Series and Parallel ac Circuits

15.1 INTRODUCTION

Using the phasors algebra provides methods to **ac circuits** analysis very similar to the methods developed for the dc circuit analysis:

- They differ only in using complex numbers algebra for ac circuit.

Time domain	Phasor domain
$v = V_m \sin(\omega t \pm \theta)$ $i = I_m \sin(\omega t \pm \theta)$	$V_{eff}\angle \pm \theta = V_m/\sqrt{2}\angle \pm \theta$ $I_{eff}\angle \pm \theta = I_m/\sqrt{2}\angle \pm \theta$

15.2 IMPEDANCE AND THE PHASOR DIAGRAM

Resistive Elements

The current i and the voltage v are in phase:

Their magnitude:

$$I_m = \frac{V_m}{R} \quad or \quad V_m = I_m R$$

$$v = V_m \sin \omega t \qquad \Longrightarrow \boldsymbol{V} = V\angle 0°$$

Where $V = 0.707\ V_m$

Applying ohm's Law and using phasor algebra:

$$\boldsymbol{I} = \frac{V\angle 0°}{R\angle \theta_R} = \frac{V}{R}\angle(0° - \theta_R)$$

because i and v are in phase $\Longrightarrow$

$$(0° - \theta_R) = 0° \ \Longrightarrow \theta_R = 0°$$

$$\boldsymbol{I} = \frac{V\angle 0°}{R\angle 0°} = \frac{V}{R}\angle 0°$$

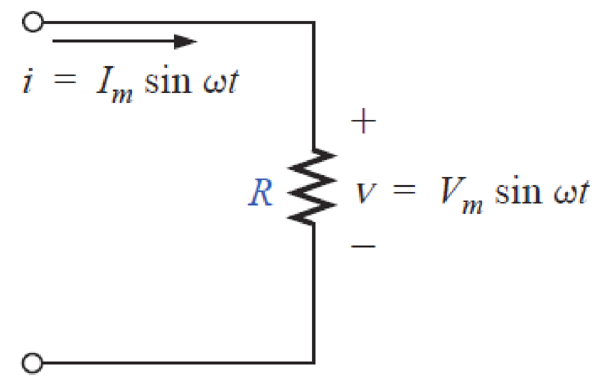


FIG. 15.1
Resistive ac circuit.

In time domain:

$$\boldsymbol{i = \sqrt{2}\left(\frac{V}{R}\right)\sin \omega t}$$

Since $\theta_R = 0°$ we can use a **polar format for the resistance** to give the proper phase relation between the current and voltage:

$$\boldsymbol{Z}_R = R\angle 0°$$

$\boldsymbol{Z}_R$ (boldface have both magnitude and associated angle) is called ***impedance*** of resistive element.

$\boldsymbol{Z}_R$ is not a phasor

The term phasor is reserved for the quantities that vary with time

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit of Fig. 15.4. Sketch the waveforms of v and i.

Solution: Note Fig. 15.5:

$$i = 4\sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A} \angle 30^\circ$$

$$\begin{aligned}\mathbf{V} = \mathbf{I}\mathbf{Z}_R &= (I\angle\theta)(R\angle 0^\circ) = (2.828 \text{ A} \angle 30^\circ)(2\ \Omega \angle 0^\circ)\\ &= 5.656 \text{ V} \angle 30^\circ\end{aligned}$$

and $$v = \sqrt{2}(5.656)\sin(\omega t + 30^\circ) = \mathbf{8.0\ sin(\omega t + 30^\circ)}$$

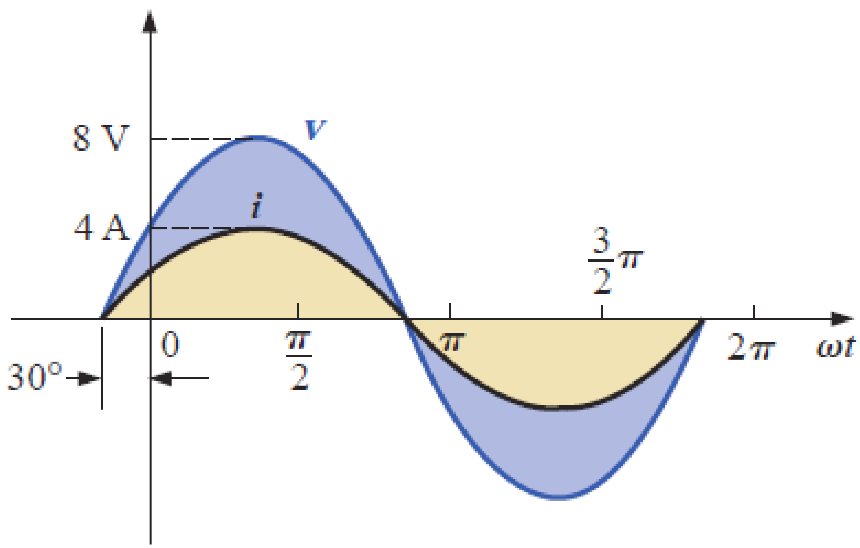


FIG. 15.5
Waveforms for Example 15.2.

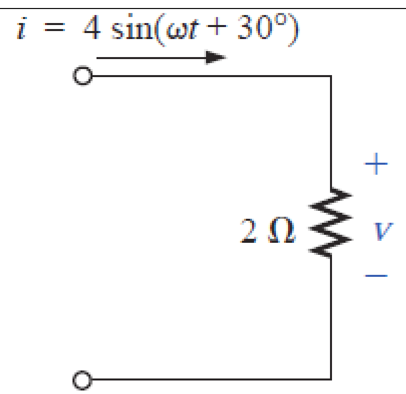


FIG. 15.4
Example 15.2.

Phasor diagram very helpful in network analysis:

- Shows magnitudes
- Shows phase relations

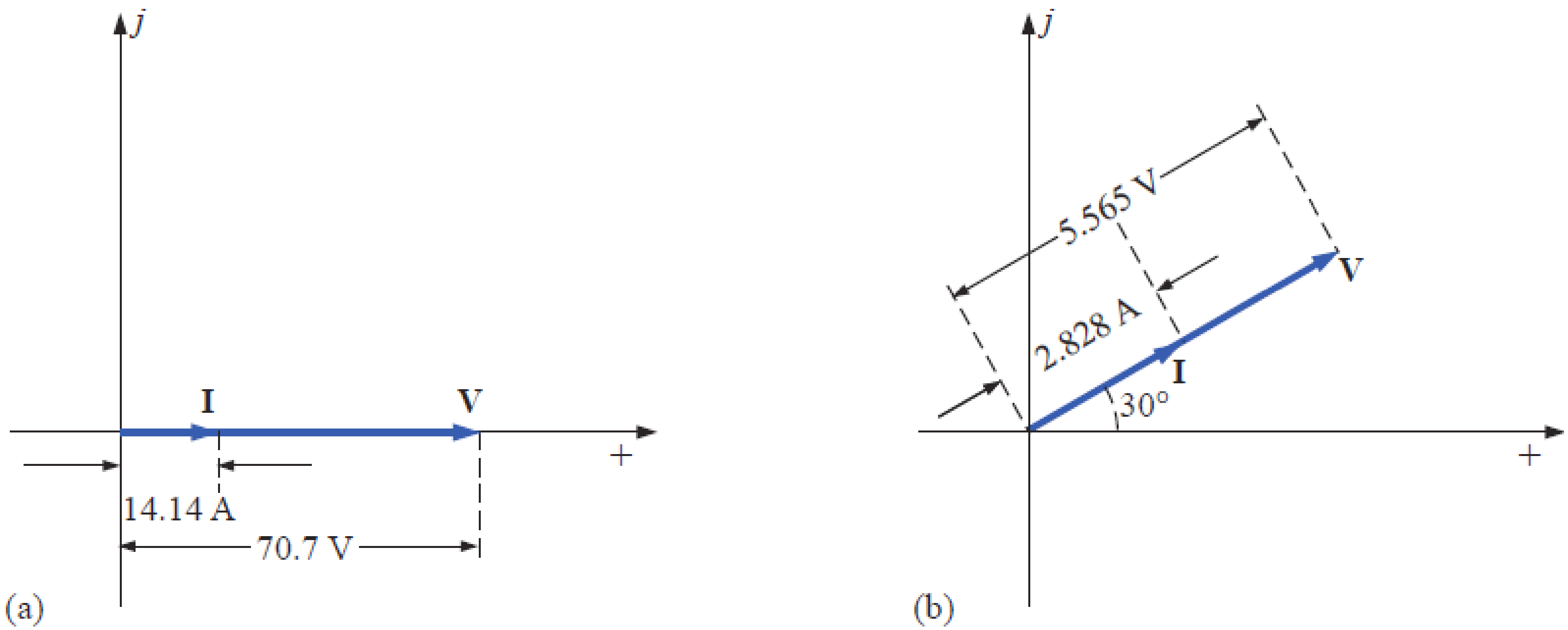


FIG. 15.6
Phasor diagrams for Examples 15.1 and 15.2.

