The Basic Elements and Phasors

14.6 COMPLEX NUMBERS

- In dc circuit analysis we need to find the algebraic sum of voltages and currents
- For ac circuit analysis we need also to find the algebraic sum of voltages and currents.

$$v_1 = V_{m1} \sin(\omega t + \theta_1)$$

$$v_2 = V_{m2} \sin(\omega t + \theta_2)$$

$$v_T = v_1 + v_2 = V_{m1} \sin(\omega t + \theta_1) + V_{m2} \sin(\omega t + \theta_2)$$

- How to do these summations when the voltages are sinusoidal in time?
  - It can be done point by point basis. This is very long and not practical
  - It can be done by employing a system of complex numbers that will be related to the sinusoidal waveform.
  - Then we can find very easily and accurately the sum of two (or more) sine waves.

- A **complex number** represents a point in a two-dimensional plane located with reference to two distinct axes.
- This point can also determine a radius vector drawn from the origin to the point.
- The symbol *j* (or sometimes *i*) is used to denote the imaginary component
- Two forms are used to represent a complex number: **rectangular** and **polar**

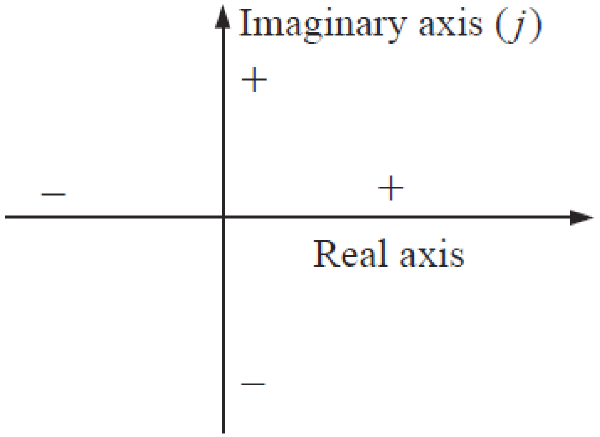


FIG. 14.35
Defining the real and imaginary axes of a complex plane.

14.7 RECTANGULAR FORM

The **rectangular form** is:

$$\boldsymbol{C} = X + jY$$

Boldface ≡ vector (magnitude and direction)

Italic ≡ magnitude only

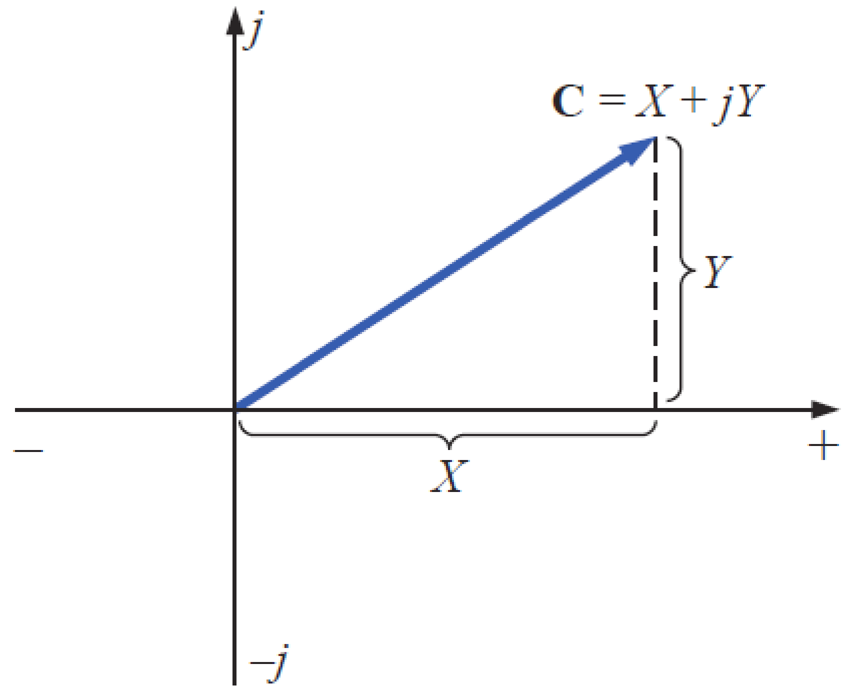


FIG. 14.36
Defining the rectangular form.

EXAMPLE 14.13 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 3 + j4$
b. $\mathbf{C} = 0 - j6$
c. $\mathbf{C} = -10 - j20$

Solutions:

a. See Fig. 14.37.
b. See Fig. 14.38.
c. See Fig. 14.39.

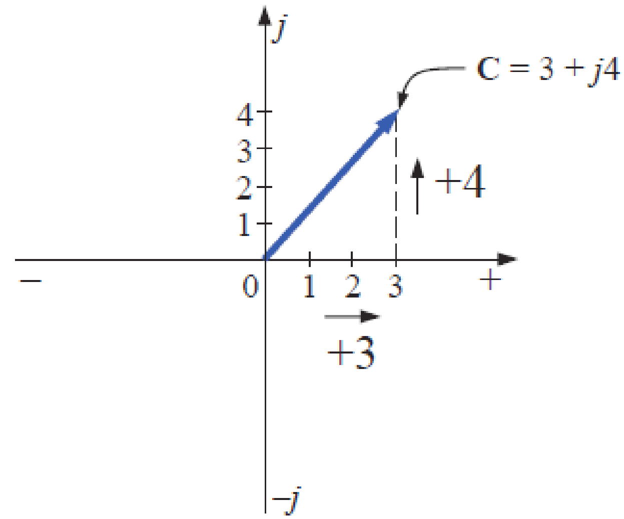


FIG. 14.37

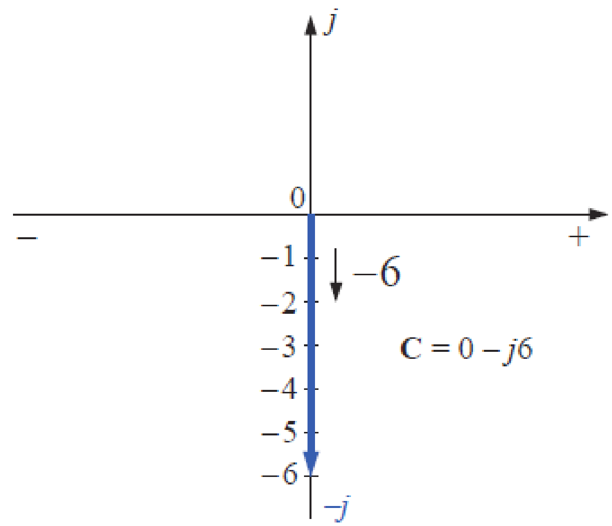


FIG. 14.38

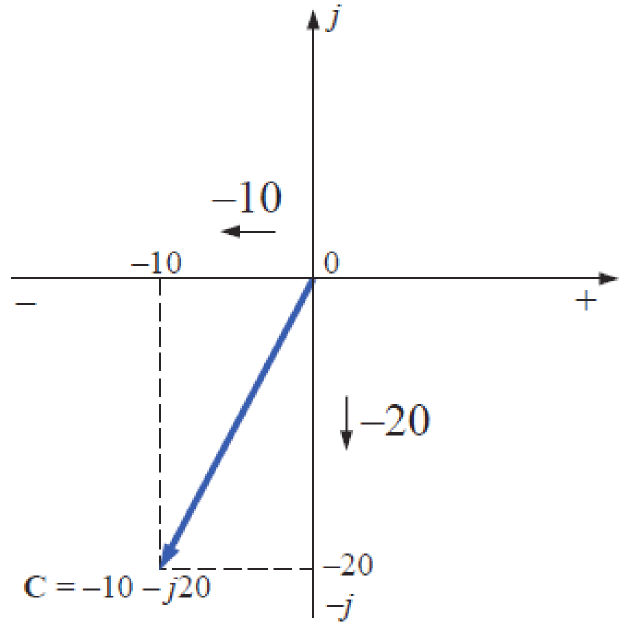


FIG. 14.39

14.8 POLAR FORM

The **polar form** is:

$$\boldsymbol{C} = Z\,\angle\theta$$

Z ≡ magnitude only always positive

θ ≡ angle measured counter-clockwise (CCW) from the positive real axis

Angle measured clockwise must have a minus sign associated

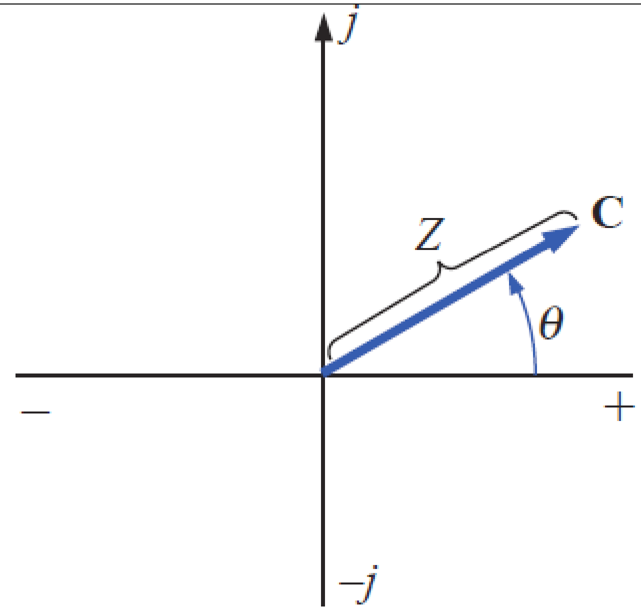


FIG. 14.40
Defining the polar form.

