

The Basic Elements and Phasors

14.1 INTRODUCTION

- Response of R, L, and C elements to a sinusoidal voltage and current with the effect of the frequency.
- Phasor notation will be introduced and employed in the analysis.

14.2 THE DERIVATIVE

The derivative: $\frac{dx}{dt}$ of the variable x is defined as the rate of change of x with respect to time.

The sine wave and its derivative:

- $\frac{dx}{dt} = 0$ at $\omega t = \frac{\pi}{2}$ and $\frac{3\pi}{2}$
- $\frac{dx}{dt} = \text{max (positive)}$ at $\omega t = 0, 2\pi$
- $\frac{dx}{dt} = \text{max (negative)}$ at $\omega t = \pi$.
- $\frac{dx}{dt}$ will change gradually between these values in between.

The derivative of a sine wave is a cosine wave

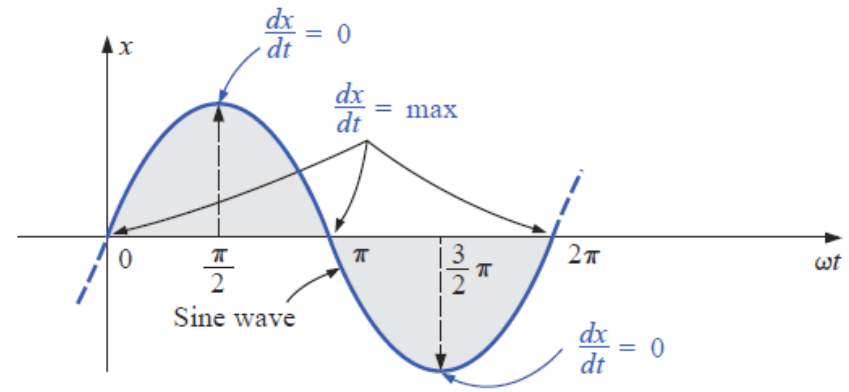


FIG. 14.1

Defining those points in a sinusoidal waveform that have maximum and minimum derivatives.

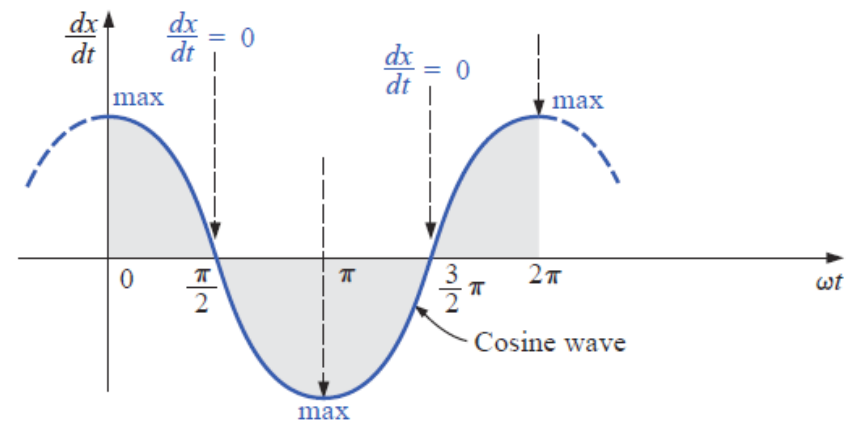


FIG. 14.2

Derivative of the sine wave of Fig. 14.1.

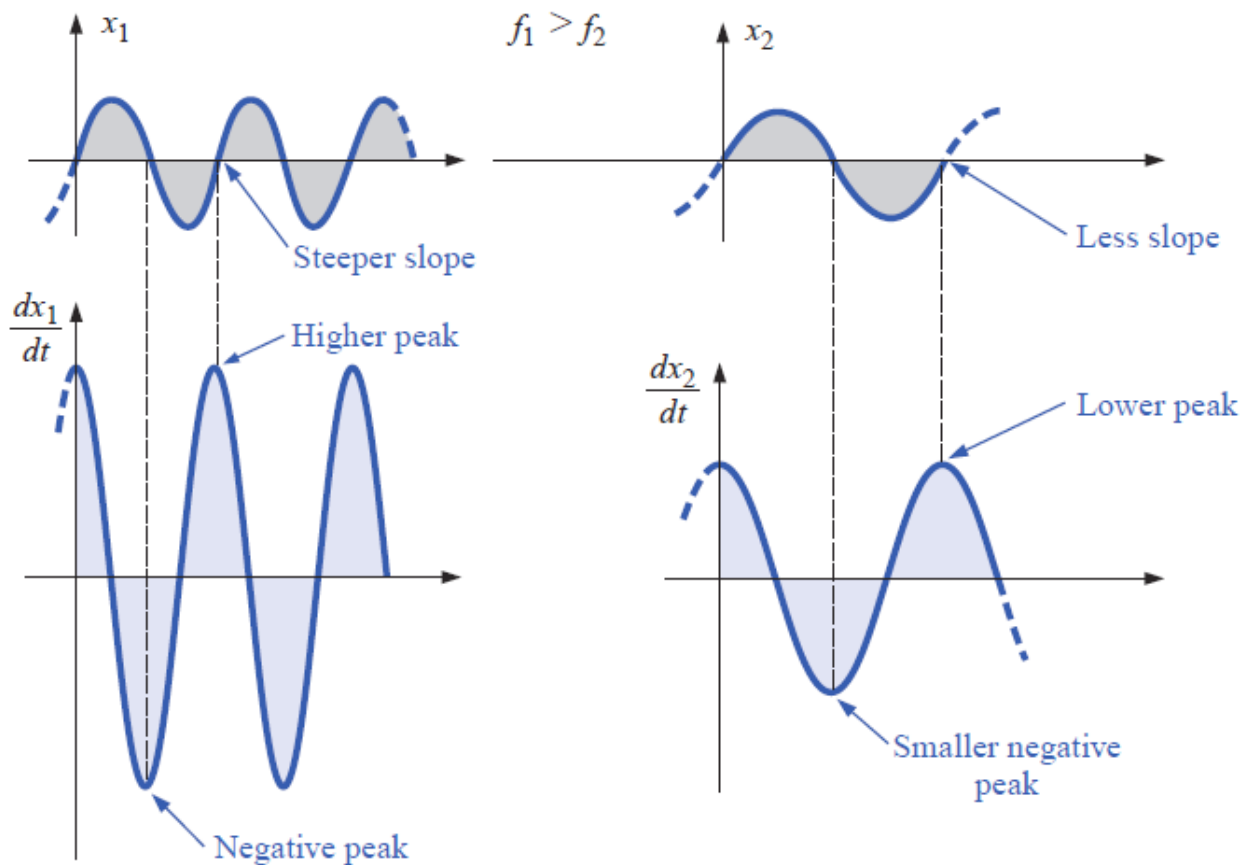


FIG. 14.3

Effect of frequency on the peak value of the derivative.