Inductive Reactance

The current i lags the voltage v by 90° and
$X_L = \omega L$

$$v = V_m \sin \omega t \quad \Longrightarrow \boldsymbol{V} = V\angle 0°$$

Applying ohm's Law and using phasor algebra:

$$\boldsymbol{I} = \frac{V\angle 0°}{X_L\angle\theta_L} = \frac{V}{X_L}\angle(0° - \theta_L)$$

because i lags v by 90° $\Longrightarrow$

$$(0° - \theta_L) = -90° \;\Longrightarrow \theta_L = 90°$$

$$\boldsymbol{I} = \frac{V\angle 0°}{X_L\angle 90°} = \frac{V}{X_L}\angle -90°$$

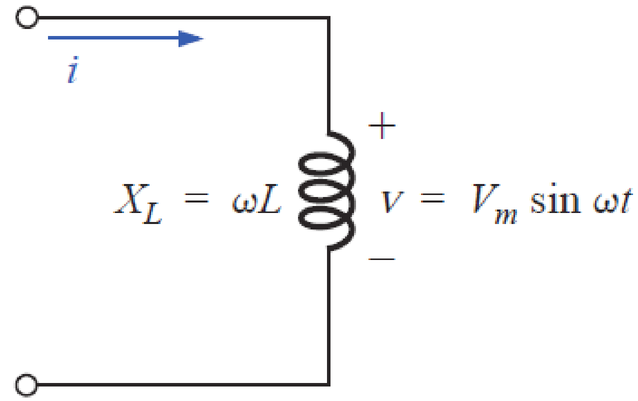


FIG. 15.7
Inductive ac circuit.

In time domain:

$$\boldsymbol{i = \sqrt{2}\left(\frac{V}{X_L}\right)\sin(\omega t - 90°)}$$

The fact that $\theta_L = 90°$ is used to define the impedance of the inductive reactance:

$$\boldsymbol{Z}_L = X_L\angle 90°$$

$\boldsymbol{Z}_L$ is not a phasor

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit of Fig. 15.10. Sketch the v and i curves.

Solution: Note Fig. 15.11:

$$i = 5\sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 3.535\text{ A}\angle 30°$$

$$\begin{aligned}\mathbf{V} = \mathbf{IZ}_L &= (I\angle\theta)(X_L\angle 90°) = (3.535\text{ A}\angle 30°)(4\,\Omega\angle{+90°})\\ &= 14.140\text{ V}\angle 120°\end{aligned}$$

and $$v = \sqrt{2}(14.140)\sin(\omega t + 120°) = \mathbf{20\sin(\omega t + 120°)}$$

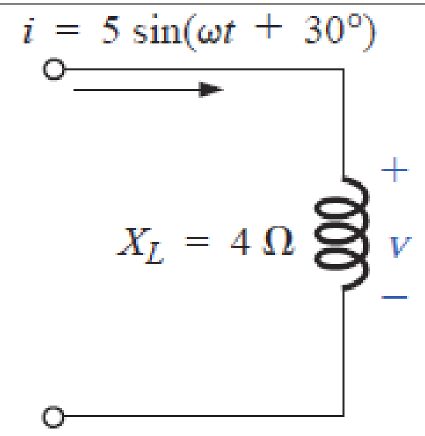


FIG. 15.10
Example 15.4.

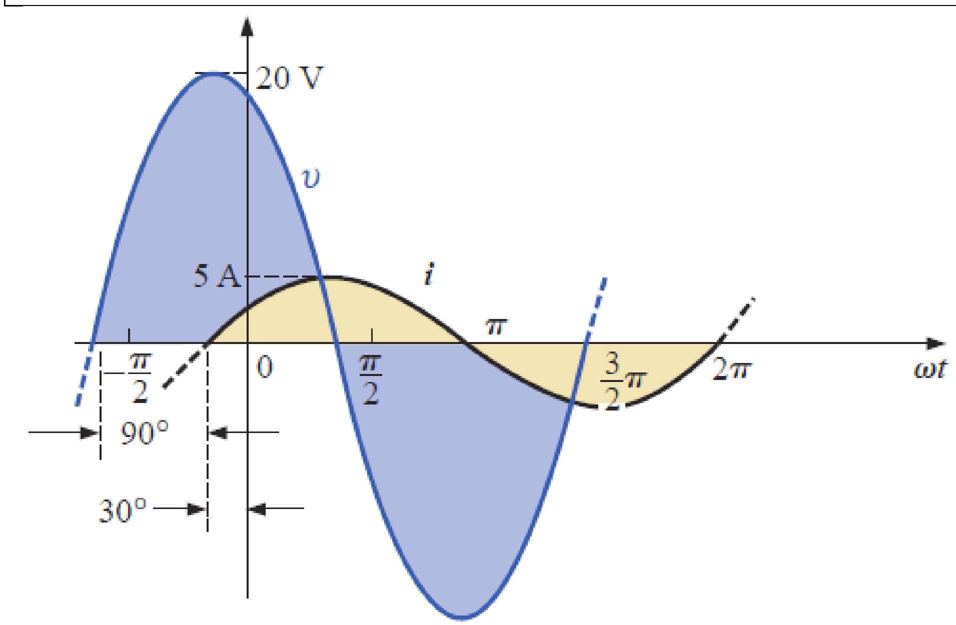


FIG. 15.11
Waveforms for Example 15.4.

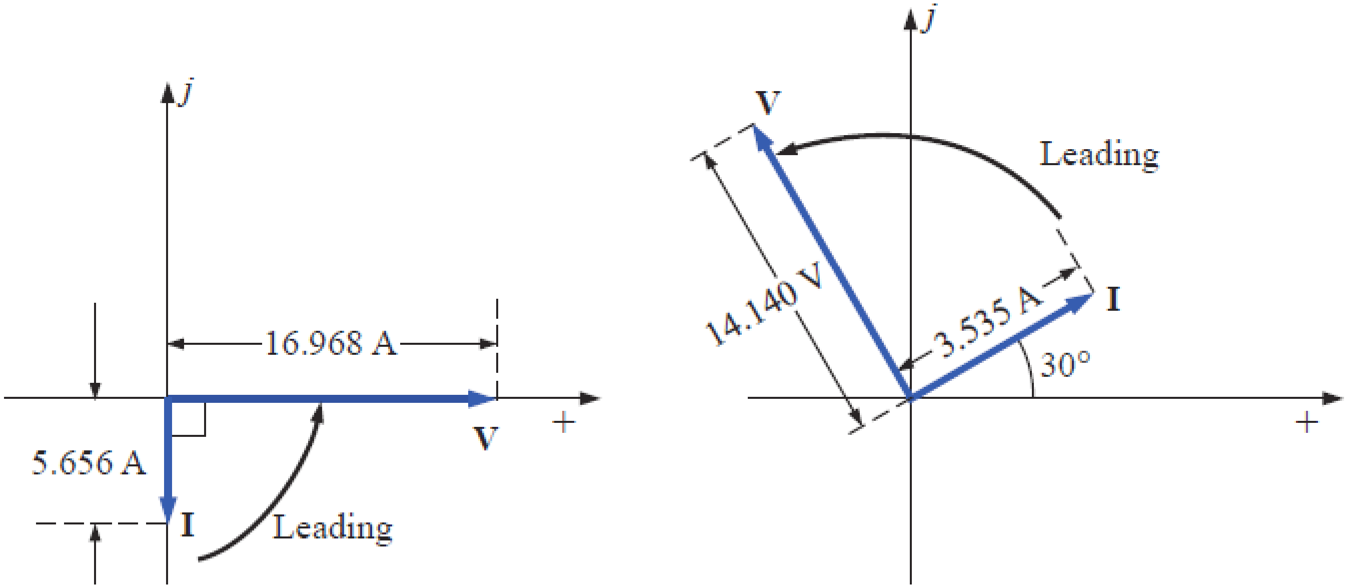


FIG. 15.12
Phasor diagrams for Examples 15.3 and 15.4.