$$-\boldsymbol{C} = -Z\angle\theta = Z\angle(\theta \pm 180^\circ)$$

$$= Z\angle(\theta \pm \pi)$$

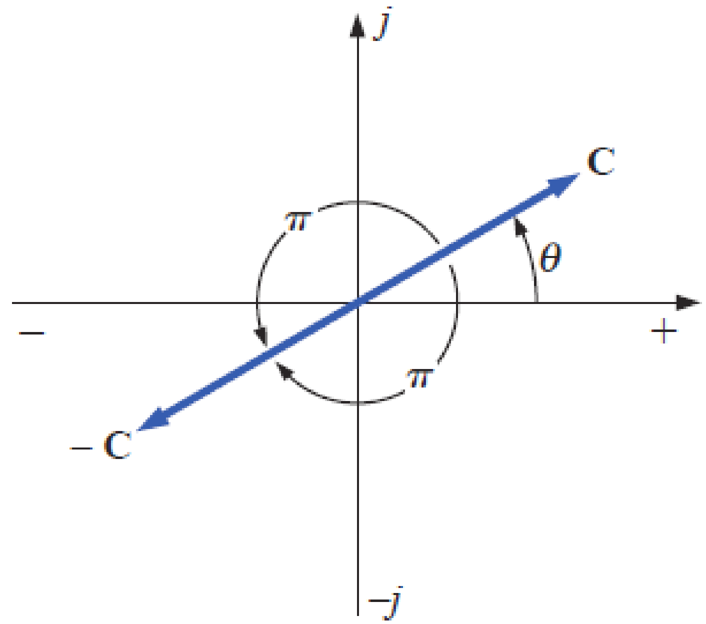


FIG. 14.41

Demonstrating the effect of a negative sign on the polar form.

EXAMPLE 14.14 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 5 \angle 30°$
b. $\mathbf{C} = 7 \angle -120°$
c. $\mathbf{C} = -4.2 \angle 60°$

Solutions:

a. See Fig. 14.42.
b. See Fig. 14.43.
c. See Fig. 14.44.

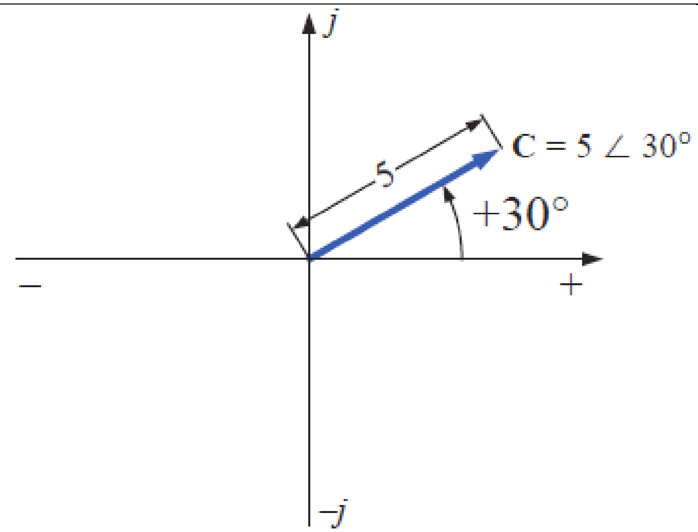


FIG. 14.42

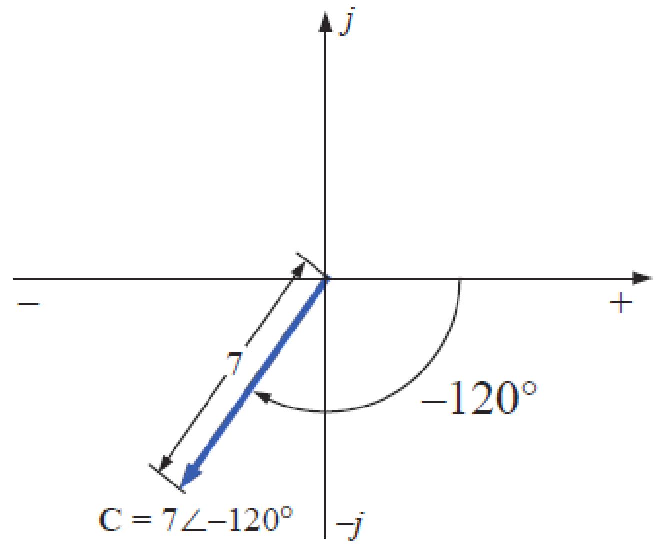


FIG. 14.43

$$\mathbf{C} = -4.2 \angle 60° = 4.2 \angle 60° + 180°$$
$$= 4.2 \angle +240°$$

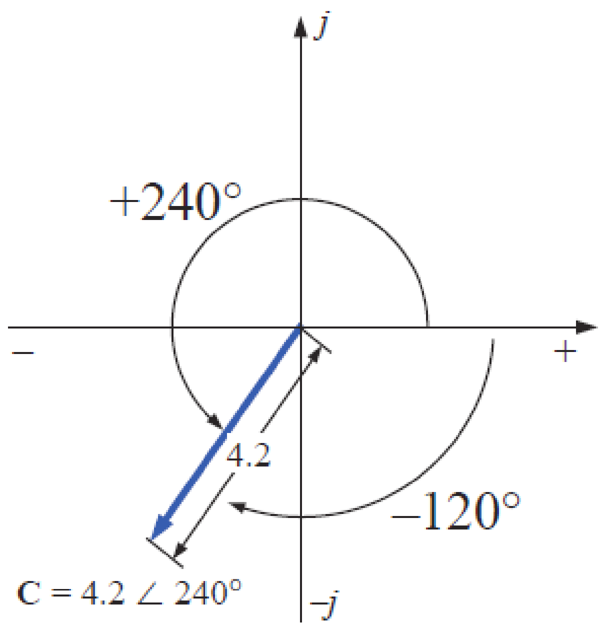


FIG. 14.44

14.9 CONVERSION BETWEEN FORMS

Rectangular to Polar

$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1} \frac{Y}{X}$$

Polar to Rectangular

$$X = Z \cos \theta$$

$$Y = Z \sin \theta$$

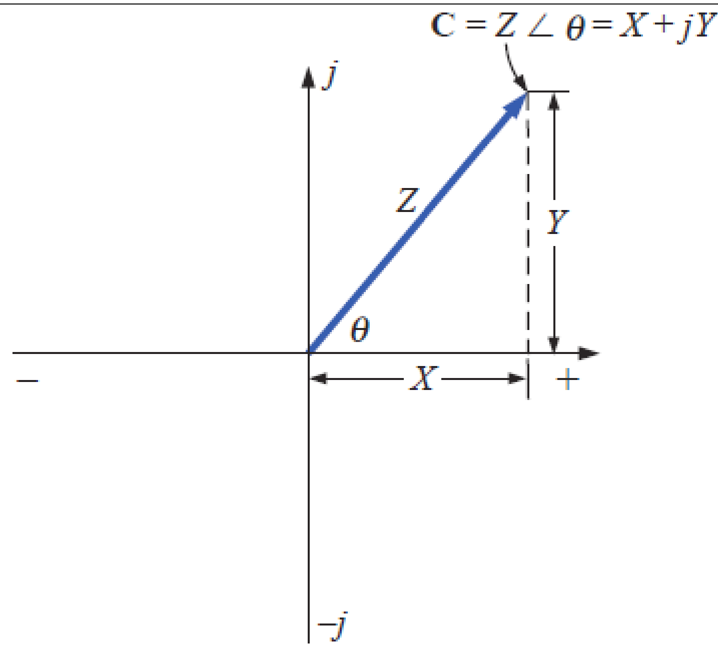


FIG. 14.45
Conversion between forms.

