Parallel ac Circuits

15.7 ADMITTANCE AND SUSCEPTANCE

In dc circuit we used the ***conductance*** G, $G = \frac{1}{R}$

In ac circuit the ***admittance Y*** is defined as the reciprocal of the impedance ***Z***.

$$\boldsymbol{Y = \frac{1}{Z}}$$ (S) Siemens

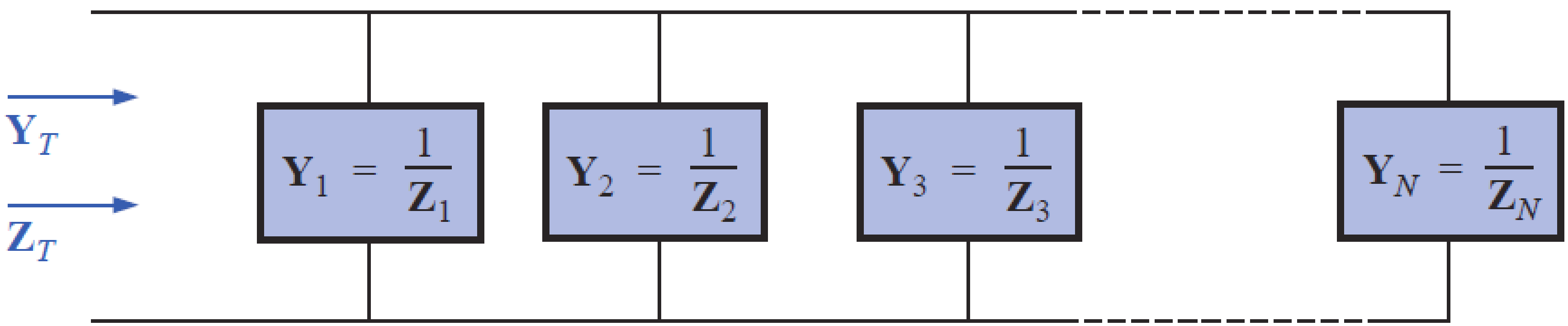


FIG. 15.54
Parallel ac network.

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$$

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}$$

For two impedances in parallel:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

For three impedances in parallel:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Resistor:

Reciprocal of resistance R is called conductance G

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ$$

Reciprocal of reactance (***1/X***) is called ***susceptance*** symbol

B* = *1/X (S)

Inductor:

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ$$

Defining

$$B_L = \frac{1}{X_L}$$

(siemens, S)

$$\mathbf{Y}_L = B_L \angle -90^\circ$$

Capacitor:

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ$$

Defining

$$B_C = \frac{1}{X_C} \quad \text{(siemens, S)}$$

we have

$$\mathbf{Y}_C = B_C \angle 90^\circ$$

Admittance Diagram:

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total admittance is the angle by which the source current leads the applied voltage.

- ***inductive* networks, θ_T is *negative*,**
- ***capacitive* networks, θ_T is *positive*.**

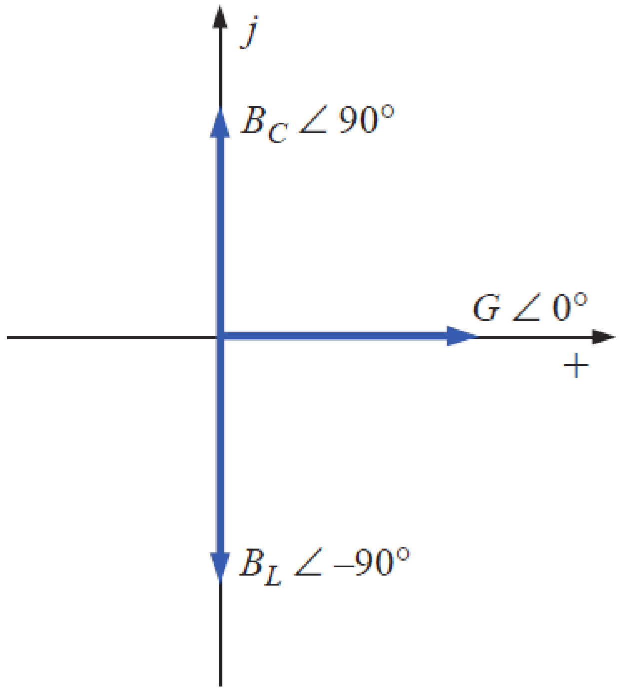


FIG. 15.55
Admittance diagram.