Capacitive Reactance

The current i leads the voltage v by 90° and

$X_C = \frac{1}{\omega C}$

$$v = V_m \sin \omega t \quad \Longrightarrow \boldsymbol{V} = V\angle 0°$$

Applying ohm's Law and using phasor algebra:

$$\boldsymbol{I} = \frac{V\angle 0°}{X_C \angle \theta_C} = \frac{V}{X_C}\angle(0° - \theta_C)$$

because i leads v by 90° $\Longrightarrow$

$$(0° - \theta_C) = +90° \Longrightarrow \theta_C = -90°$$

$$\boldsymbol{I} = \frac{V\angle 0°}{X_C \angle -90°} = \frac{V}{X_C}\angle\big(0° - (-90°)\big) = \frac{V}{X_C}\angle 90°$$

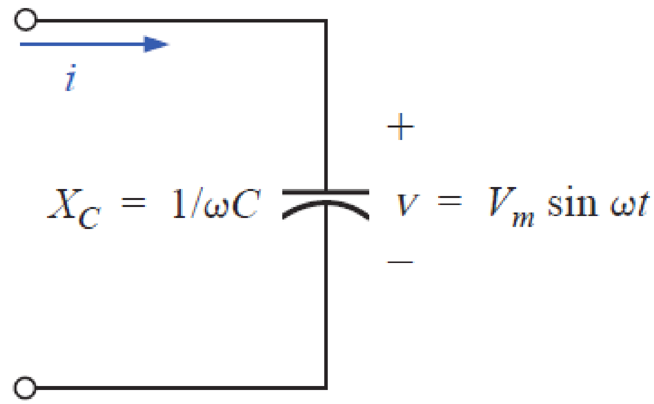


FIG. 15.13
Capacitive ac circuit.

In time domain:

$$\boldsymbol{i = \sqrt{2}\left(\frac{V}{X_C}\right)\sin(\omega t + 90°)}$$

The fact that $\theta_C = -90°$ is used to define the impedance of the capacitive reactance:

$$\boldsymbol{Z}_C = X_C\angle -90°$$

$\boldsymbol{Z}_C$ is not a phasor

EXAMPLE 15.6 Using complex algebra, find the voltage v for the circuit of Fig. 15.16. Sketch the v and i curves.

Solution: Note Fig. 15.17:

$$i = 6\sin(\omega t - 60°) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242\text{ A} \angle -60°$$

$$\begin{aligned}\mathbf{V} = \mathbf{I}\mathbf{Z}_C &= (I \angle \theta)(X_C \angle -90°) = (4.242\text{ A} \angle -60°)(0.5\ \Omega \angle -90°)\\ &= 2.121\text{ V} \angle -150°\end{aligned}$$

and $\quad v = \sqrt{2}(2.121)\sin(\omega t - 150°) = \mathbf{3.0\ sin(\boldsymbol{\omega} t - 150°)}$

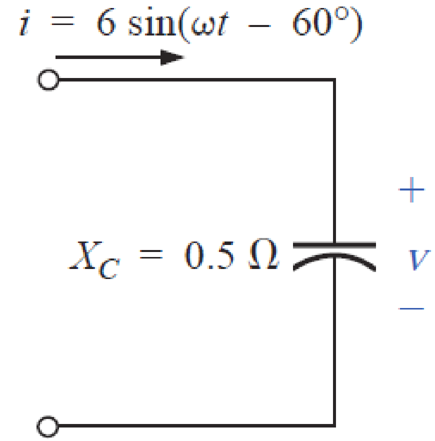


FIG. 15.16
Example 15.6.

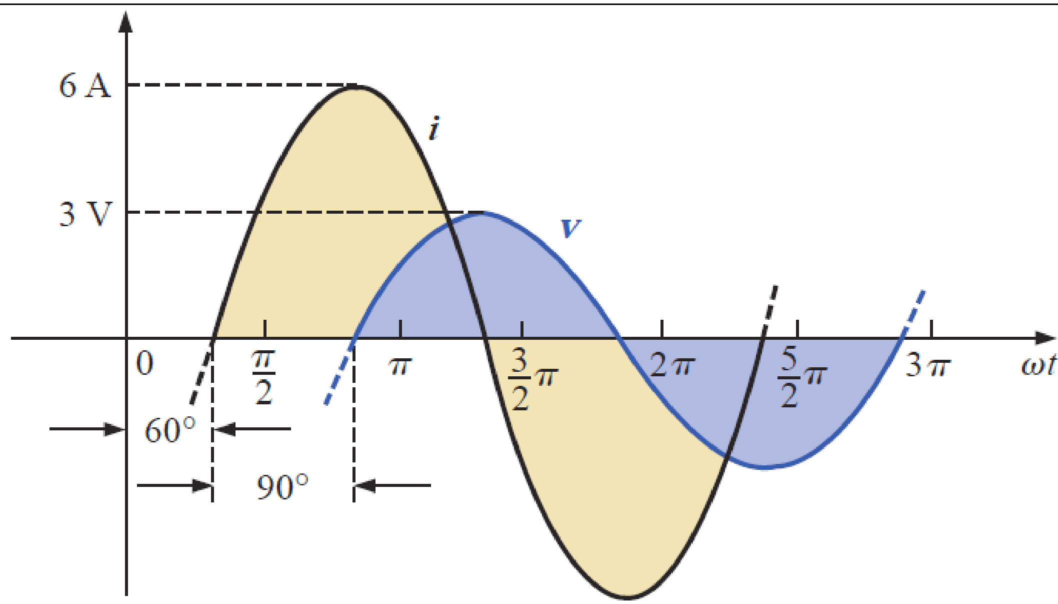
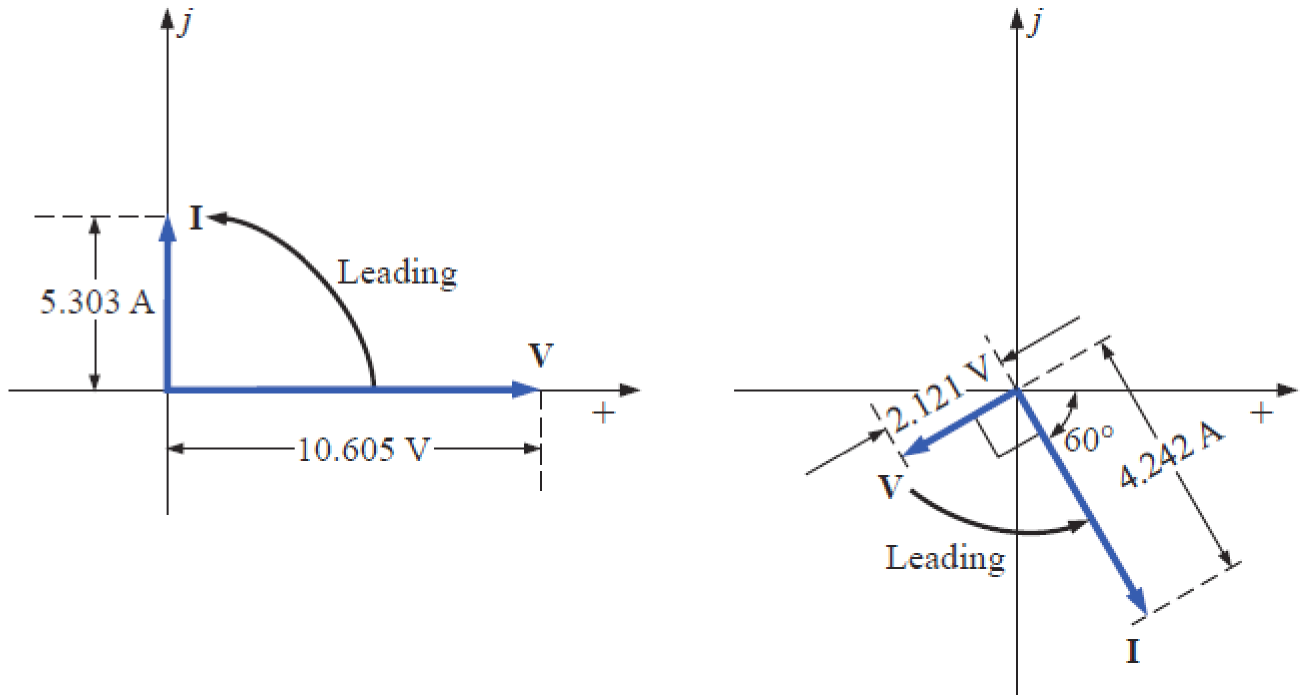


FIG. 15.17
Waveforms for Example 15.6.



(a) (b)

FIG. 15.18
Phasor diagrams for Examples 15.5 and 15.6.

