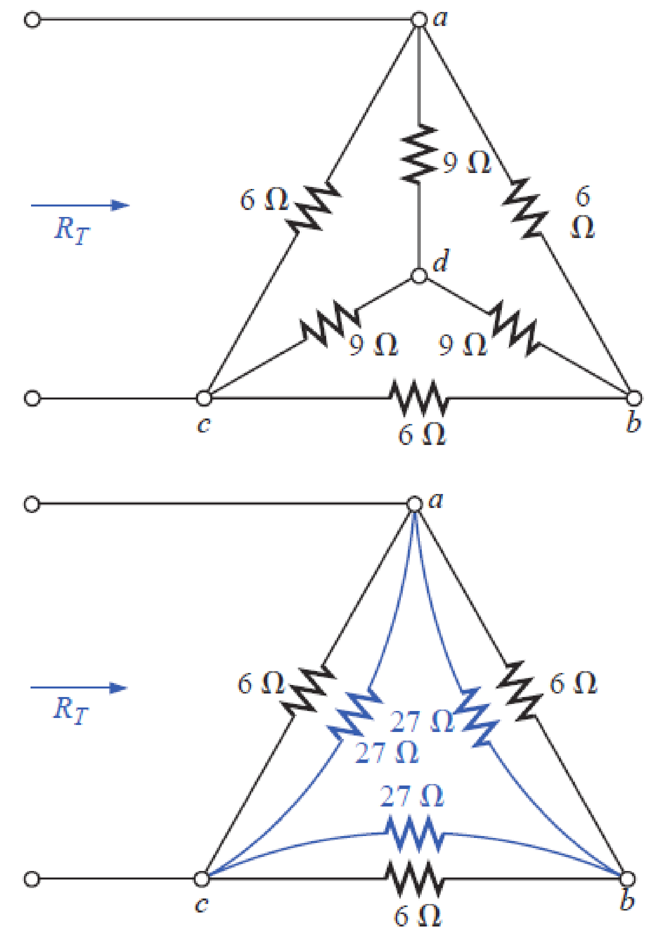
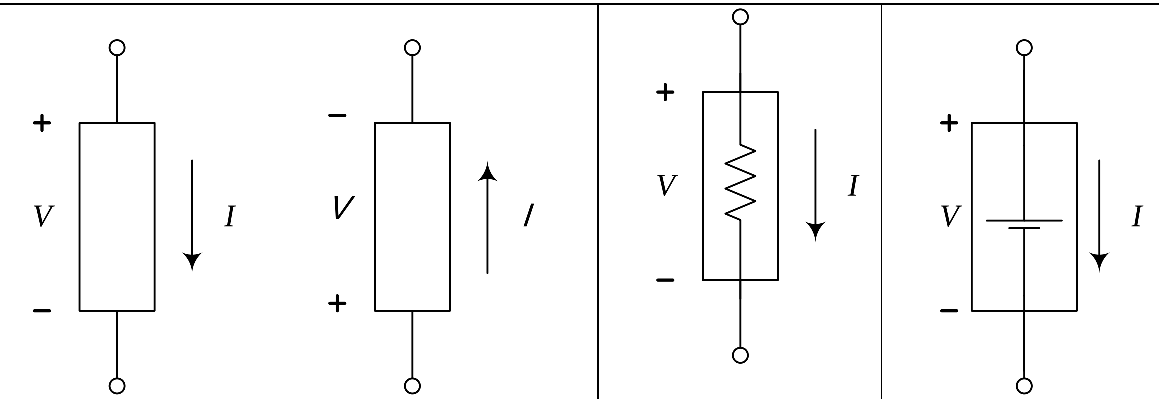


FIG. 8.85
Substituting the converted Y configuration into the network of Fig. 8.82.

General rules concerning Power:

P = VI

Power **absorbed** by the Element



P = VI

Power **delivered** by the Element