EXAMPLE 14.15 Convert the following from rectangular to polar form:

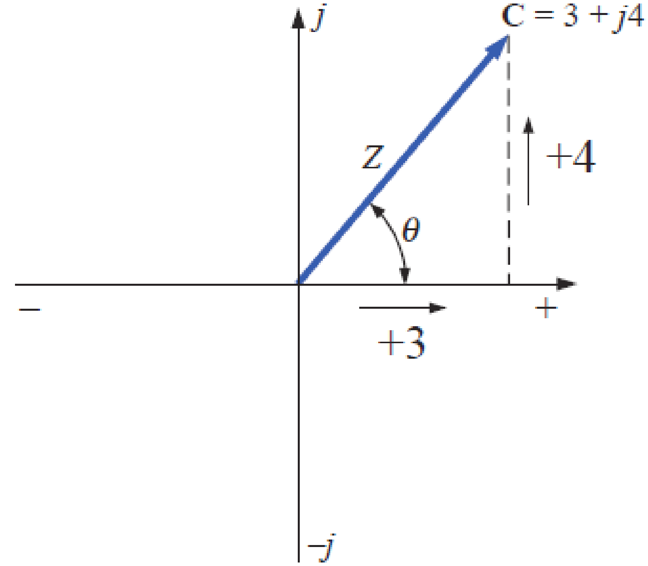
$$\mathbf{C} = 3 + j\,4 \qquad \text{(Fig. 14.46)}$$

Solution:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

and $$\mathbf{C = 5 \angle 53.13°}$$



EXAMPLE 14.16 Convert the following from polar to rectangular form:

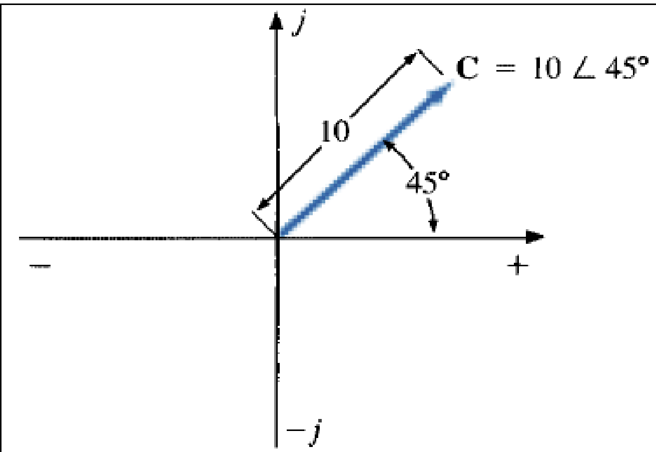
$$\mathbf{C} = 10 \angle 45° \qquad \text{(Fig. 14.47)}$$

Solution:

$$X = 10 \cos 45° = (10)(0.707) = 7.07$$

$$Y = 10 \sin 45° = (10)(0.707) = 7.07$$

and $$\mathbf{C = 7.07 + j7.07}$$



EXAMPLE 14.17 Convert the following from rectangular to polar form:

$$\mathbf{C} = -6 + j\,3 \qquad \text{(Fig. 14.48)}$$

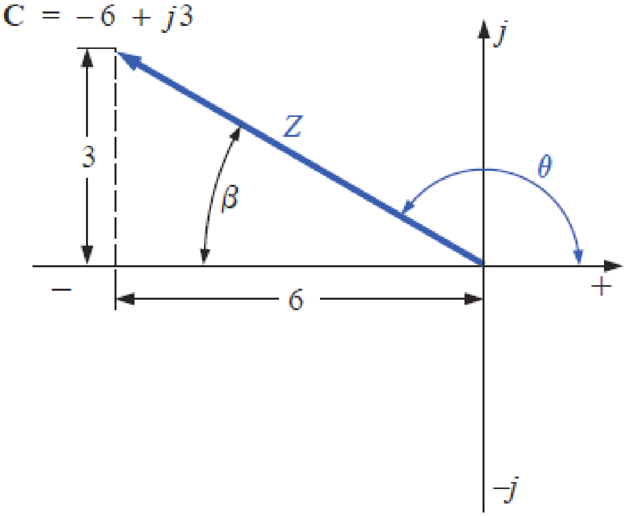
Solution:

$$Z = \sqrt{(6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\beta = \tan^{-1}\left(\frac{3}{6}\right) = 26.57^\circ$$

$$\theta = 180^\circ - 26.57^\circ = 153.43^\circ$$

and $$\mathbf{C} = \mathbf{6.71\ \angle 153.43^\circ}$$



EXAMPLE 14.18 Convert the following from polar to rectangular form:

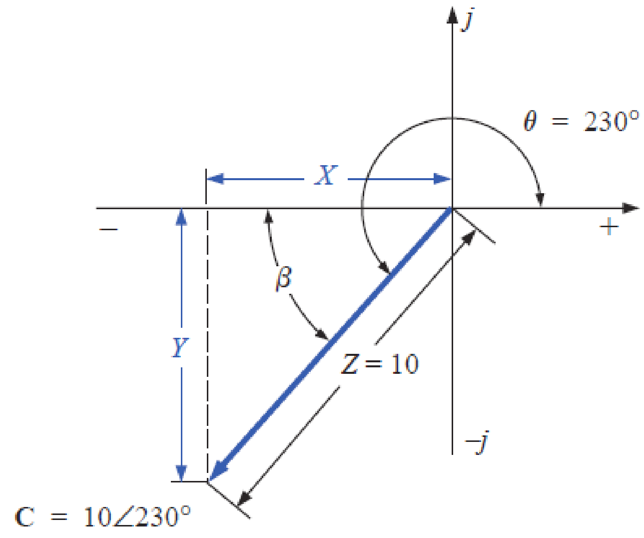
$$\mathbf{C} = 10\ \angle 230^\circ \qquad \text{(Fig. 14.49)}$$

Solution:

$$X = Z\cos\beta = 10\cos(230^\circ - 180^\circ) = 10\cos 50^\circ$$
$$= (10)(0.6428) = 6.428$$

$$Y = Z\sin\beta = 10\sin 50^\circ = (10)(0.7660) = 7.660$$

and $$\mathbf{C} = \mathbf{-6.428 - j7.660}$$



14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

$$j = \sqrt{-1}$$

Thus,

$$j^2 = -1$$

and

$$j^3 = j^2 j = -1j = -j$$

with

$$j^4 = j^2 j^2 = (-1)(-1) = +1$$

$$j^5 = j$$

and so on. Further,

$$\frac{1}{j} = (1)\left(\frac{1}{j}\right) = \left(\frac{j}{j}\right)\left(\frac{1}{j}\right) = \frac{j}{j^2} = \frac{j}{-1}$$

and

$$\frac{1}{j} = -j$$

Complex Conjugate

The **conjugate** or **complex conjugate** of a complex number can be found by simply changing the sign of the imaginary part in the rectangular form or by using the negative of the angle of the polar form:

The conjugate of:

$$\boldsymbol{C} = 2 + j3$$

is

$$= 2 - j3$$

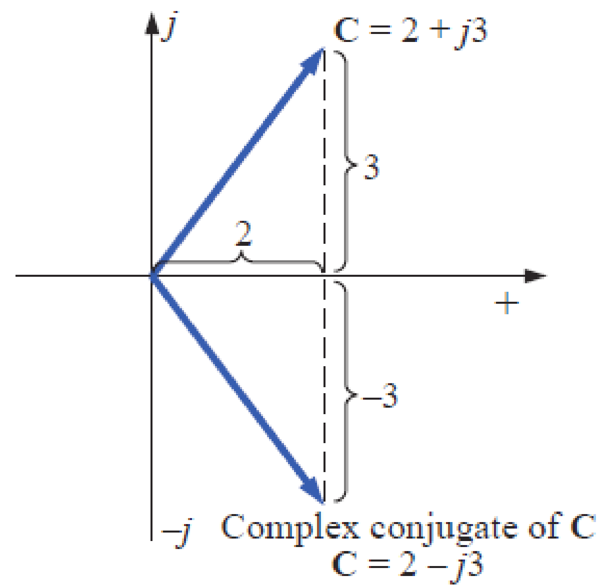
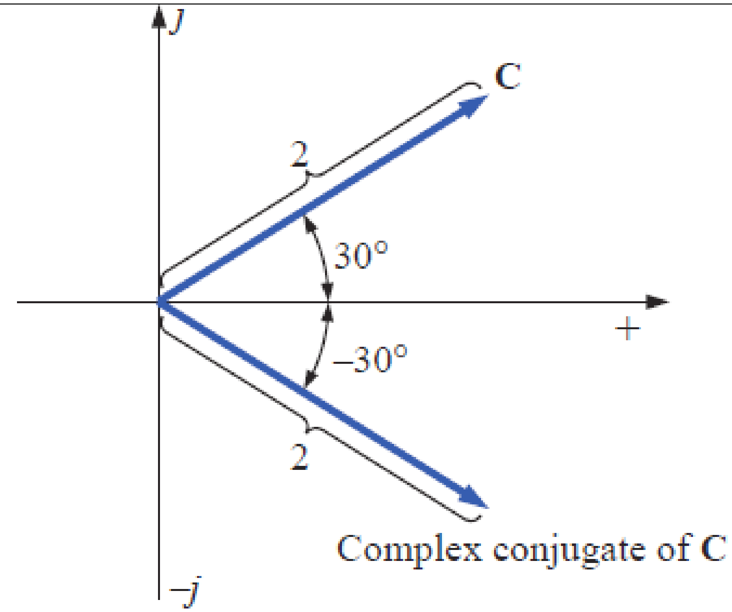


FIG. 14.50

Defining the complex conjugate of a complex number in rectangular form.