Impedance Diagram

Resistor $\Longrightarrow$ $\boldsymbol{Z}_R = R\angle 0°$

Inductor $\Longrightarrow$ $\boldsymbol{Z}_L = X_L\angle 90°$

Capacitor $\Longrightarrow$ $\boldsymbol{Z}_C = X_C\angle -90°$

Each can be placed in a complex plane diagram:

- Resistor appears positive real axis
- Inductive reactance appears positive imaginary axis
- Capacitive reactance appears negative imaginary axis

The result is an **Impedance Diagram** that can be used to find total impedance of networks.

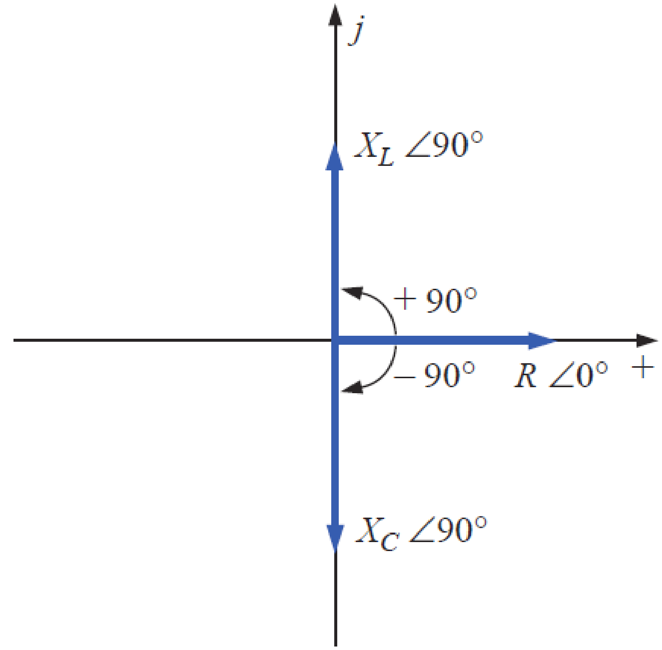


FIG. 15.19
Impedance diagram.

Networks combining different types of elements will have total impedance $\mathbf{Z}_t$ having angle that extends from $-90° \ to \ 90°$

- $\mathbf{Z}_t$ has angle = 0° $\longrightarrow$ it is resistive in nature
- $\mathbf{Z}_t$ has positive angle $\longrightarrow$ it is inductive in nature
- $\mathbf{Z}_t$ has negative angle $\longrightarrow$ it is capacitive in nature

Once $\mathbf{Z}_t$ of a network is determined:

- its magnitude will define the resulting current level (through Ohm's law),
- its angle will reveal whether the network is primarily inductive or capacitive or simply resistive.

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

15.3 SERIES CONFIGURATION

similar to dc network the total impedance of a series ac network is the sum of the individual impedances.

- In dc we were dealing with resistances (real numbers)
- In ac we are dealing with impedances (complex numbers)

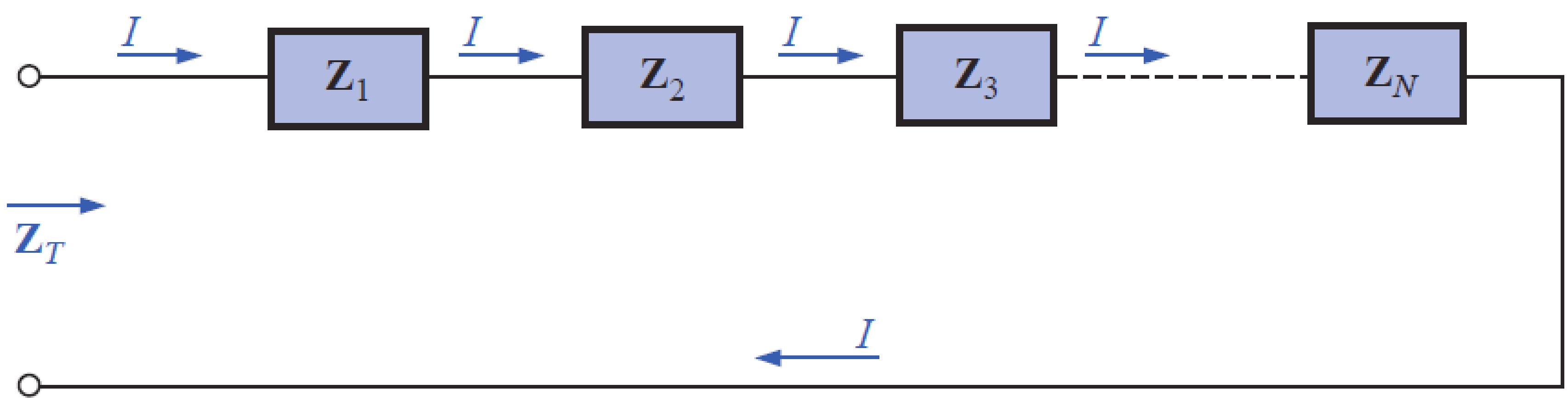


FIG. 15.20
Series impedances.

$$\boxed{\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N}$$

EXAMPLE 15.7 Draw the impedance diagram for the circuit of Fig. 15.21, and find the total impedance.

Solution: As indicated by Fig. 15.22, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain

$$
\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\
&= R \angle 0^\circ + X_L \angle 90^\circ \\
&= R + jX_L = 4\ \Omega + j8\ \Omega \\
\mathbf{Z}_T &= \mathbf{8.944\ \Omega \angle 63.43^\circ}
\end{aligned}
$$

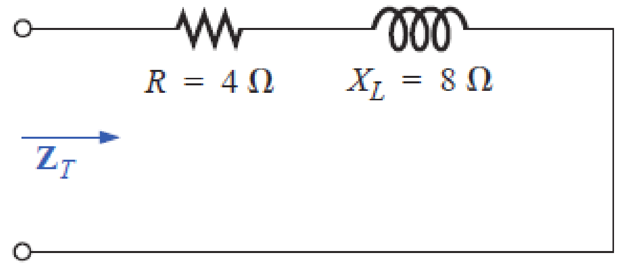


FIG. 15.21
Example 15.7.

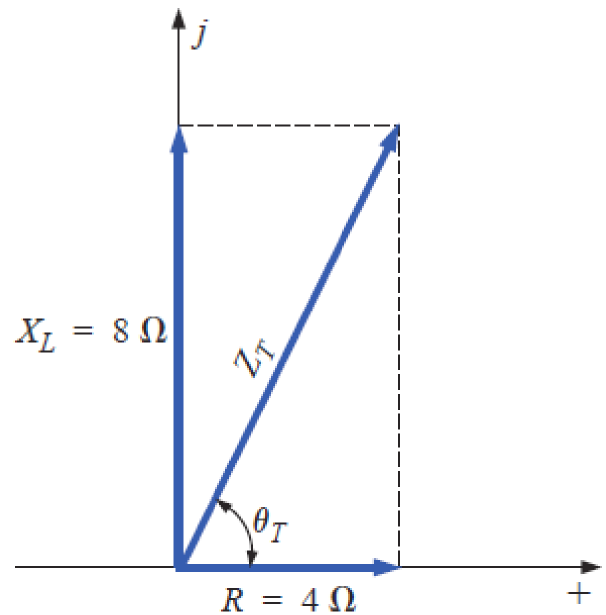


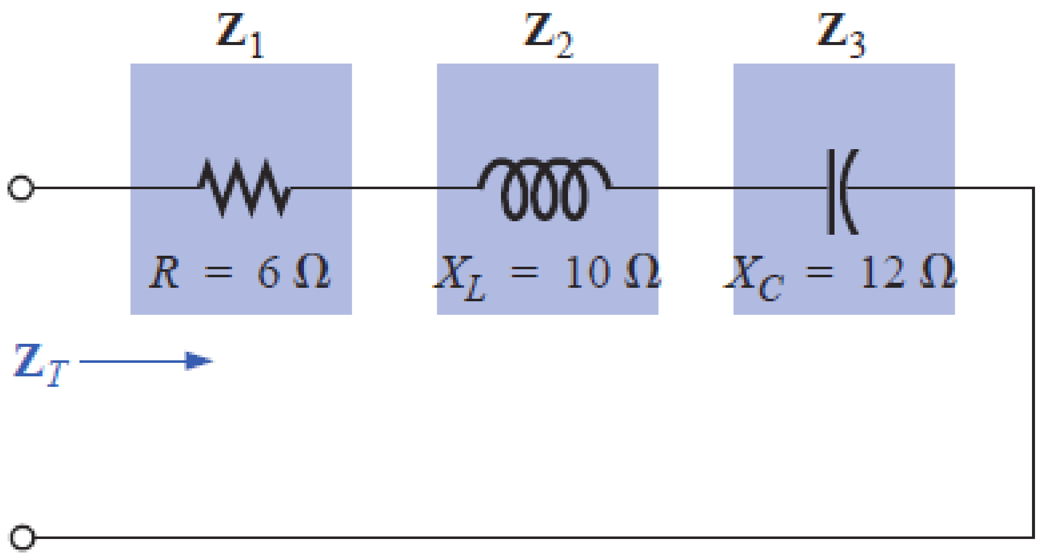
FIG. 15.22
Impedance diagram for Example 15.7.