The **peak value of the cosine** wave is proportional to the **frequency** of the original wave.

*The derivative of a sine wave has the **same period and frequency** as the original sinusoidal waveform.*

For $e(t) = E_m \sin(\omega t \pm \theta)$, the derivative is:

$$\begin{aligned} \frac{d}{dt} e(t) &= \omega E_m \cos(\omega t \pm \theta) \\ &= 2\pi f E_m \cos(\omega t \pm \theta) \end{aligned}$$

14.3 RESPONSE OF BASIC R, L, C

1- Resistor R

For low and medium frequency up to ~ 100 kHz
Ohm' Law apply even for sinusoidal voltage and
current, for $v = V_m \sin \omega t$

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

$$V_m = I_m R$$

*for a purely resistive element, the voltage across
and the current through the element are in phase,
with their peak values related by Ohm's law.*

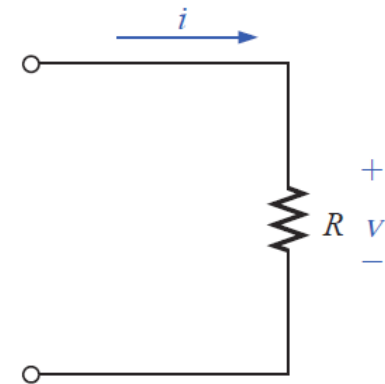


FIG. 14.4

Determining the sinusoidal response for a resistive element.

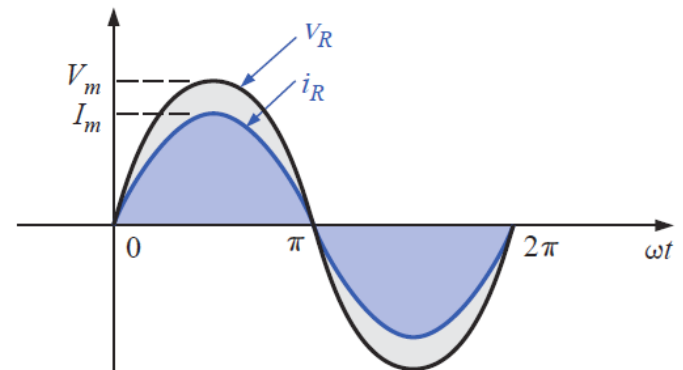


FIG. 14.5

The voltage and current of a resistive element are in phase.

2- Inductor L

The voltage across an inductor is directly related to the rate of change of current through the coil.

► Higher frequency \Rightarrow higher magnitude of v_L .

$$v_L = L \frac{di_L}{dt}$$

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

Therefore, $v_L = L \frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$

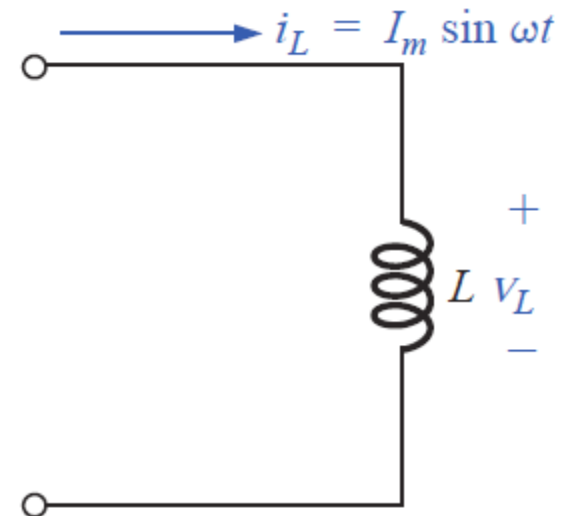
or $v_L = V_m \sin(\omega t + 90^\circ)$

where $V_m = \omega L I_m$

V_m is proportional to ω

For an inductor, v_L leads i_L by 90° .

i_L lags v_L by 90° .



Investigating the sinusoidal response of an inductive element.

$$i_L = I_m \sin(\omega t \pm \theta)$$

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90^\circ)$$

The quantity X_L :

$$X_L = \frac{V_m}{I_m} = \omega L \quad (\text{ohms, } \Omega)$$

is called reactance.

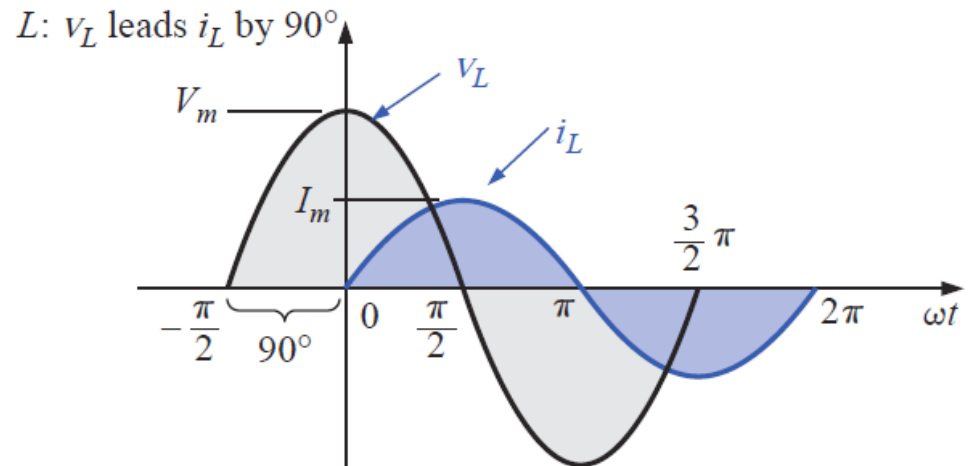


FIG. 14.9

For a pure inductor, the voltage across the coil leads the current through the coil by 90° .

- Inductive reactance is the opposition to the flow of current
- Inductors do not dissipate energy (not like resistors)
- There is continual interchange of energy between the source and the inductor

3- Capacitor C

The current of a capacitor is directly related to the frequency and the capacitance of the capacitor

► Higher frequency \Rightarrow higher magnitude of i_C .

$$i_C = C \frac{dv_C}{dt}$$

$$v_C = V_m \sin(\omega t)$$

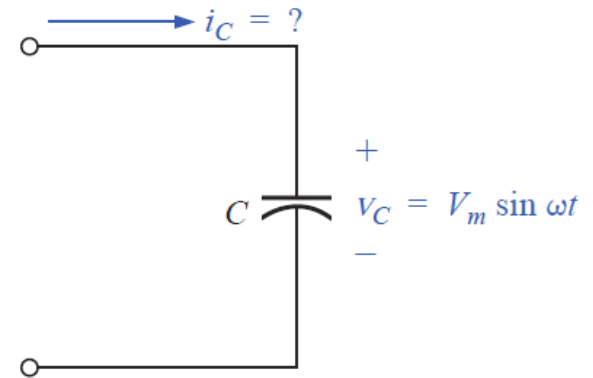
$$i_C = C \frac{dv_C}{dt} = C(\omega V_m \cos(\omega t)) = \omega C V_m \cos(\omega t)$$

$$i_C = I_m \sin(\omega t + 90^\circ)$$

$$I_m = \omega C V_m$$

For a capacitor, i_C leads v_C by 90° .

v_C lags i_C by 90° .