The conjugate of: $$\boldsymbol{C} = 2\angle 30°$$ is $$2\angle -30°$$	 **FIG. 14.51** *Defining the complex conjugate of a complex number in polar form.*

Reciprocal

The **reciprocal** of a complex number is 1 divided by the complex number. For example, the reciprocal of

$$\mathbf{C} = X + jY$$

is

$$\frac{1}{X + jY}$$

and of $Z \angle \theta$,

$$\frac{1}{Z \angle \theta}$$

Addition

To add two or more complex numbers, simply add the real and imaginary parts separately. For example, if

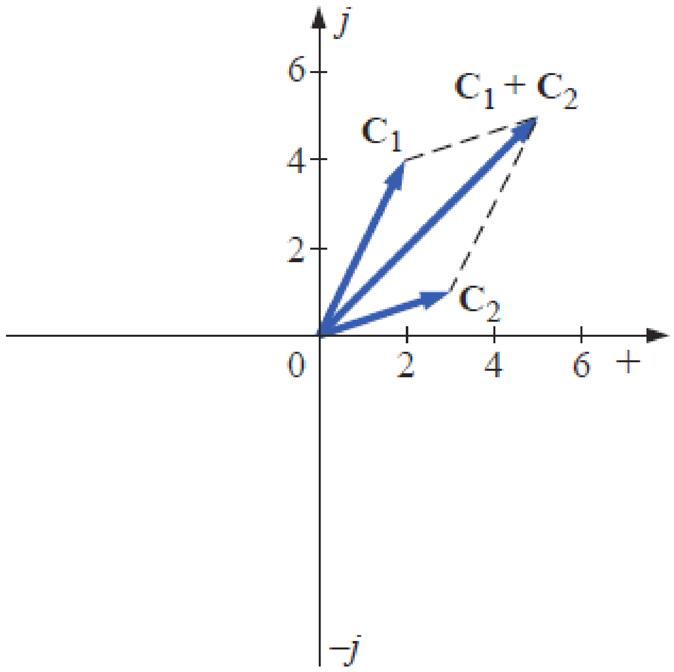
$$\mathbf{C}_1 = \pm X_1 \pm jY_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm jY_2$$

then

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)} \qquad \textbf{(14.30)}$$

EXAMPLE 14.19

a. Add $\mathbf{C}_1 = 2 + j4$ and $\mathbf{C}_2 = 3 + j1$.
b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.



Solutions:

a. By Eq. (14.30),

$$\mathbf{C}_1 + \mathbf{C}_2 = (2 + 3) + j(4 + 1) = \mathbf{5 + \mathit{j}5}$$

Note Fig. 14.52. An alternative method is

$$\begin{array}{r} 2 + j4 \\ \underline{3 + j1} \\ \downarrow \quad \downarrow \\ \mathbf{5 + \mathit{j}5} \end{array}$$

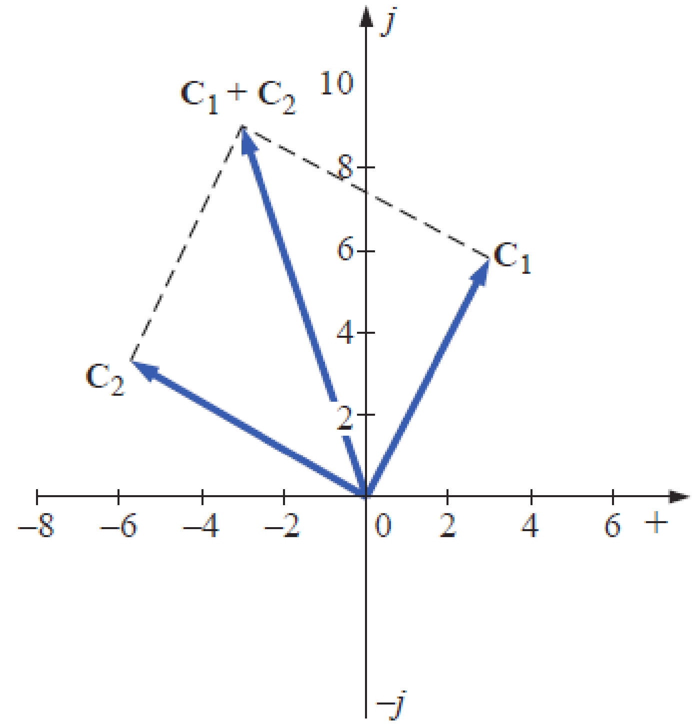
b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

b. By Eq. (14.30),

$$\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$$

Note Fig. 14.53. An alternative method is

$$\begin{array}{r} 3 + j6 \\ \underline{-6 + j3} \\ \downarrow \quad \downarrow \\ \mathbf{-3 + j9} \end{array}$$



Subtraction

In subtraction, the real and imaginary parts are again considered separately. For example, if

$$\mathbf{C}_1 = \pm X_1 \pm j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm j\,Y_2$$

then

$$\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_2 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)] \tag{14.31}$$

EXAMPLE 14.20

a. Subtract $\mathbf{C}_2 = 1 + j4$ from $\mathbf{C}_1 = 4 + j6$.

b. Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

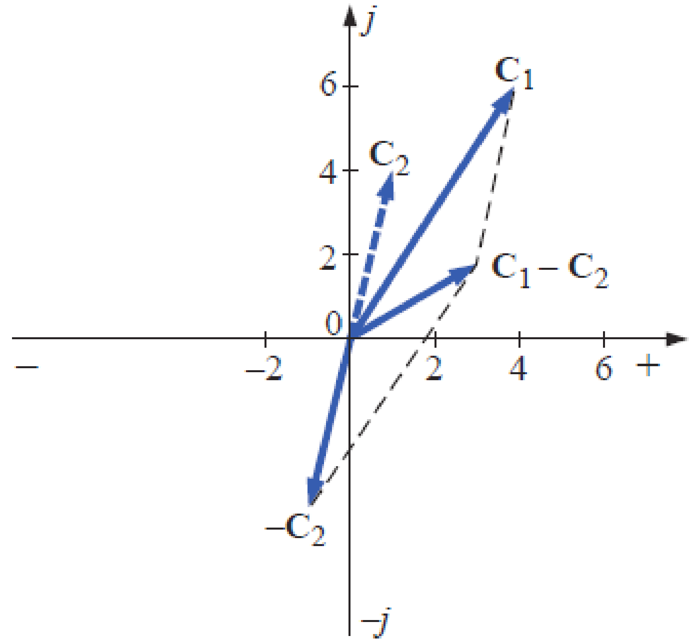
Solutions:

a. By Eq. (14.31),

$$\mathbf{C}_1 - \mathbf{C}_2 = (4 - 1) + j(6 - 4) = \mathbf{3 + j2}$$

Note Fig. 14.54. An alternative method is

$$\begin{array}{r} 4 + j6 \\ \underline{-(1 + j4)} \\ \downarrow \quad \downarrow \\ \mathbf{3 + j2} \end{array}$$



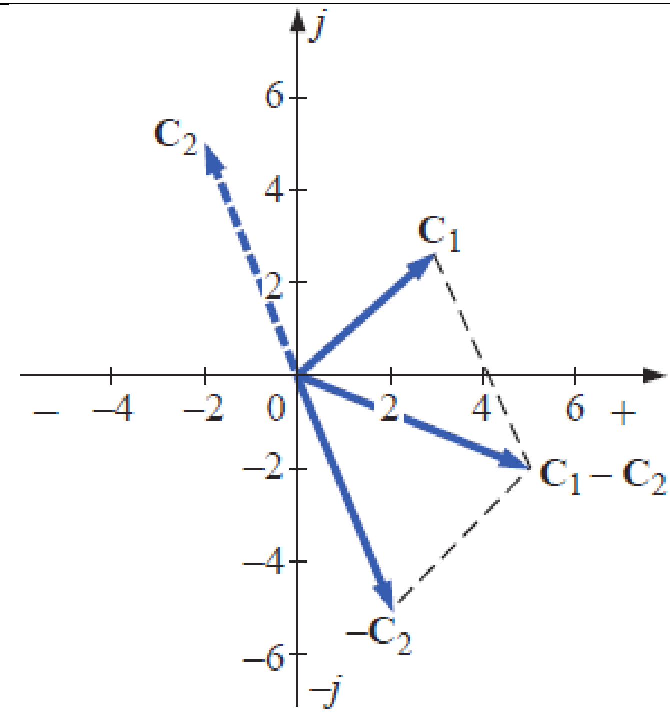
b. Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

b. By Eq. (14.31),

$$\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$$

Note Fig. 14.55. An alternative method is

$$\begin{array}{r} 3 + j3 \\ \underline{-(-2 + j5)} \\ \downarrow \quad \downarrow \\ \mathbf{5 - j2} \end{array}$$



Addition or subtraction cannot be performed in polar form unless the complex numbers have the same angle θ or unless they differ only by multiples of 180°.