EXAMPLE 15.8 Determine the input impedance to the series network of Fig. 15.23. Draw the impedance diagram.

Solution:

$$\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \\
&= R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\
&= R + jX_L - jX_C \\
&= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega \\
\mathbf{Z}_T &= \mathbf{6.325\ \Omega \angle -18.43^\circ}
\end{aligned}$$

The impedance diagram appears in Fig. 15.24. Note that in this example, series inductive and capacitive reactances are in direct opposition. For the circuit of Fig. 15.23, if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive. We will have more to say about this particular condition in a later chapter.



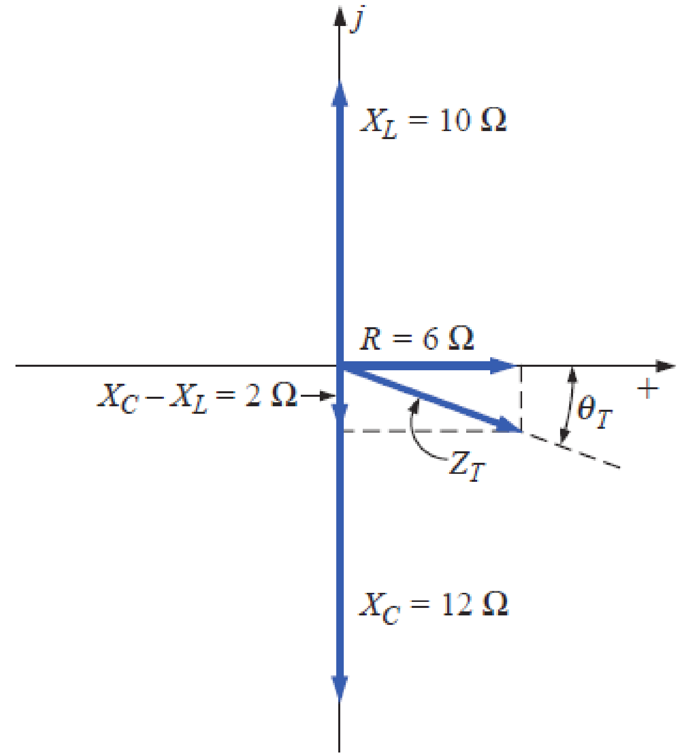


FIG. 15.23

Example 15.8

Consider the **series ac networks** having two impedances:

- The current is the same

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\boxed{\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}}$$

$$\boxed{\mathbf{V}_1 = \mathbf{I}\mathbf{Z}_1} \qquad \boxed{\mathbf{V}_2 = \mathbf{I}\mathbf{Z}_2}$$

All the variables in these relations are complex numbers (vectors) not just numbers.

$$\mathbf{E} - \mathbf{V}_1 - \mathbf{V}_2 = 0 \Longrightarrow \boxed{\mathbf{E} = \mathbf{V}_1 + \mathbf{V}_2}$$

The power to the circuit can be determined by

$$\boxed{P = EI \cos \theta_T}$$

where θ_T is the phase angle between **E** and **I**.

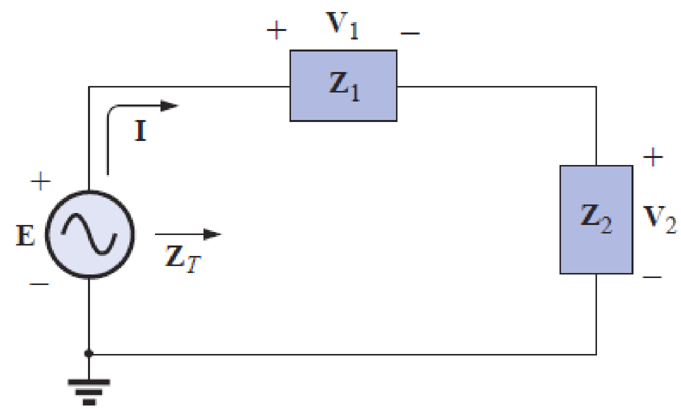


FIG. 15.25
Series ac circuit.

R-L

Phasor Notation:

$e = 141.4 \sin(\omega t) \implies \boldsymbol{E} = 100\ V \angle 0°$

Z_T

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 =$$
$$= 3\ \Omega \angle 0° + 4\ \Omega \angle 90° = 3\ \Omega + j4\ \Omega$$

$$\boldsymbol{Z}_T = 5\ \Omega \angle 53.13°$$

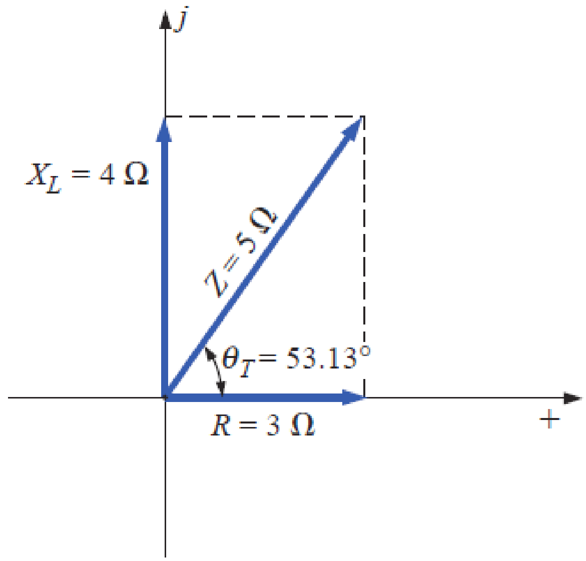


FIG. 15.28
Impedance diagram for the series R-L circuit of Fig. 15.26.

Impedance diagram

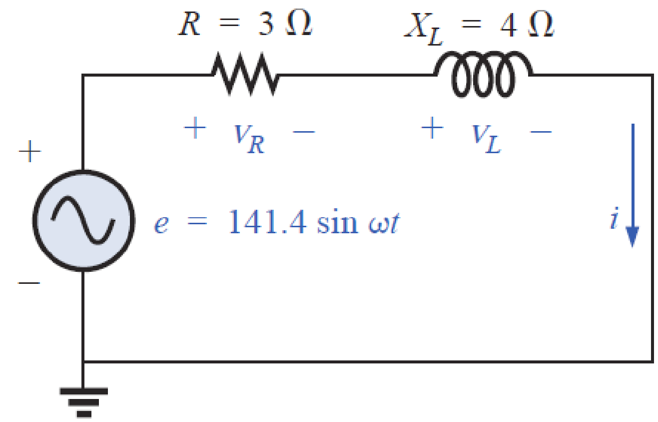


FIG. 15.26
Series R-L circuit.

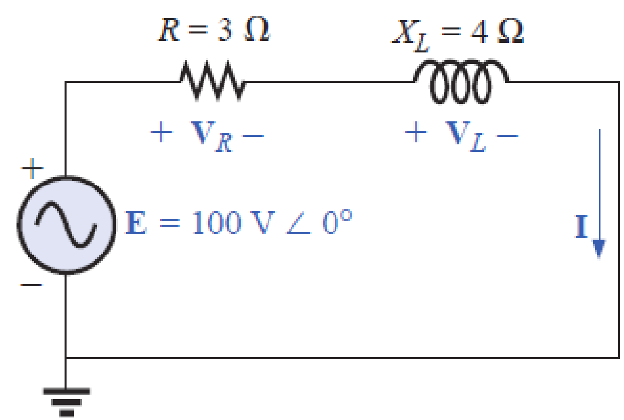


FIG. 15.27
Applying phasor notation to the network

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{20\ A\ \angle -53.13^\circ}$$

V_R and V_L

Ohm's law:

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20\text{ A}\ \angle -53.13^\circ)(3\ \Omega\ \angle 0^\circ)$$
$$= \mathbf{60\ V\angle -53.13^\circ}$$
$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20\text{ A}\ \angle -53.13^\circ)(4\ \Omega\ \angle 90^\circ)$$
$$= \mathbf{80\ V\ \angle 36.87^\circ}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L$$

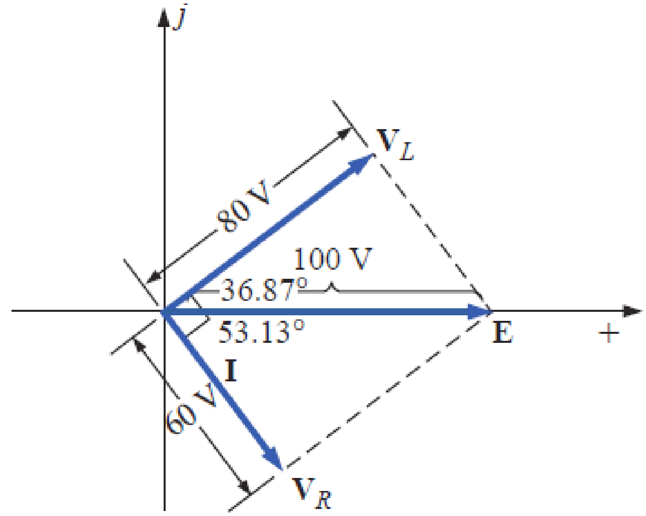


FIG. 15.29
Phasor diagram for the series R-L circuit of Fig. 15.26.