Investigating the sinusoidal response of a capacitive element.

$$v_C = V_m \sin(\omega t \pm \theta)$$

$$i_C = \omega C V_m \sin(\omega t \pm \theta + 90^\circ)$$

The quantity X_C :

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C} \quad (\text{ohms, } \Omega)$$

is called reactance.

$$X_C = \frac{1}{\omega C} \quad (\text{ohms, } \Omega)$$

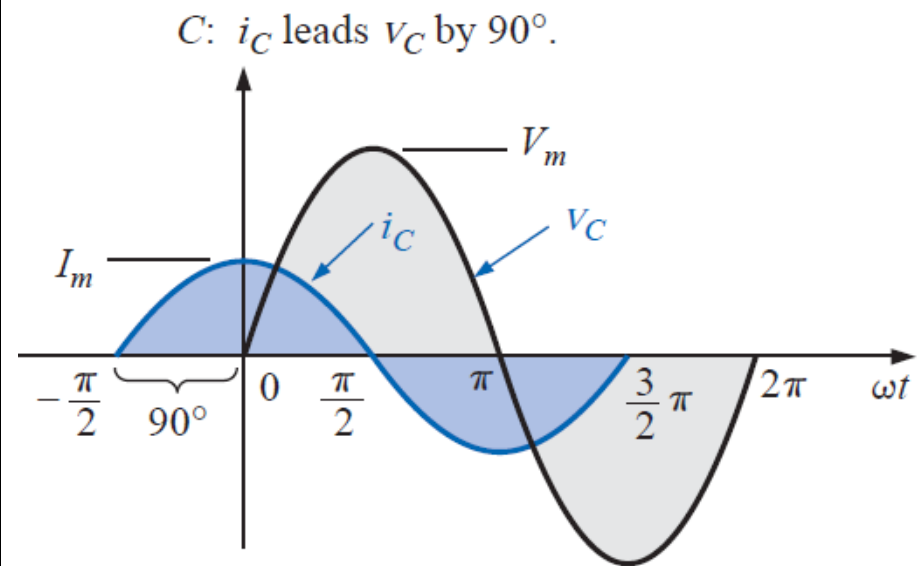


FIG. 14.12

The current of a purely capacitive element leads the voltage across the element by 90° .

- Capacitive reactance is the opposition to the change in the flow of charge
- Capacitor does not dissipate energy
- There is continual interchange of energy between the source and the capacitor

$$v_L = L \frac{di_L}{dt} \quad \Rightarrow \quad i_L = \frac{1}{L} \int v_L dt$$
$$i_C = C \frac{dv_C}{dt} \quad \Rightarrow \quad v_C = \frac{1}{C} \int i_C dt$$

If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is $10\ \Omega$. Sketch the curves for v and i .

a. $v = 100 \sin 377t$

b. $v = 25 \sin(377t + 60^\circ)$

Solutions:

a. Eq. (14.2): $I_m = \frac{V_m}{R} = \frac{100\ \text{V}}{10\ \Omega} = 10\ \text{A}$

(v and i are in phase), resulting in

$$i = 10 \sin 377t$$

The curves are sketched in Fig. 14.13.

b. Eq. (14.2): $I_m = \frac{V_m}{R} = \frac{25\ \text{V}}{10\ \Omega} = 2.5\ \text{A}$

(v and i are in phase), resulting in

$$i = 2.5 \sin(377t + 60^\circ)$$

The curves are sketched in Fig. 14.14.

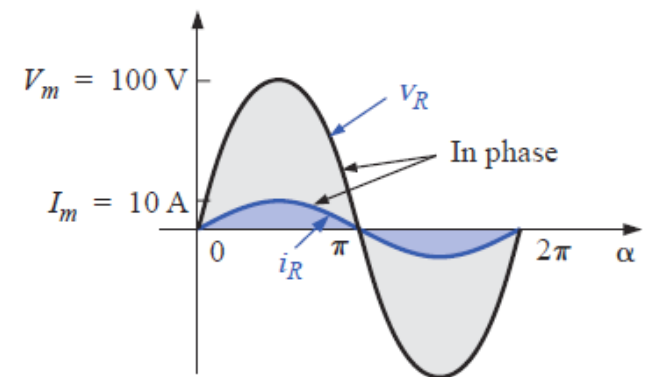


FIG. 14.13
Example 14.1(a).

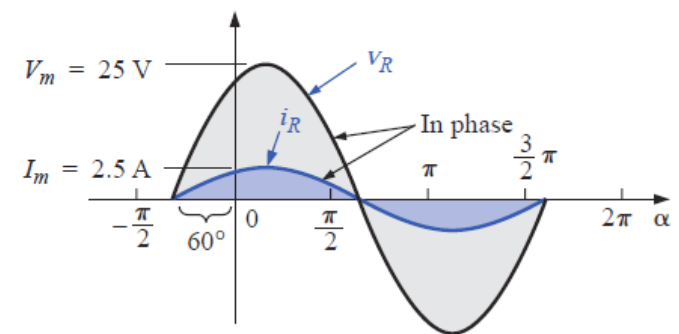


FIG. 14.14
Example 14.1(b).

EXAMPLE 14.2 The current through a $5\text{-}\Omega$ resistor is given. Find the sinusoidal expression for the voltage across the resistor for $i = 40 \sin(377t + 30^\circ)$.

Solution: Eq. (14.3): $V_m = I_m R = (40 \text{ A})(5 \text{ }\Omega) = 200 \text{ V}$

(v and i are in phase), resulting in

$$v = 200 \sin(377t + 30^\circ)$$

EXAMPLE 14.3 The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$

b. $i = 7 \sin(377t - 70^\circ)$

Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7 \text{ }\Omega$

Eq. (14.5): $V_m = I_m X_L = (10 \text{ A})(37.7 \text{ }\Omega) = 377 \text{ V}$

and we know that for a coil v leads i by 90° . Therefore,

$$v = 377 \sin(377t + 90^\circ)$$

The curves are sketched in Fig. 14.15.