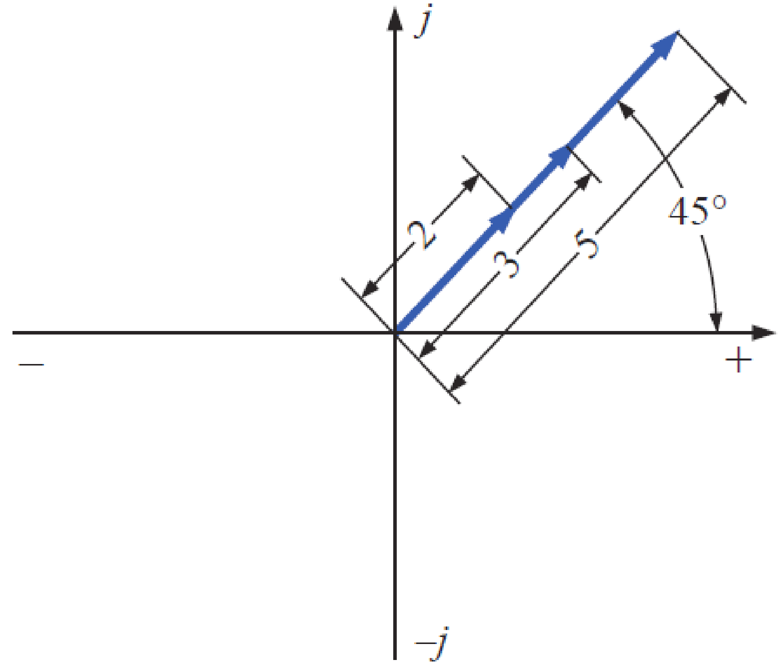
EXAMPLE 14.21

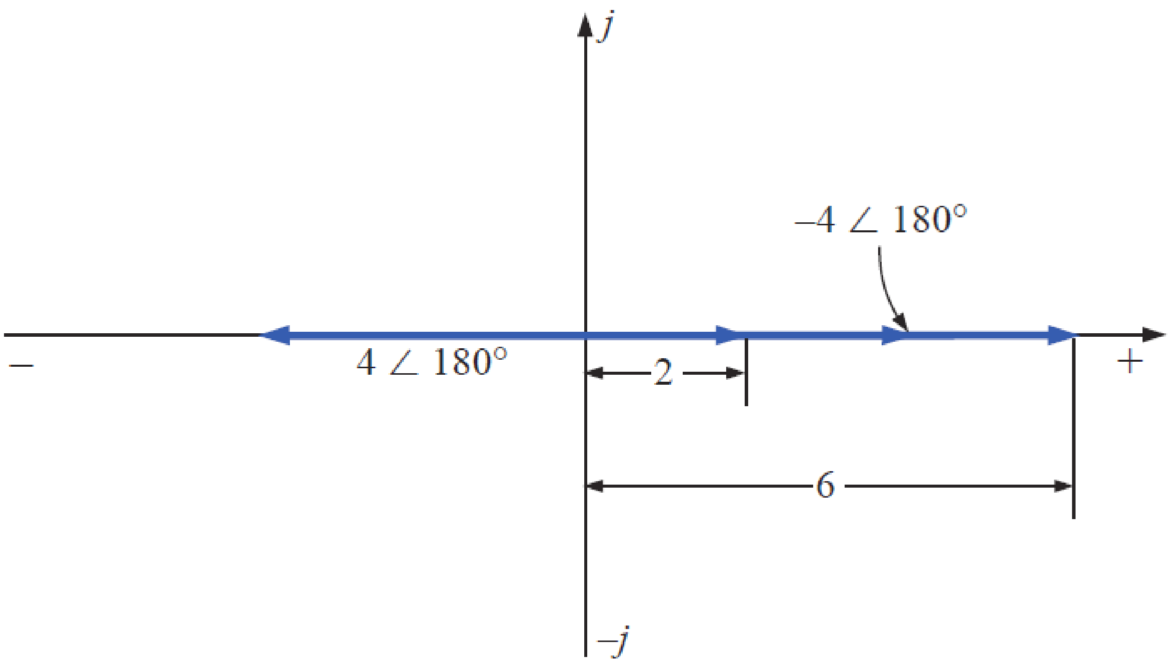
a. $2\ \angle 45^\circ + 3\ \angle 45^\circ = \mathbf{5\ \angle 45^\circ}$

Note Fig. 14.56. Or

b. $2\ \angle 0^\circ - 4\ \angle 180^\circ = \mathbf{6\ \angle 0^\circ}$

Note Fig. 14.57.





Multiplication

To multiply two complex numbers in *rectangular* form, multiply the real and imaginary parts of one in turn by the real and imaginary parts of the other. For example, if

$$\mathbf{C}_1 = X_1 + j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + j\,Y_2$$

then $\mathbf{C}_1 \cdot \mathbf{C}_2$:

$$\begin{array}{r} X_1 + j\,Y_1 \\ \underline{X_2 + j\,Y_2} \\ X_1X_2 + j\,Y_1X_2 \\ \underline{+ j\,X_1Y_2 + j^2Y_1Y_2} \\ X_1X_2 + j\,(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1) \end{array}$$

and

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j\,(Y_1X_2 + X_1Y_2)}$$

EXAMPLE 14.22

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 2 + j3 \quad \text{and} \quad \mathbf{C}_2 = 5 + j10$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = -2 - j3 \quad \text{and} \quad \mathbf{C}_2 = +4 - j6$$

Solutions:

a. Using the format above, we have

$$\begin{aligned}\mathbf{C}_1 \cdot \mathbf{C}_2 &= [(2)(5) - (3)(10)] + j\,[(3)(5) + (2)(10)] \\ &= \mathbf{-20 + j35}\end{aligned}$$

b. Without using the format, we obtain

$$\begin{array}{l} -2 - j3 \\ \underline{+4 - j6 \qquad\qquad\quad} \\ -8 - j12 \\ \underline{\quad\;\, + j12 + j^2 18} \\ -8 + j(-12 + 12) - 18 \end{array}$$

and $$\mathbf{C}_1 \cdot \mathbf{C}_2 = \mathbf{-26 = 26 \angle 180^\circ}$$

In *polar* form, the magnitudes are multiplied and the angles added algebraically. For example, for

$$\mathbf{C}_1 = Z_1 \angle \theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle \theta_2$$

we write

$$\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1 Z_2 \underline{/\theta_1 + \theta_2}$$

EXAMPLE 14.23

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 5 \angle 20^\circ \quad \text{and} \quad \mathbf{C}_2 = 10 \angle 30^\circ$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 2 \angle -40^\circ \quad \text{and} \quad \mathbf{C}_2 = 7 \angle +120^\circ$$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20^\circ)(10 \angle 30^\circ) = (5)(10) \underline{/20^\circ + 30^\circ} = \mathbf{50 \angle 50^\circ}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40^\circ)(7 \angle +120^\circ) = (2)(7) \underline{/-40^\circ + 120^\circ}$

$= \mathbf{14 \angle +80^\circ}$

To multiply a complex number in rectangular form by a real number requires that both the real part and the imaginary part be multiplied by the real number. For example,

$$(10)(2 + j3) = 20 + j30$$

and $$50 \angle 0°(0 + j6) = j300 = 300 \angle 90°$$

Division

To divide two complex numbers in *rectangular* form, multiply the numerator and denominator by the conjugate of the denominator and the resulting real and imaginary parts collected. That is, if

$$\mathbf{C}_1 = X_1 + jY_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + jY_2$$

then $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)}$$

$$= \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

and $$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}} \qquad \textbf{(14.34)}$$

EXAMPLE 14.24

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 1 + j4$ and $\mathbf{C}_2 = 4 + j5$.
b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = -4 - j8$ and $\mathbf{C}_2 = +6 - j1$.