In rectangular form,

$$\mathbf{V}_R = 60\text{ V}\ \angle -53.13^\circ = 36\text{ V} - j\,48\text{ V}$$
$$\mathbf{V}_L = 80\text{ V}\ \angle +36.87^\circ = 64\text{ V} + j\,48\text{ V}$$

and

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L = (36\text{ V} - j\,48\text{ V}) + (64\text{ V} + j\,48\text{ V}) = 100\text{ V} + j\,0$$
$$= 100\text{ V}\ \angle 0^\circ$$

Power: The total power in watts delivered to the circuit is

$$\begin{aligned}P_T &= EI\cos\theta_T \\ &= (100\text{ V})(20\text{ A})\cos 53.13^\circ = (2000\text{ W})(0.6) \\ &= \mathbf{1200\ W}\end{aligned}$$

where E and I are effective values and θ_T is the phase angle between E and I, or

$$\begin{aligned}P_T &= I^2R \\ &= (20\text{ A})^2(3\ \Omega) = (400)(3) \\ &= \mathbf{1200\ W}\end{aligned}$$

where I is the effective value, or, finally,

$$\begin{aligned}P_T = P_R + P_L &= V_R I\cos\theta_R + V_L I\cos\theta_L \\ &= (60\text{ V})(20\text{ A})\cos 0^\circ + (80\text{ V})(20\text{ A})\cos 90^\circ \\ &= 1200\text{ W} + 0 \\ &= \mathbf{1200\ W}\end{aligned}$$

Power factor: The power factor F_p of the circuit is $\cos 53.13^\circ =$ **0.6 lagging,** where 53.13° is the phase angle between **E** and **I.**

Power Factor:

$$\cos\theta = \frac{P}{EI}$$

$$\cos\theta = \frac{P}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

$$\boxed{F_p = \cos\theta_T = \frac{R}{Z_T}}$$

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

R-C

Phasor Notation:

$$i = 7.07\sin(\omega t + 53.13°) \implies \boldsymbol{I} = 5\ A\ \angle 53.13°$$

$\mathbf{Z_T}$

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 =$$

$$= 6\ \Omega\ \angle 0° + 8\ \Omega\ \angle -90° = 6\ \Omega - j8\ \Omega$$

$$\boldsymbol{Z}_T = 10\ \Omega\ \angle -53.13°$$

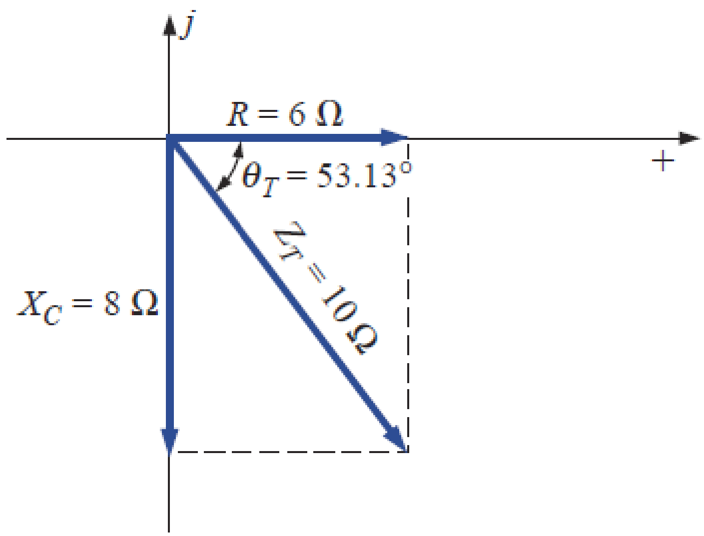


FIG. 15.32

Impedance diagram

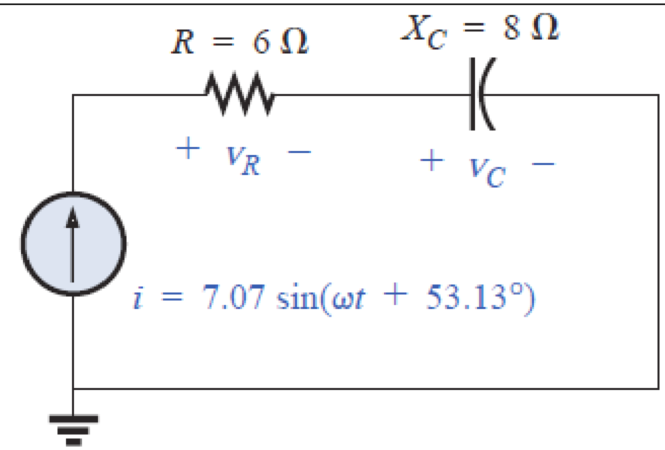


FIG. 15.30

Series R-C ac circuit.

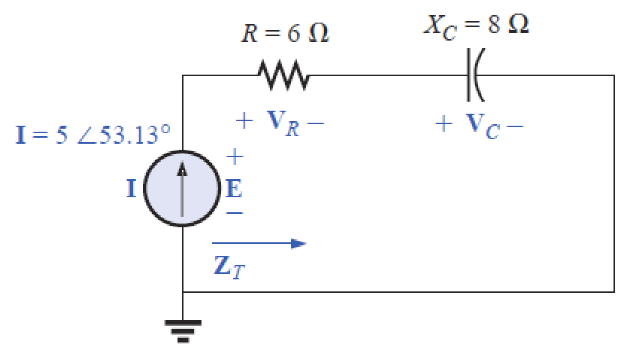


FIG. 15.31

Applying phasor notation to the circuit of Fig. 15.30.

E

$$\mathbf{E} = \mathbf{IZ}_T = (5\text{ A}\angle 53.13°)(10\ \Omega\angle -53.13°) = \mathbf{50\ V\angle 0°}$$

V_R and V_C

$$\mathbf{V}_R = \mathbf{IZ}_R = (I\angle\theta)(R\angle 0°) = (5\text{ A}\angle 53.13°)(6\ \Omega\angle 0°) = \mathbf{30\ V\angle 53.13°}$$

$$\mathbf{V}_C = \mathbf{IZ}_C = (I\angle\theta)(X_C\angle -90°) = (5\text{ A}\angle 53.13°)(8\ \Omega\angle -90°) = \mathbf{40\ V\angle -36.87°}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright}\mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_C = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_C$$

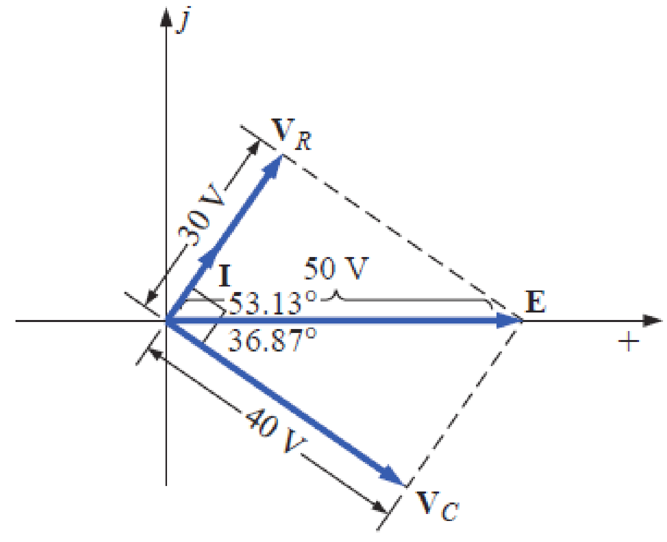


FIG. 15.33
Phasor diagram for the series R-C circuit of Fig. 15.30.