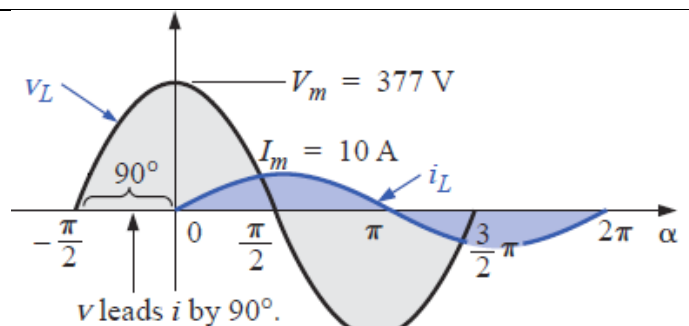


FIG. 14.15
Example 14.3(a).

b. X_L remains at 37.7Ω .

$$V_m = I_m X_L = (7 \text{ A})(37.7 \Omega) = 263.9 \text{ V}$$

and we know that for a coil v leads i by 90° . Therefore,

$$v = 263.9 \sin(377t - 70^\circ + 90^\circ)$$

and

$$v = 263.9 \sin(377t + 20^\circ)$$

The curves are sketched in Fig. 14.16.

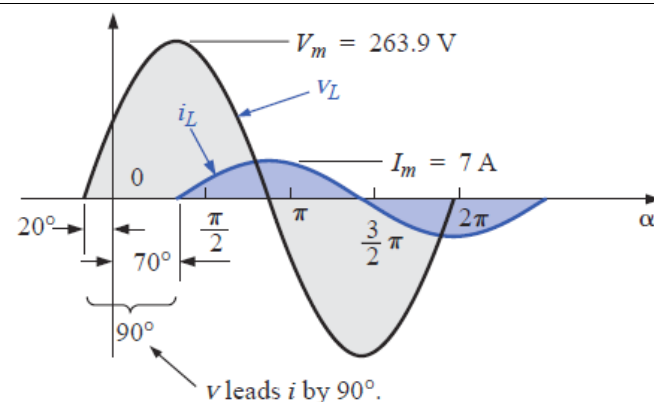


FIG. 14.16

Example 14.3(b).

EXAMPLE 14.4 The voltage across a 0.5-H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100 \sin 20t$$

Solution:

$$X_L = \omega L = (20 \text{ rad/s})(0.5 \text{ H}) = 10 \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100 \text{ V}}{10 \Omega} = 10 \text{ A}$$

and we know that i lags v by 90° . Therefore,

$$i = 10 \sin(20t - 90^\circ)$$

EXAMPLE 14.5 The voltage across a $1\text{-}\mu\text{F}$ capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

Solution:

$$\text{Eq. (14.6): } X_C = \frac{1}{\omega C} = \frac{1}{(400 \text{ rad/s})(1 \times 10^{-6} \text{ F})} = \frac{10^6 \Omega}{400} = 2500 \Omega$$

$$\text{Eq. (14.7): } I_m = \frac{V_m}{X_C} = \frac{30 \text{ V}}{2500 \Omega} = 0.0120 \text{ A} = 12 \text{ mA}$$

and we know that for a capacitor i leads v by 90° . Therefore,

$$i = 12 \times 10^{-3} \sin(400t + 90^\circ)$$

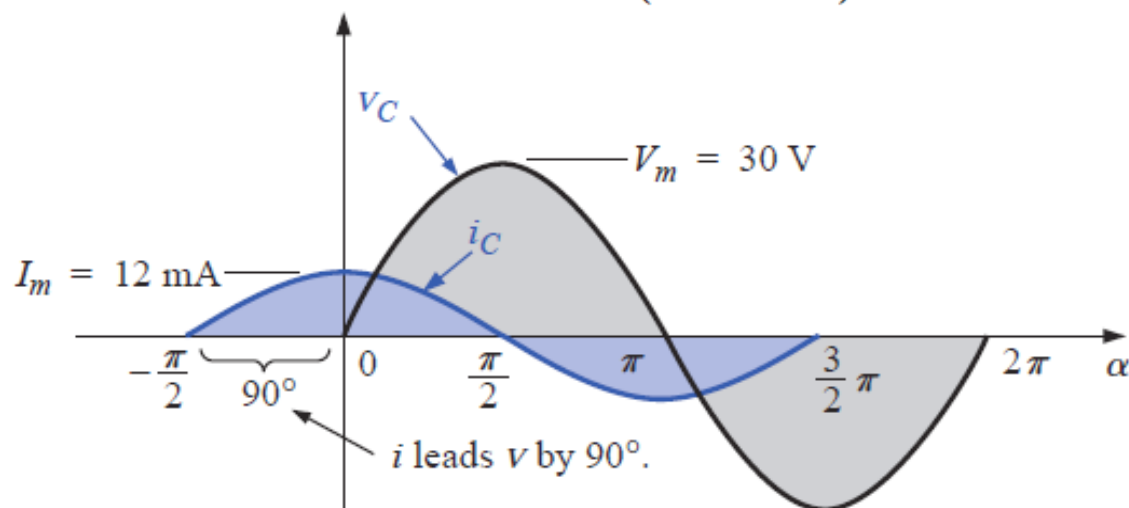


FIG. 14.17
Example 14.5.

EXAMPLE 14.6 The current through a $100\text{-}\mu\text{F}$ capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60^\circ)$$

Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{(500 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6 \Omega}{5 \times 10^4} = \frac{10^2 \Omega}{5} = 20 \Omega$$

$$V_m = I_m X_C = (40 \text{ A})(20 \Omega) = 800 \text{ V}$$

and we know that for a capacitor, v lags i by 90° . Therefore,

$$v = 800 \sin(500t + 60^\circ - 90^\circ)$$

and

$$v = \mathbf{800 \sin(500t - 30^\circ)}$$

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of C , L , or R if sufficient data are provided (Fig. 14.18):

- $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 40^\circ)$
- $v = 1000 \sin(377t + 10^\circ)$
 $i = 5 \sin(377t - 80^\circ)$
- $v = 500 \sin(157t + 30^\circ)$
 $i = 1 \sin(157t + 120^\circ)$
- $v = 50 \cos(\omega t + 20^\circ)$
 $i = 5 \sin(\omega t + 110^\circ)$

Solutions:

- a. Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{100 \text{ V}}{20 \text{ A}} = \mathbf{5 \Omega}$$

- b. Since v *leads* i by 90° , the element is an *inductor*, and

$$X_L = \frac{V_m}{I_m} = \frac{1000 \text{ V}}{5 \text{ A}} = 200 \Omega$$

so that $X_L = \omega L = 200 \Omega$ or

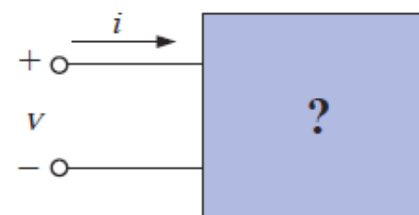


FIG. 14.18
Example 14.7.