Solutions:

a. By Eq. (14.34),

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(1)(4) + (4)(5)}{4^2 + 5^2} + j\,\frac{(4)(4) - (1)(5)}{4^2 + 5^2}$$

$$= \frac{24}{41} + \frac{j11}{41} \cong \mathbf{0.585 + j\,0.268}$$

b. Using an alternative method, we obtain

$$\begin{array}{l} \quad -4 - j8 \\ \quad \underline{+6 + j1} \\ -24 - j48 \\ \quad\quad \underline{- j4 - j^2 8} \\ -24 - j52 + 8 = -16 - j52 \end{array}$$

$$\begin{array}{l} +6 - j1 \\ \underline{+6 + j1} \\ 36 + j6 \\ \quad \underline{- j6 - j^2 1} \\ 36 + 0 + 1 \quad = 37 \end{array}$$

and $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{-16}{37} - \frac{j52}{37} = \mathbf{-0.432 - j1.405}$$

To divide a complex number in rectangular form by a real number, both the real part and the imaginary part must be divided by the real number. For example,

$$\frac{8 + j\,10}{2} = 4 + j\,5$$

and

$$\frac{6.8 - j\,0}{2} = 3.4 - j\,0 = 3.4\,\angle 0^\circ$$

In *polar* form, division is accomplished by simply dividing the magnitude of the numerator by the magnitude of the denominator and subtracting the angle of the denominator from that of the numerator. That is, for

$$\mathbf{C}_1 = Z_1\,\angle\theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2\,\angle\theta_2$$

we write

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2}\,\angle\theta_1 - \theta_2 \qquad \textbf{(14.35)}$$

EXAMPLE 14.25

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10°$ and $\mathbf{C}_2 = 2 \angle 7°$.

b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120°$ and $\mathbf{C}_2 = 16 \angle -50°$.

Solutions:

a. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{15 \angle 10°}{2 \angle 7°} = \frac{15}{2} \underline{/10° - 7°} = \mathbf{7.5 \angle 3°}$$

b. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{8 \angle 120°}{16 \angle -50°} = \frac{8}{16} \underline{/120° - (-50°)} = \mathbf{0.5 \angle 170°}$$

14.12 PHASORS

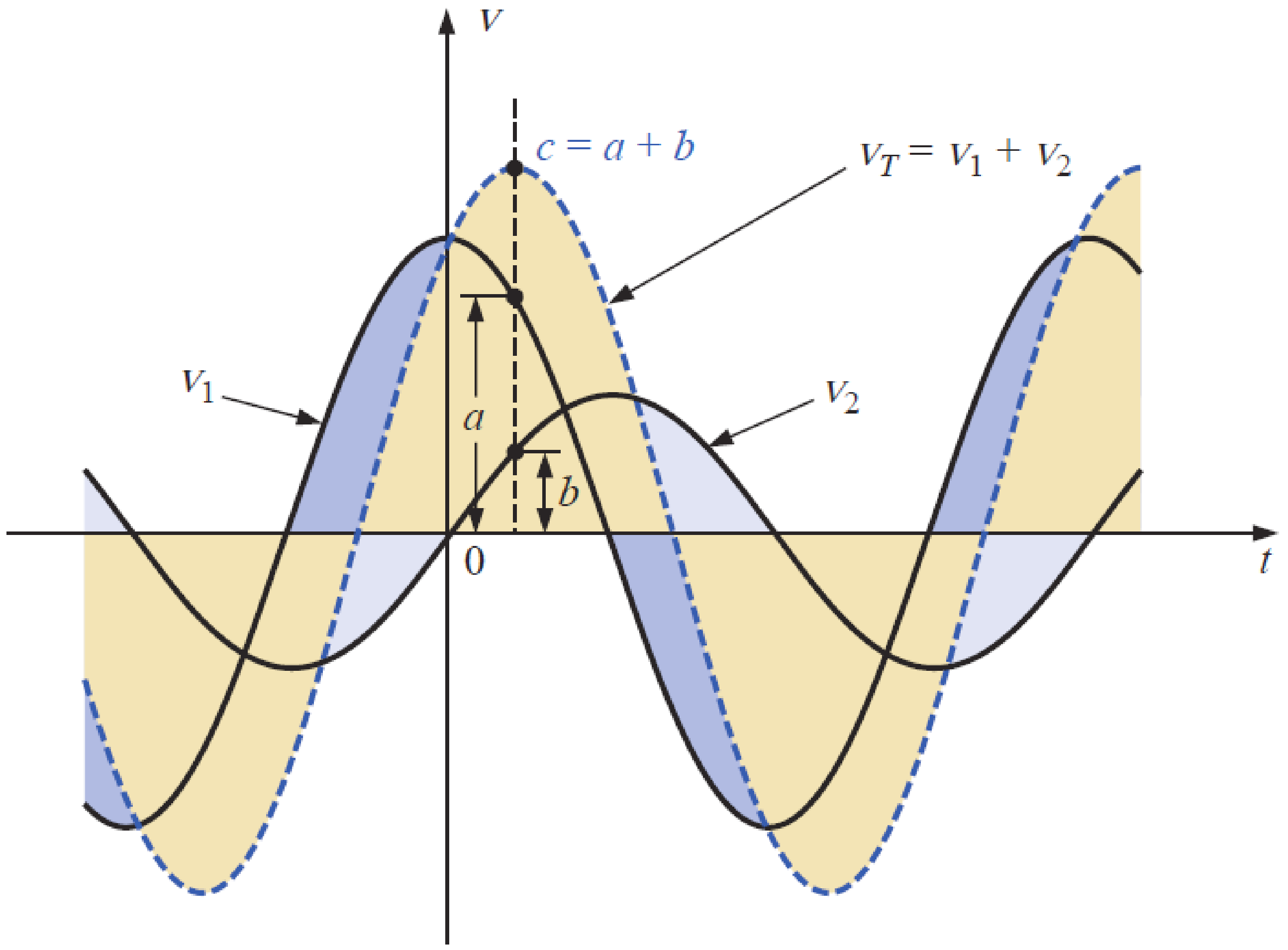


FIG. 14.62
Adding two sinusoidal waveforms on a point-by-point basis.

A better method uses the ***rotating radius vector*** (used to generate the sine wave)

- This **radius vector**:
  - has a *constant magnitude*
  - *one end is fixed at the origin*

is the phasor when applied to electric circuit

$$v_1 = V_{m1} \sin(\omega t \pm \theta_1) \quad \longrightarrow \quad V_{m1} \angle \pm \theta_1$$

$$v_2 = V_{m2} \sin(\omega t \pm \theta_2) \quad \longrightarrow \quad V_{m2} \angle \pm \theta_2$$

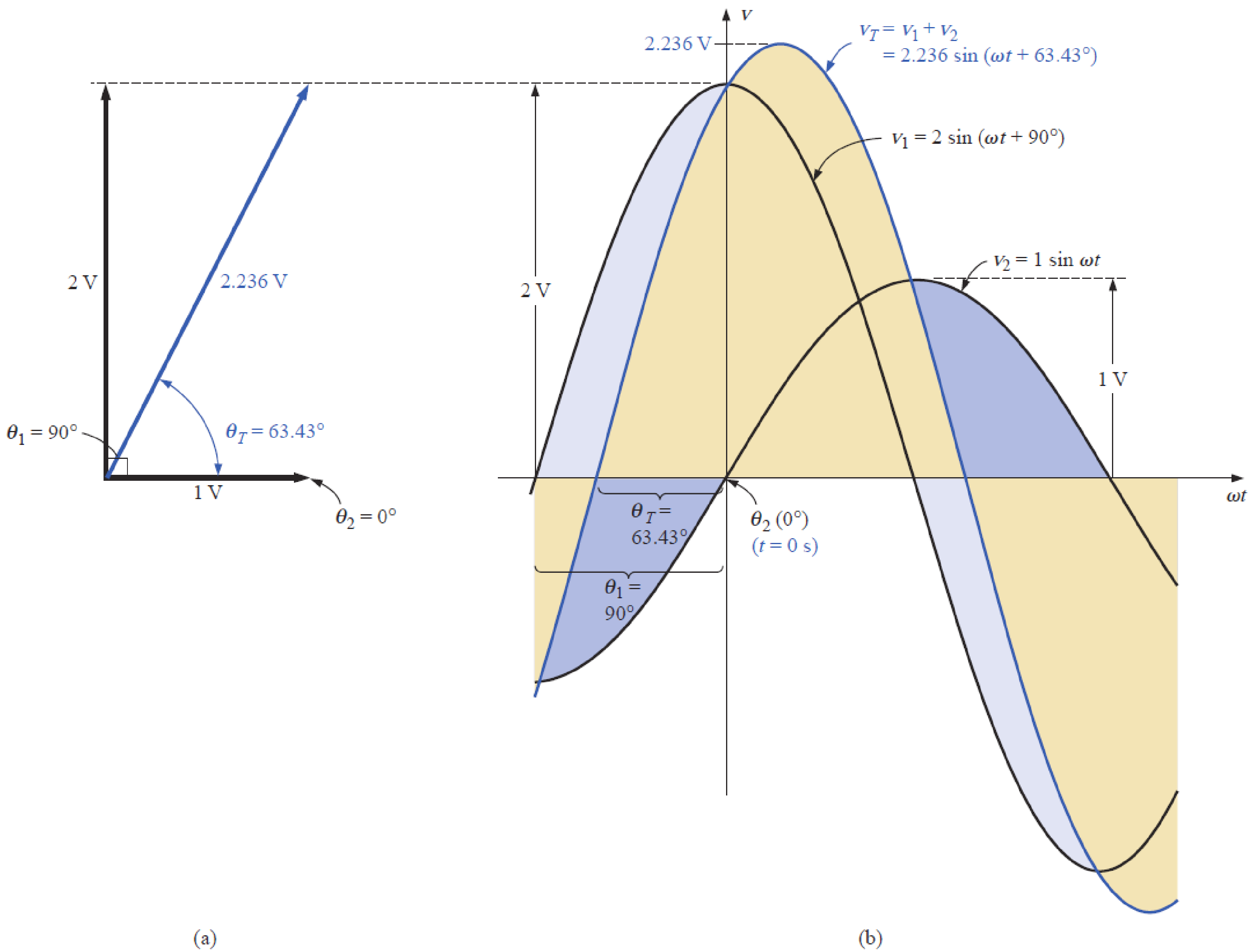


FIG. 14.63

(a) The phasor representation of the sinusoidal waveforms of Fig. 14.63(b); (b) finding the sum of two sinusoidal waveforms of v_1 and v_2.