Time domain: In the time domain,

$$e = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\ sin\ \omega t}$$

$$v_R = \sqrt{2}(30)\sin(\omega t + 53.13°) = \mathbf{42.42\ sin(\omega t + 53.13°)}$$

$$v_C = \sqrt{2}(40)\sin(\omega t - 36.87°) = \mathbf{56.56\ sin(\omega t - 36.87°)}$$

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(5\text{ A})\cos 53.13^\circ$$
$$= (250)(0.6) = \mathbf{150\ W}$$

or

$$P_T = I^2R = (5\text{ A})^2(6\ \Omega) = (25)(6)$$
$$= \mathbf{150\ W}$$

or, finally,

$$\begin{aligned} P_T = P_R + P_C &= V_R I\cos\theta_R + V_C I\cos\theta_C \\ &= (30\text{ V})(5\text{ A})\cos 0^\circ + (40\text{ V})(5\text{ A})\cos 90^\circ \\ &= 150\text{ W} + 0 \\ &= \mathbf{150\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta = \cos 53.13^\circ = \mathbf{0.6\ leading}$$

Using Eq. (15.9), we obtain

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{6\ \Omega}{10\ \Omega}$$
$$= \mathbf{0.6\ leading}$$

R-L-C

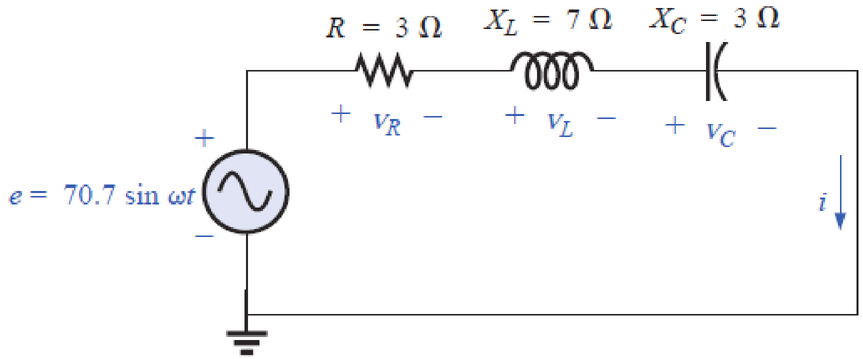
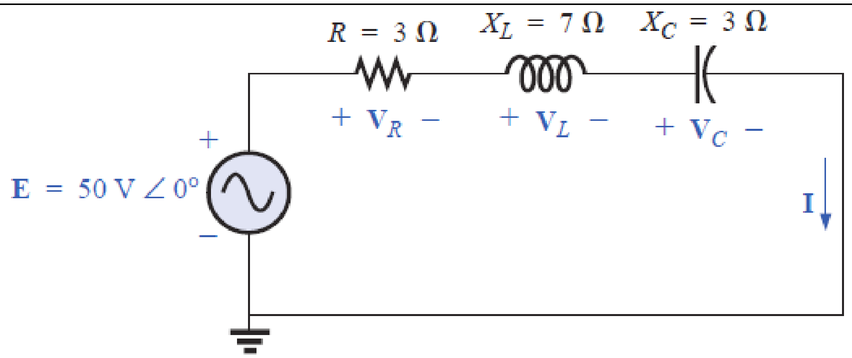


FIG. 15.35
Series R-L-C ac circuit.



Phasor notation

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$

and $$\mathbf{Z}_T = \mathbf{5\ \Omega \angle 53.13^\circ}$$

Impedance diagram: As shown in Fig. 15.37.

$\mathbf{I}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V} \angle 0^\circ}{5\ \Omega \angle 53.13^\circ} = \mathbf{10\ A \angle -53.13^\circ}$$

$\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$

$$\mathbf{V}_R = \mathbf{IZ}_R = (I \angle \theta)(R \angle 0^\circ) = (10\ \text{A} \angle -53.13^\circ)(3\ \Omega \angle 0^\circ)$$
$$= \mathbf{30\ V \angle -53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{IZ}_L = (I \angle \theta)(X_L \angle 90^\circ) = (10\ \text{A} \angle -53.13^\circ)(7\ \Omega \angle 90^\circ)$$
$$= \mathbf{70\ V \angle 36.87^\circ}$$

$$\mathbf{V}_C = \mathbf{IZ}_C = (I \angle \theta)(X_C \angle -90^\circ) = (10\ \text{A} \angle -53.13^\circ)(3\ \Omega \angle -90^\circ)$$
$$= \mathbf{30\ V \angle -143.13^\circ}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

or $$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$

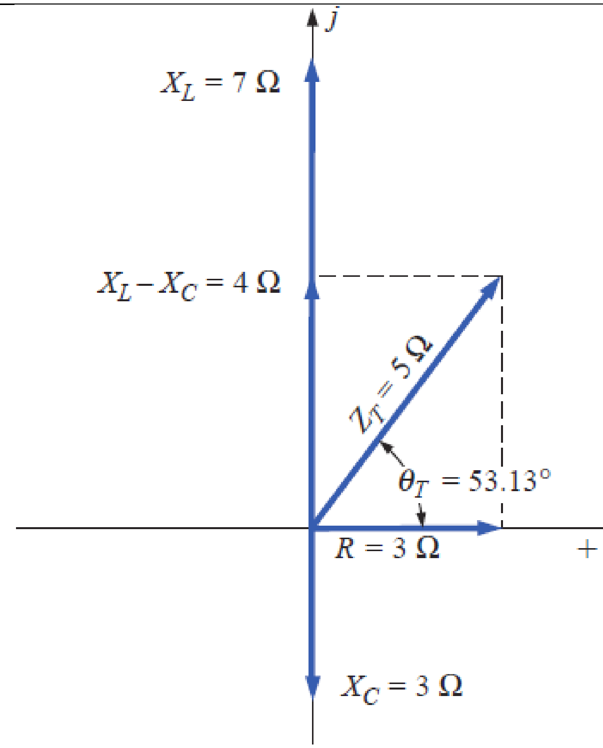


FIG. 15.37
Impedance diagram for the series R-L-C circuit of Fig. 15.35.

Time domain:

$$i = \sqrt{2}(10)\sin(\omega t - 53.13^\circ) = \mathbf{14.14\sin(\omega t - 53.13^\circ)}$$
$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13^\circ) = \mathbf{42.42\sin(\omega t - 53.13^\circ)}$$
$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87^\circ) = \mathbf{98.98\sin(\omega t + 36.87^\circ)}$$
$$v_C = \sqrt{2}(30)\sin(\omega t - 143.13^\circ) = \mathbf{42.42\sin(\omega t - 143.13^\circ)}$$

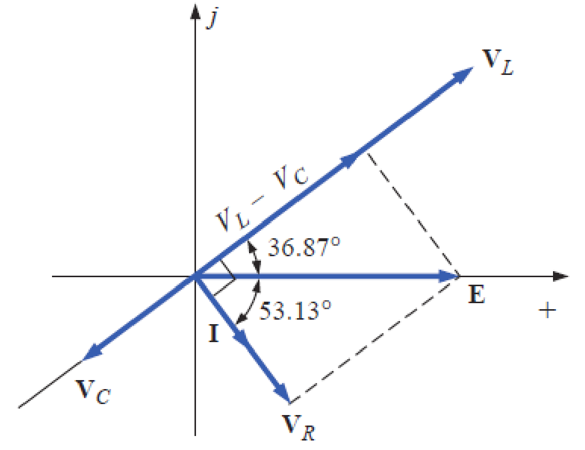


FIG. 15.38
Phasor diagram for the series R-L-C circuit of Fig. 15.35.