$$L = \frac{200 \Omega}{\omega} = \frac{200 \Omega}{377 \text{ rad/s}} = \mathbf{0.531 \text{ H}}$$

c. Since i leads v by 90° , the element is a *capacitor*, and

$$X_C = \frac{V_m}{I_m} = \frac{500 \text{ V}}{1 \text{ A}} = 500 \Omega$$

so that $X_C = \frac{1}{\omega C} = 500 \Omega$ or

$$C = \frac{1}{\omega 500 \Omega} = \frac{1}{(157 \text{ rad/s})(500 \Omega)} = \mathbf{12.74 \mu\text{F}}$$

d. $v = 50 \cos(\omega t + 20^\circ) = 50 \sin(\omega t + 20^\circ + 90^\circ)$
 $= 50 \sin(\omega t + 110^\circ)$

Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{50 \text{ V}}{5 \text{ A}} = \mathbf{10 \Omega}$$

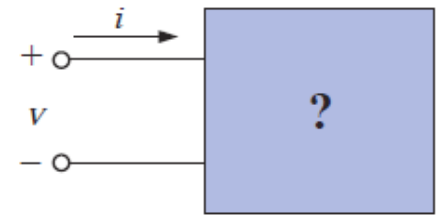


FIG. 14.18
Example 14.7.

dc, High-, and Low-Frequency Effects on L and C

1. DC (very low frequency)

At dc the frequency is zero: $f = 0$

a. Inductor: $X_L = \omega L = 2\pi f L = 0 \Omega$ (short circuit)

b. Capacitor: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \infty \Omega$ (open circuit)

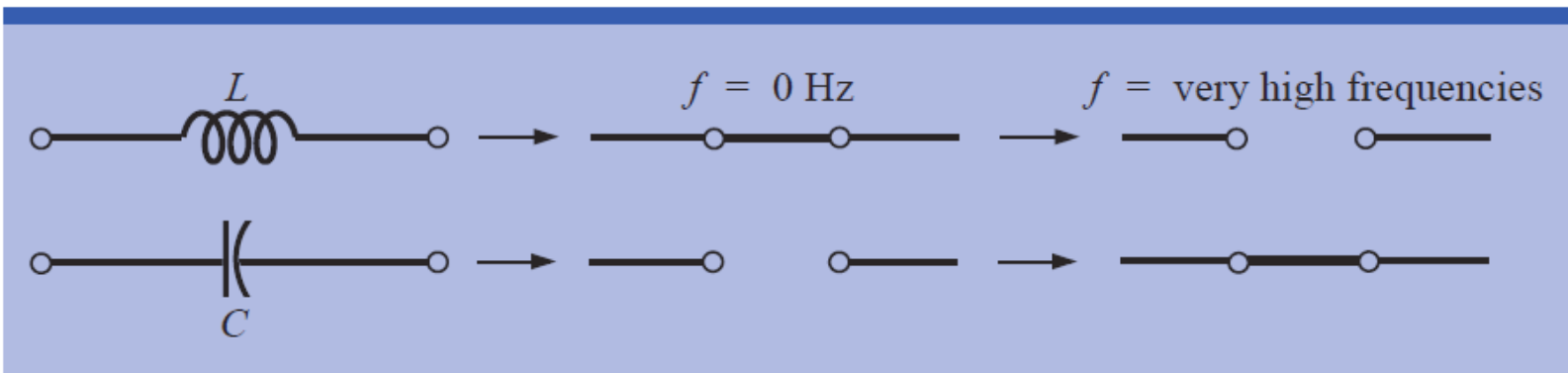
2. very high frequency ($f \rightarrow \infty$)

At very high frequency: $f \rightarrow \infty$

a. Inductor: $X_L = \omega L = 2\pi f L \Rightarrow \infty \Omega$ (open circuit)

b. Capacitor: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \Rightarrow 0 \Omega$ (short circuit)

Effect of high and low frequencies on the circuit model of an inductor and a capacitor.



Resistors $R(f)$

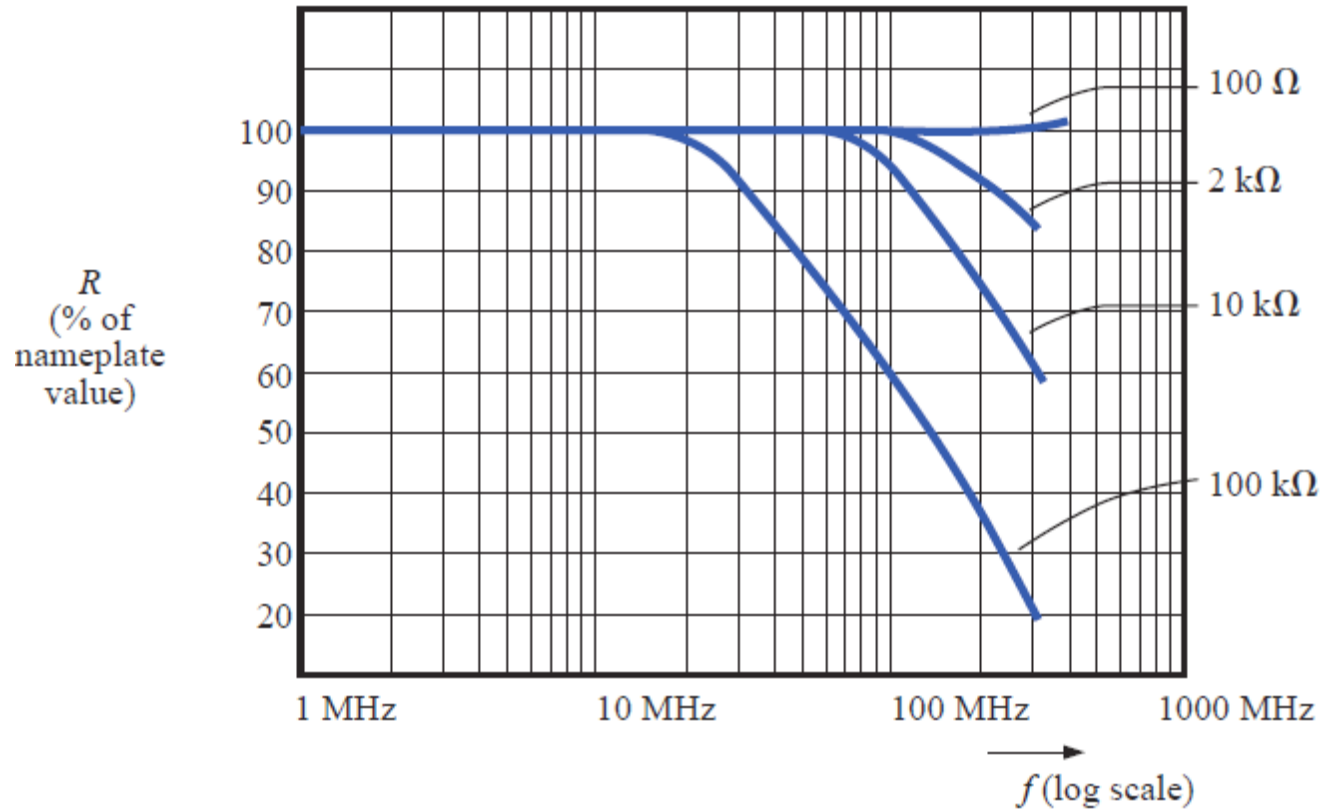


FIG. 14.20

Typical resistance-versus-frequency curves for carbon compound resistors.