$$v_1 = V_{m1}\sin(\omega t \pm \theta_1) \quad \longrightarrow \quad V_{m1}\angle \pm \theta_1$$

$$v_2 = V_{m2}\sin(\omega t \pm \theta_2) \quad \longrightarrow \quad V_{m2}\angle \pm \theta_2$$

We can find the sum (v_1+v_2) using their phasors $V_{m1}\angle \pm \theta_1$ and $V_{m2}\angle \pm \theta_2$

The phasors are added using the complex number algebra to obtain the **phasor form of the sum** $v_T = v_1+v_2$ very easily and then convert it back to sinusoidal form.

Position of the various phasors, is called phasor diagram.

It is actually a "snapshot" of the rotating vector at t = 0

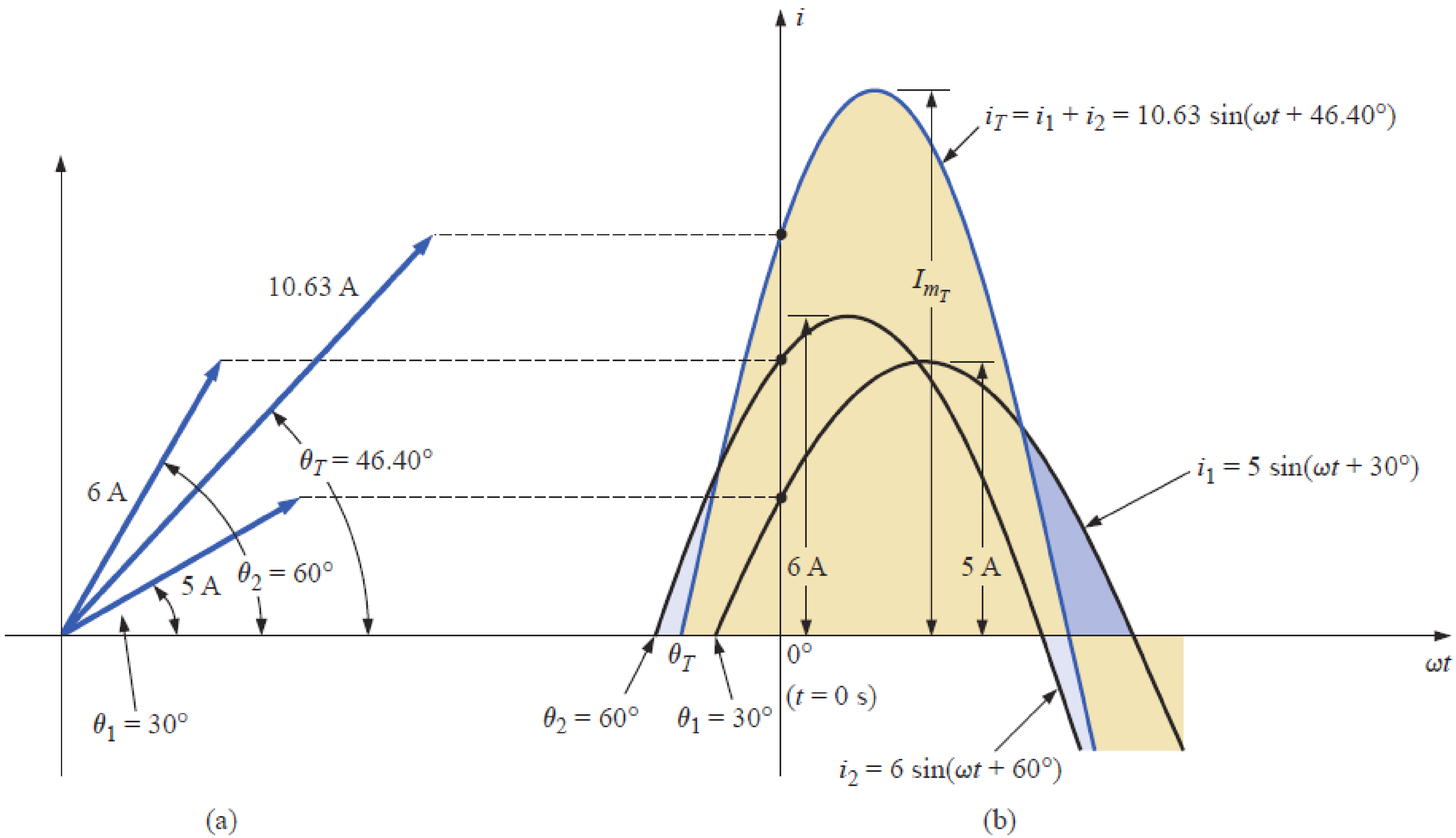


FIG. 14.64

Adding two sinusoidal currents with phase angles other than 90°.

Because the rms values are more used in ac circuits then in all future notations:

The phasor used will have **magnitude equal to the effective** (rms) value of the sign wave it represents.

$$\boldsymbol{V} = V\angle\theta \text{ and } \boldsymbol{I} = I\angle\theta$$

Where V and I are the rms values and θ is the phase angle.

Phasor algebra for sinusoidal quantities is applicable only for waveforms having the same frequency.

Time domain	Phasor domain
$v = V_m \, sin(\omega t \pm \theta)$ $i = I_m \, sin(\omega t \pm \theta)$	$V_{eff}\angle \pm \theta = V_m/\sqrt{2}\angle \pm \theta$ $I_{eff}\angle \pm \theta = I_m/\sqrt{2}\angle \pm \theta$

EXAMPLE 14.29 Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50) \sin \omega t$	$\mathbf{50 \angle 0^\circ}$
b. $69.6 \sin(\omega t + 72^\circ)$	$(0.707)(69.6) \angle 72^\circ = \mathbf{49.21 \angle 72^\circ}$
c. $45 \cos \omega t$	$(0.707)(45) \angle 90^\circ = \mathbf{31.82 \angle 90^\circ}$

EXAMPLE 14.30 Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $\mathbf{I} = 10 \angle 30^\circ$	$i = \sqrt{2}(10) \sin(2\pi 60t + 30^\circ)$ and $i = \mathbf{14.14 \sin(377t + 30^\circ)}$
b. $\mathbf{V} = 115 \angle -70^\circ$	$v = \sqrt{2}(115) \sin(377t - 70^\circ)$ and $v = \mathbf{162.6 \sin(377t - 70^\circ)}$

EXAMPLE 14.31 Find the input voltage of the circuit of Fig. 14.65 if

$$\left.\begin{aligned} v_a &= 50 \sin(377t + 30°) \\ v_b &= 30 \sin(377t + 60°) \end{aligned}\right\} f = 60 \text{ Hz}$$

Solution: Applying Kirchhoff's voltage law, we have

$$e_{in} = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50 \sin(377t + 30°) \Rightarrow \mathbf{V}_a = 35.35 \text{ V} \angle 30°$$
$$v_b = 30 \sin(377t + 60°) \Rightarrow \mathbf{V}_b = 21.21 \text{ V} \angle 60°$$

Converting from polar to rectangular form for addition yields

$$\mathbf{V}_a = 35.35 \text{ V} \angle 30° = 30.61 \text{ V} + j\,17.68 \text{ V}$$
$$\mathbf{V}_b = 21.21 \text{ V} \angle 60° = 10.61 \text{ V} + j\,18.37 \text{ V}$$

Then

$$\begin{aligned} \mathbf{E}_{in} = \mathbf{V}_a + \mathbf{V}_b &= (30.61 \text{ V} + j\,17.68 \text{ V}) + (10.61 \text{ V} + j\,18.37 \text{ V}) \\ &= 41.22 \text{ V} + j\,36.05 \text{ V} \end{aligned}$$

Converting from rectangular to polar form, we have

$$\mathbf{E}_{in} = 41.22 \text{ V} + j\,36.05 \text{ V} = 54.76 \text{ V} \angle 41.17°$$