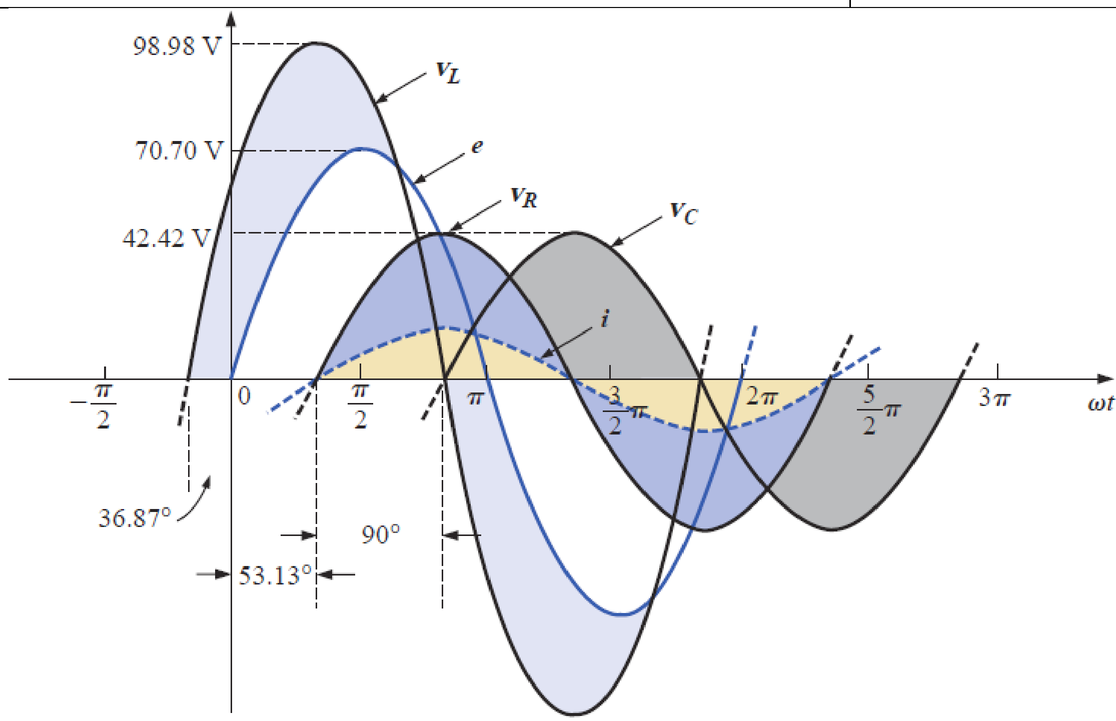


FIG. 15.39
Waveforms for the series R-L circuit of Fig. 15.35.

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(10\text{ A})\cos 53.13^\circ = (500)(0.6) = \mathbf{300\ W}$$

or
$$P_T = I^2R = (10\text{ A})^2(3\ \Omega) = (100)(3) = \mathbf{300\ W}$$

or

$$\begin{aligned}P_T &= P_R + P_L + P_C\\ &= V_RI\cos\theta_R + V_LI\cos\theta_L + V_CI\cos\theta_C\\ &= (30\text{ V})(10\text{ A})\cos 0^\circ + (70\text{ V})(10\text{ A})\cos 90^\circ + (30\text{ V})(10\text{ A})\cos 90^\circ\\ &= (30\text{ V})(10\text{ A}) + 0 + 0 = \mathbf{300\ W}\end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13^\circ = \mathbf{0.6\ lagging}$$

Using Eq. (15.9), we obtain

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

15.4 VOLTAGE DIVIDER RULE

For dc we found:

$$\boxed{V_x = \frac{R_x E}{R_T}} \qquad \text{(voltage divider rule)}$$

The same rule is valid in ac network:

$$\boxed{\mathbf{V}_x = \frac{\mathbf{Z}_x \mathbf{E}}{\mathbf{Z}_T}} \qquad \text{(voltage divider rule)}$$

$\boldsymbol{V_x}$ is the voltage across one or more elements:

$\boldsymbol{E}$ is the total voltage across the series circuit:

$\boldsymbol{Z_T}$ is the total impedance of the series network:

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit of Fig. 15.41.

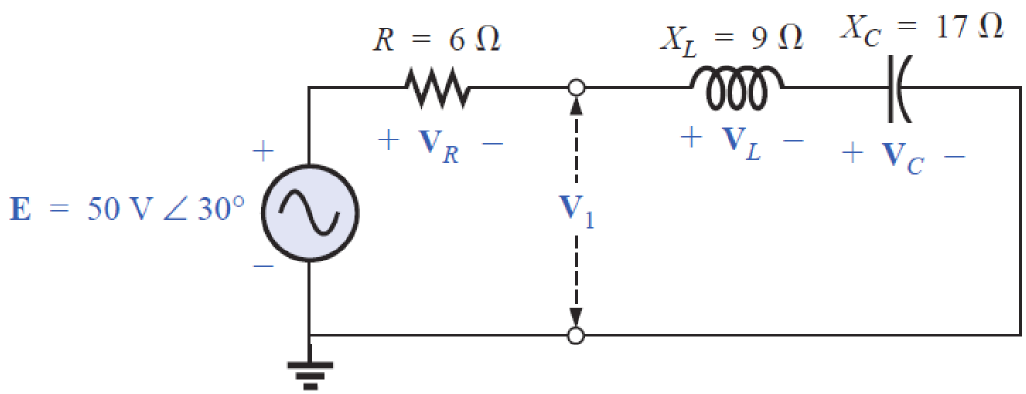


FIG. 15.41
Example 15.10.

Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0^\circ)(50\ \text{V}\ \angle 30^\circ)}{6\ \Omega\ \angle 0^\circ + 9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ}$$

$$= \frac{300\ \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300\ \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300\ \angle 30^\circ}{10\ \angle -53.13^\circ} = \mathbf{30\ V\ \angle 83.13^\circ}$$

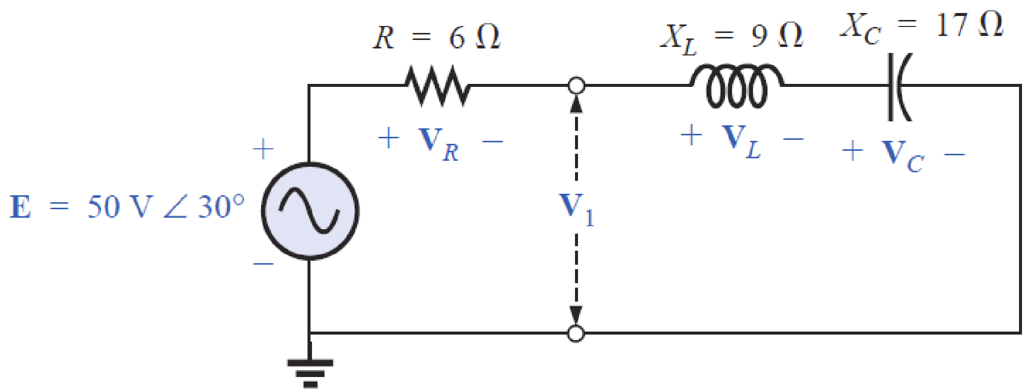


FIG. 15.41
Example 15.10.

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90^\circ)(50\ \text{V} \angle 30^\circ)}{10\ \Omega \angle -53.13^\circ} = \frac{450\ \text{V} \angle 120^\circ}{10 \angle -53.13^\circ}$$

$$= \mathbf{45\ V \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega \angle -90^\circ)(50\ \text{V} \angle 30^\circ)}{10\ \Omega \angle -53.13^\circ} = \frac{850\ \text{V} \angle -60^\circ}{10 \angle -53^\circ}$$

$$= \mathbf{85\ V \angle -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90^\circ + 17\ \Omega \angle -90^\circ)(50\ \text{V} \angle 30^\circ)}{10\ \Omega \angle -53.13^\circ}$$

$$= \frac{(8 \angle -90^\circ)(50 \angle 30^\circ)}{10 \angle -53.13^\circ}$$

$$= \frac{400 \angle -60^\circ}{10 \angle -53.13^\circ} = \mathbf{40\ V \angle -6.87^\circ}$$

15.6 SUMMARY: SERIES ac CIRCUITS

For series ac circuits with reactive elements:

1. ***The total impedance will be frequency dependent.***

2. ***The impedance of any one element can be greater than the total impedance of the network.***

3. ***The inductive and capacitive reactances are always in direct opposition on an impedance diagram.***

4. ***Depending on the frequency applied, the same circuit can be either predominantly inductive or predominantly capacitive.***

5. ***At lower frequencies the capacitive elements will usually have the most impact on the total impedance, while at high frequencies the inductive elements will usually have the most impact.***

6. ***The magnitude of the voltage across any one element can be greater than the applied voltage.***

7. ***The magnitude of the voltage across an element compared to the other elements of the circuit is directly related to the magnitude of its impedance; that is, the larger the impedance of an element, the larger the magnitude of the voltage across the element.***

8. ***The voltages across a coil or capacitor are always in direct opposition on a phasor diagram.***

9. ***The current is always in phase with the voltage across the resistive elements, lags the voltage across all the inductive elements by 90°, and leads the voltage across all the capacitive elements by 90°.***

10. ***The larger the resistive element of a circuit compared to the net reactive impedance, the closer the power factor is to unity.***