Inductors $X_L(f)$

$$X_L = \omega L = 2\pi fL = 2\pi Lf$$

$$y = mx + b = (2\pi L)f + 0$$

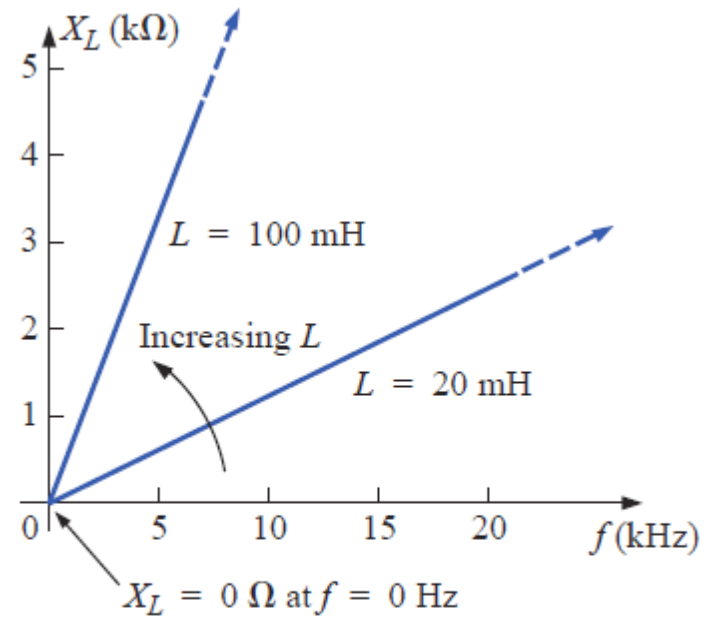


FIG. 14.22

X_L versus frequency.

Capacitors $X_C(f)$

$$X_C = \frac{1}{2\pi fC}$$

$$y = \frac{a}{x}$$

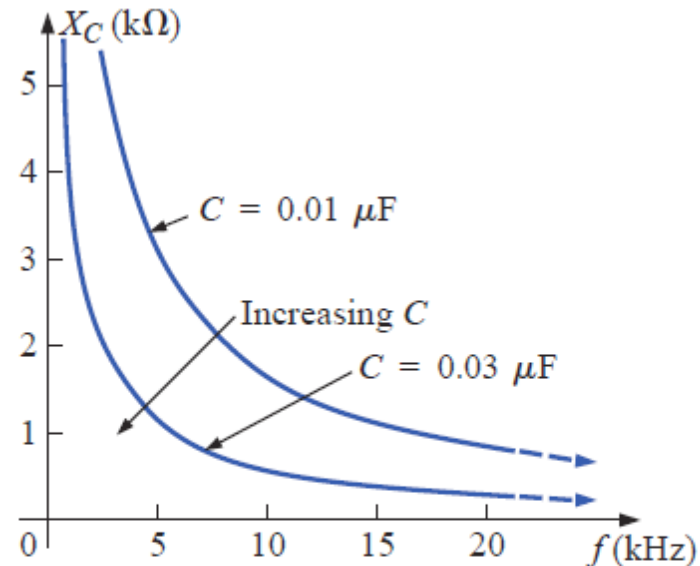


FIG. 14.23

X_C versus frequency.

In summary, therefore, as the applied frequency increases, the resistance of a resistor remains constant, the reactance of an inductor increases linearly, and the reactance of a capacitor decreases nonlinearly.

EXAMPLE 14.8 At what frequency will the reactance of a 200-mH inductor match the resistance level of a 5-k Ω resistor?

Solution: The resistance remains constant at 5 k Ω for the frequency range of the inductor. Therefore,

$$\begin{aligned} R = 5000 \, \Omega &= X_L = 2\pi fL = 2\pi Lf \\ &= 2\pi(200 \times 10^{-3} \text{ H})f = 1.257f \end{aligned}$$

and
$$f = \frac{5000 \text{ Hz}}{1.257} \cong \mathbf{3.98 \text{ kHz}}$$

EXAMPLE 14.9 At what frequency will an inductor of 5 mH have the same reactance as a capacitor of $0.1 \mu\text{F}$?

Solution:

$$X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

and $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3} \text{ H})(0.1 \times 10^{-6} \text{ F})}}$

$$= \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})}$$

$$f = \frac{10^5 \text{ Hz}}{14.05} \cong \mathbf{7.12 \text{ kHz}}$$

14.5 AVERAGE POWER AND POWER FACTOR

$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

then the power is defined by

$$\begin{aligned} p = vi &= V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i) \\ &= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i) \end{aligned}$$

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

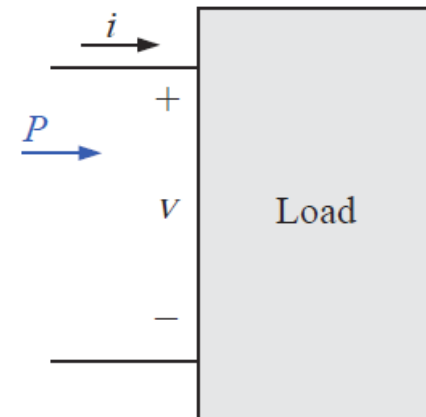


FIG. 14.28

Determining the power delivered in a sinusoidal ac network.

the function $\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$ becomes

$$\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$

$$= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2}$$

$$= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2}$$

so that

$$p = \left[\overbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}^{\text{Fixed value}} \right] - \left[\overbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)}^{\text{Time-varying (function of } t)} \right]$$