Converting from the phasor to the time domain, we obtain

$$\mathbf{E}_{in} = 54.76 \text{ V} \angle 41.17° \Rightarrow e_{in} = \sqrt{2}(54.76) \sin(377t + 41.17°)$$

and $$e_{in} = \mathbf{77.43 \sin(377}t \mathbf{+ 41.17°)}$$

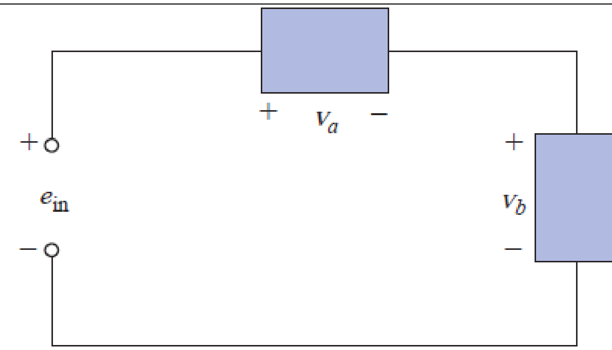
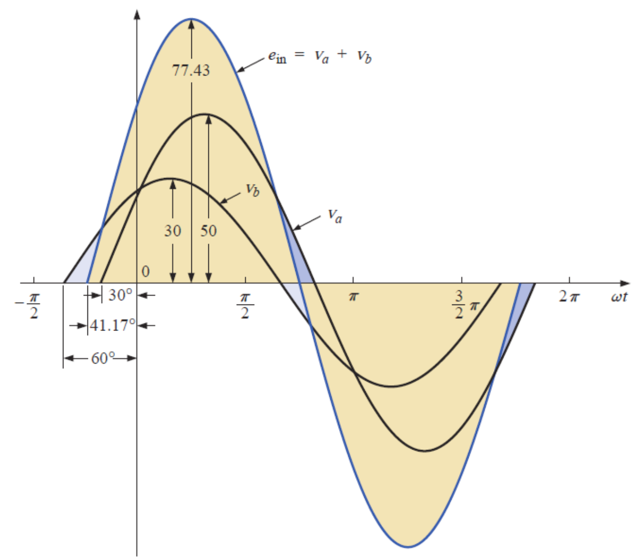


FIG. 14.65
Example 14.31.



EXAMPLE 14.32 Determine the current i_2 for the network of Fig. 14.67.

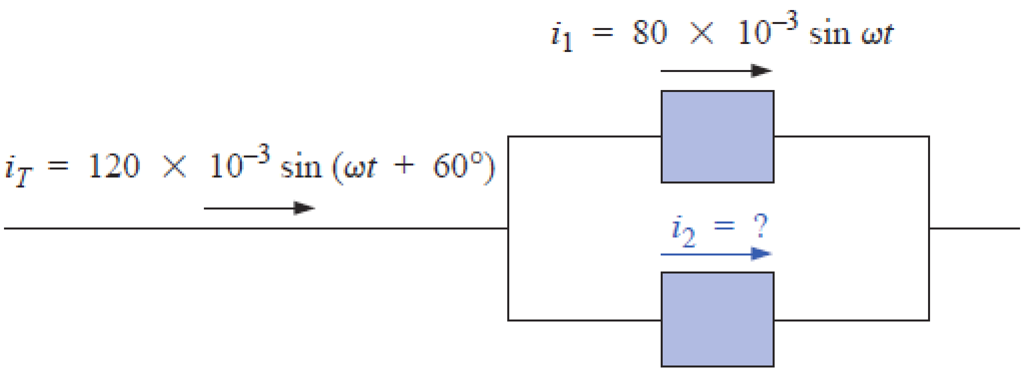


FIG. 14.67
Example 14.32.

Solution: Applying Kirchhoff's current law, we obtain

$$i_T = i_1 + i_2 \quad \text{or} \quad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60°) \Rightarrow 84.84 \text{ mA } \angle 60°$$
$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA } \angle 0°$$

Converting from polar to rectangular form for subtraction yields

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60° = 42.42 \text{ mA} + j\,73.47 \text{ mA}$$
$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0° = 56.56 \text{ mA} + j\,0$$

Then

$$\mathbf{I}_2 = \mathbf{I}_T - \mathbf{I}_1$$
$$= (42.42 \text{ mA} + j\,73.47 \text{ mA}) - (56.56 \text{ mA} + j\,0)$$

and $$\mathbf{I}_2 = -14.14 \text{ mA} + j\,73.47 \text{ mA}$$

Converting from rectangular to polar form, we have

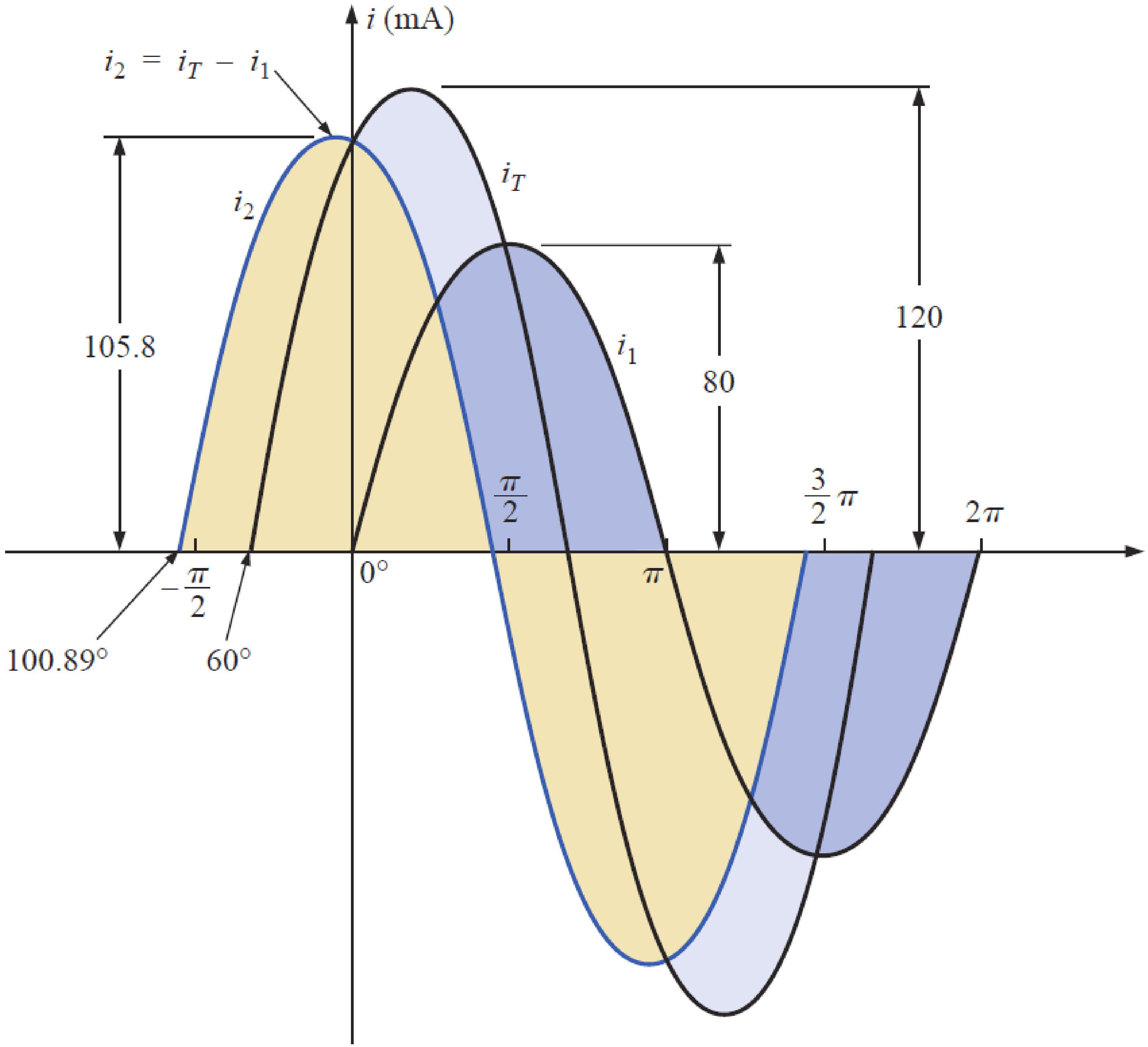
$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89°$$

Converting from the phasor to the time domain, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89° \Rightarrow$$
$$i_2 = \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89°)$$

and $$i_2 = \mathbf{105.8 \times 10^{-3} \sin(\omega t + 100.89°)}$$

A plot of the three waveforms appears in Fig. 14.68. The waveforms clearly indicate that $i_T = i_1 + i_2$.



i (mA)
$i_2 = i_T - i_1$
i_T
i_2
i_1
105.8
80
120
$\frac{\pi}{2}$
$\frac{3}{2}\pi$
2π
0°
π
$-\frac{\pi}{2}$
100.89°
60°