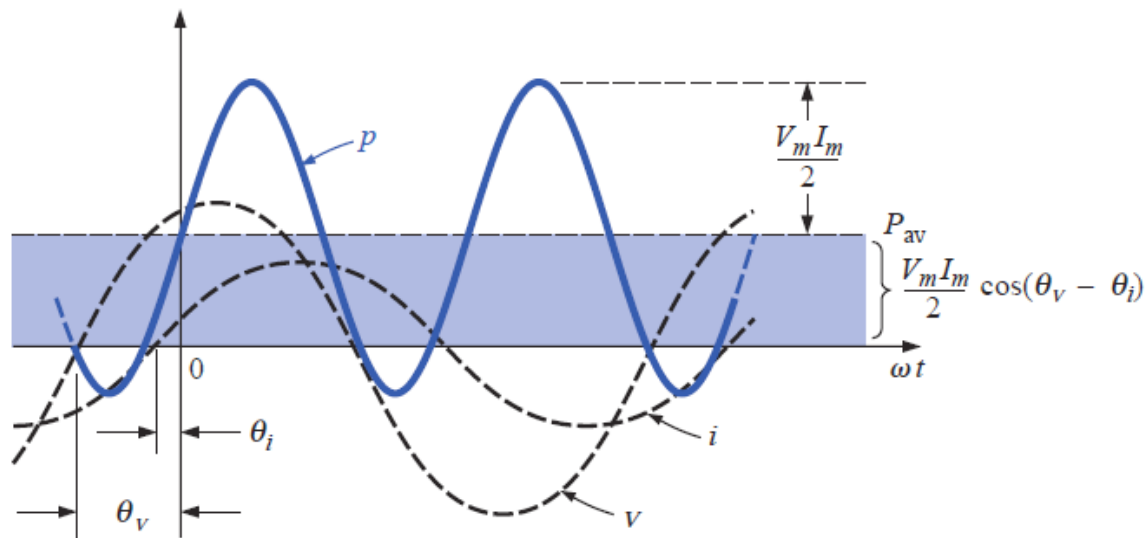


FIG. 14.29

Defining the average power for a sinusoidal ac network.

The average value of the power is:

$$P_{avr} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

It is also called the **real power**,

it is the power delivered to and dissipated by the load.

The angle $(\theta_v - \theta_i)$ is the phase angle between v and i .

the magnitude of average power delivered is independent of whether v leads i or i leads v . $\cos(-\alpha) = \cos(\alpha)$

define: $\theta = |\theta_v - \theta_i|$

$$P = \frac{V_m I_m}{2} \cos \theta$$

(watts, W)

$$P = \left(\frac{V_m}{\sqrt{2}} \right) \left(\frac{I_m}{\sqrt{2}} \right) \cos \theta$$

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

$$P = V_{\text{eff}} I_{\text{eff}} \cos \theta$$

Resistor

In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0^\circ$, and $\cos \theta = 1$, so that:

$$P = \frac{V_m I_m}{2} = V_{\text{eff}} I_{\text{eff}} \quad (\text{W})$$

Or, since

$$I_{\text{eff}} = \frac{V_{\text{eff}}}{R}$$

then

$$P = \frac{V_{\text{eff}}^2}{R} = I_{\text{eff}}^2 R \quad (\text{W})$$

Inductor

In a purely inductive circuit, since v leads i by 90° , $|\theta_v - \theta_i| = \theta = 90^\circ$, and $\cos \theta = 0$, so that:

$$P = \frac{V_m I_m}{2} \cos 90^\circ = \frac{V_m I_m}{2} (0) = 0 \text{ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90° , $|\theta_v - \theta_i| = \theta = |-90^\circ| = 90^\circ$, and $\cos \theta = 0$, so that:

$$P = \frac{V_m I_m}{2} \cos(90^\circ) = \frac{V_m I_m}{2} (0) = 0 \text{ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

EXAMPLE 14.10 Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5 \sin(\omega t + 40^\circ)$$

$$v = 10 \sin(\omega t + 40^\circ)$$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10 \text{ V})(5 \text{ A})}{2} = \mathbf{25 \text{ W}}$$

or

$$R = \frac{V_m}{I_m} = \frac{10 \text{ V}}{5 \text{ A}} = 2 \ \Omega$$

and

$$P = \frac{V_{\text{eff}}^2}{R} = \frac{[(0.707)(10 \text{ V})]^2}{2} = \mathbf{25 \text{ W}}$$

or

$$P = I_{\text{eff}}^2 R = [(0.707)(5 \text{ A})]^2 (2) = \mathbf{25 \text{ W}}$$

EXAMPLE 14.11 Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 70^\circ)$

Solutions:

a. $V_m = 100, \quad \theta_v = 40^\circ$
 $I_m = 20, \quad \theta_i = 70^\circ$
 $\theta = |\theta_v - \theta_i| = |40^\circ - 70^\circ| = |-30^\circ| = 30^\circ$

and

$$P = \frac{V_m I_m}{2} \cos \theta = \frac{(100 \text{ V})(20 \text{ A})}{2} \cos(30^\circ) = (1000 \text{ W})(0.866)$$
$$= \mathbf{866 \text{ W}}$$

EXAMPLE 14.11 Determine the average power delivered to networks having the following input voltage and current:

b. $v = 150 \sin(\omega t - 70^\circ)$
 $i = 3 \sin(\omega t - 50^\circ)$

Solutions:

b. $V_m = 150 \text{ V}, \quad \theta_v = -70^\circ$
 $I_m = 3 \text{ A}, \quad \theta_i = -50^\circ$
 $\theta = |\theta_v - \theta_i| = |-70^\circ - (-50^\circ)|$
 $= |-70^\circ + 50^\circ| = |-20^\circ| = 20^\circ$

and

$$P = \frac{V_m I_m}{2} \cos \theta = \frac{(150 \text{ V})(3 \text{ A})}{2} \cos(20^\circ) = (225 \text{ W})(0.9397)$$
$$= \mathbf{211.43 \text{ W}}$$

Power Factor

In the expression of the power $P = (V_m I_m / 2) \cdot \cos \theta$ the factor that has significant control over the delivered power level is $\cos \theta$

This term is called **Power factor**

$$\text{Power factor} = F_p = \cos \theta$$

1- Purely resistive load:

$$\theta = 0 \Rightarrow F_p = \cos \theta = 1$$

2- Purely reactive load:

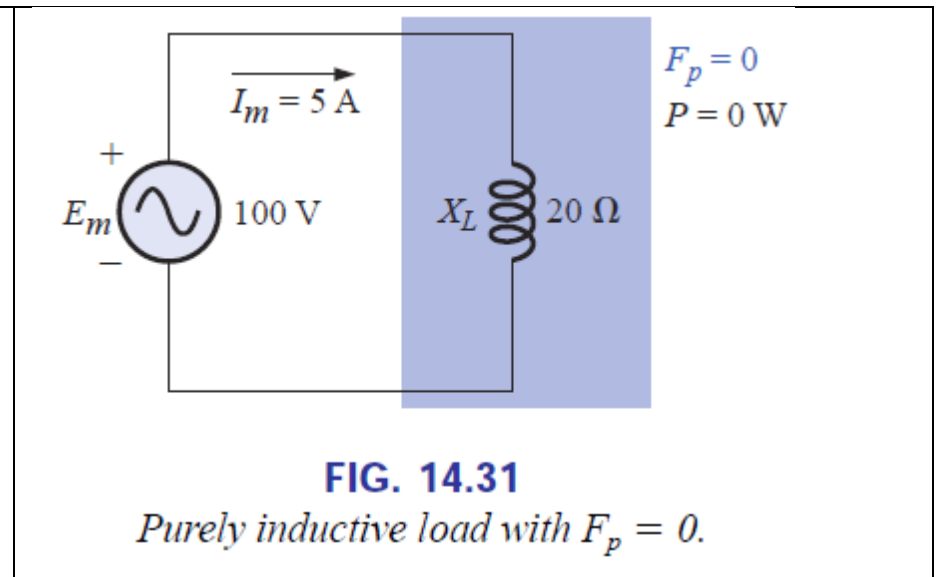
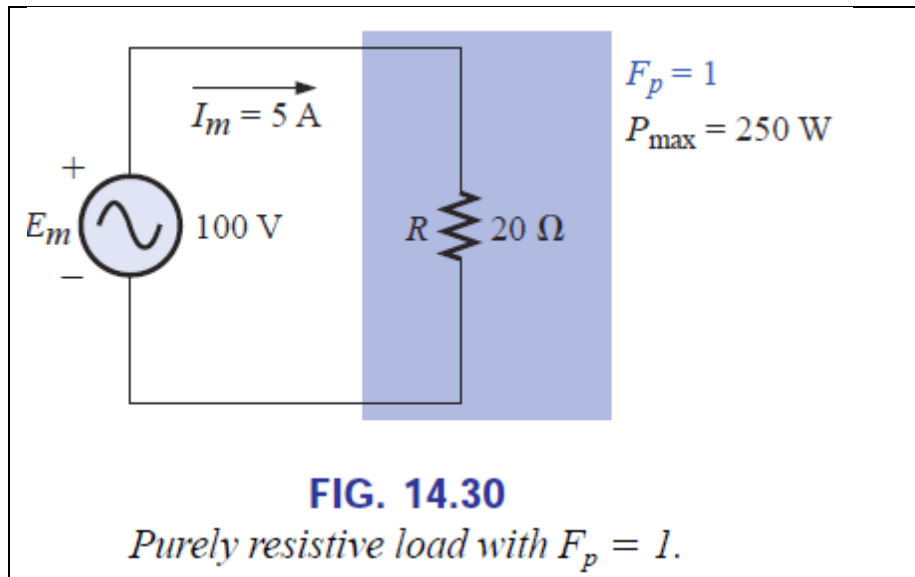
$$\theta = 90^\circ \Rightarrow F_p = \cos \theta = 0$$

The terms *leading* and *lagging* are often used with the power factor:

- i leads v : **leading Power factor**
- i lags v : **lagging Power factor**

$$F_p = \cos \theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}}$$

capacitive networks have leading power factors, and inductive networks have lagging power factors.



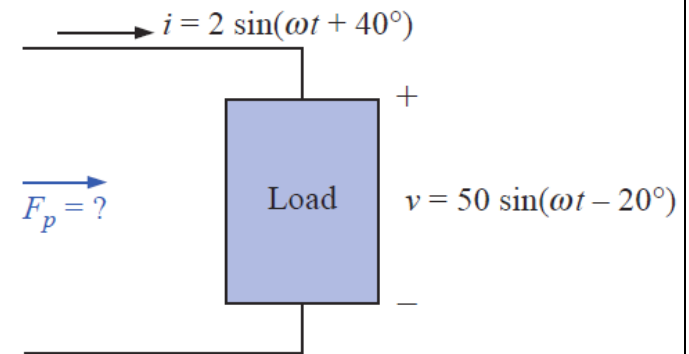
EXAMPLE 14.12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

- Fig. 14.32
- Fig. 14.33
- Fig. 14.34

Solutions:

- $F_p = \cos \theta = \cos |40^\circ - (-20^\circ)| = \cos 60^\circ = \mathbf{0.5 \text{ leading}}$
- $F_p = \cos \theta |80^\circ - 30^\circ| = \cos 50^\circ = \mathbf{0.6428 \text{ lagging}}$
- $F_p = \cos \theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}} = \frac{100 \text{ W}}{(20 \text{ V})(5 \text{ A})} = \frac{100 \text{ W}}{100 \text{ W}} = \mathbf{1}$

The load is resistive, and F_p is neither leading nor lagging.



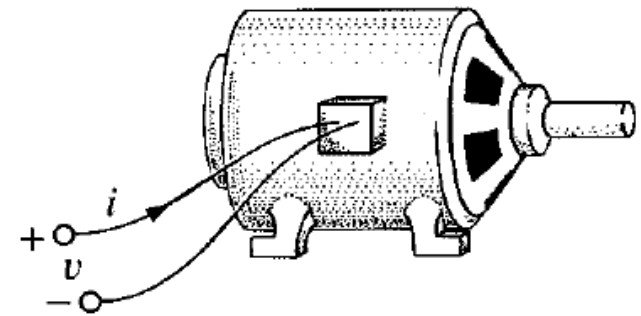
EXAMPLE 14.12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

- a. Fig. 14.32
- b. Fig. 14.33
- c. Fig. 14.34

Solutions:

- a. $F_p = \cos \theta = \cos |40^\circ - (-20^\circ)| = \cos 60^\circ = \mathbf{0.5 \text{ leading}}$
- b. $F_p = \cos \theta |80^\circ - 30^\circ| = \cos 50^\circ = \mathbf{0.6428 \text{ lagging}}$
- c. $F_p = \cos \theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}} = \frac{100 \text{ W}}{(20 \text{ V})(5 \text{ A})} = \frac{100 \text{ W}}{100 \text{ W}} = \mathbf{1}$

The load is resistive, and F_p is neither leading nor lagging.



$$v = 120 \sin(\omega t + 80^\circ)$$

$$i = 5 \sin(\omega t + 30^\circ)$$

FIG. 14.33

Example 14.12(b).

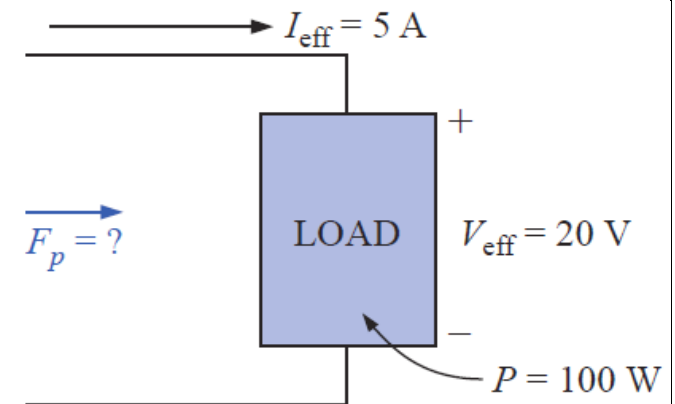


FIG. 14.34

Example 14.12(c).