EXAMPLE 15.12 For the network

a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.

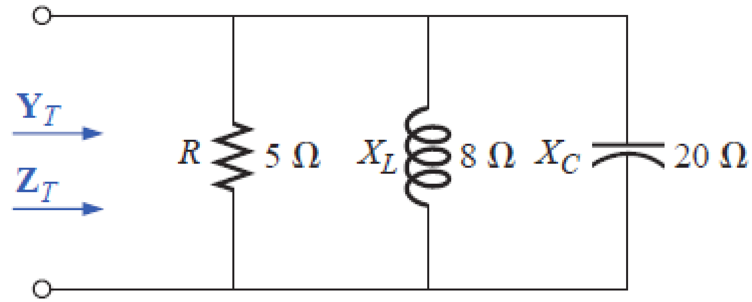


FIG. 15.58
Example 15.13.

Solutions:

a. $\mathbf{Y}_R = G \angle 0° = \dfrac{1}{R} \angle 0° = \dfrac{1}{5\ \Omega} \angle 0°$

$= \mathbf{0.2\ S \angle 0° = 0.2\ S} + \boldsymbol{j}\ \mathbf{0}$

$\mathbf{Y}_L = B_L \angle -90° = \dfrac{1}{X_L} \angle -90° = \dfrac{1}{8\ \Omega} \angle -90°$

$= \mathbf{0.125\ S \angle -90° = 0 -} \boldsymbol{j}\ \mathbf{0.125\ S}$

$\mathbf{Y}_C = B_C \angle 90° = \dfrac{1}{X_C} \angle 90° = \dfrac{1}{20\ \Omega} \angle 90°$

$= \mathbf{0.050\ S \angle +90° = 0 +} \boldsymbol{j}\ \mathbf{0.050\ S}$

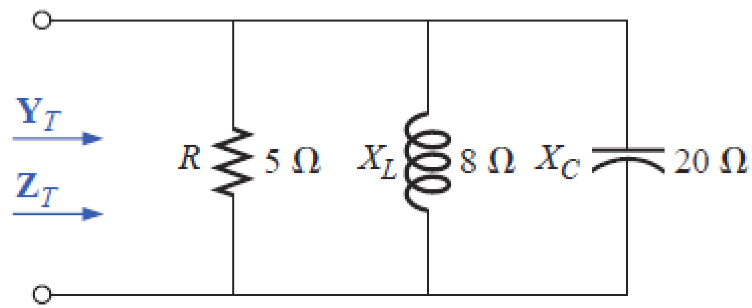


FIG. 15.58
Example 15.13.

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$

$$= (0.2\ \text{S} + j\ 0) + (0 - j\ 0.125\ \text{S}) + (0 + j\ 0.050\ \text{S})$$

$$= 0.2\ \text{S} - j\ 0.075\ \text{S} = \mathbf{0.2136\ S\ \angle{-20.56^\circ}}$$

c. $$\mathbf{Z}_T = \frac{1}{0.2136\ \text{S}\ \angle{-20.56^\circ}} = \mathbf{4.68\ \Omega\ \angle 20.56^\circ}$$

or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C}$$

$$= \frac{(5\ \Omega\ \angle 0^\circ)(8\ \Omega\ \angle 90^\circ)(20\ \Omega\ \angle{-90^\circ})}{(5\ \Omega\ \angle 0^\circ)(8\ \Omega\ \angle 90^\circ) + (8\ \Omega\ \angle 90^\circ)(20\ \Omega\ \angle{-90^\circ}) + (5\ \Omega\ \angle 0^\circ)(20\ \Omega\ \angle{-90^\circ})}$$

$$= \frac{800\ \Omega\ \angle 0^\circ}{40\ \angle 90^\circ + 160\ \angle 0^\circ + 100\ \angle{-90^\circ}}$$

$$= \frac{800\ \Omega}{160 + j\ 40 - j\ 100} = \frac{800\ \Omega}{160 - j\ 60}$$

$$= \frac{800\ \Omega}{170.88\ \angle{-20.56^\circ}}$$

$$= \mathbf{4.68\ \Omega\ \angle 20.56^\circ}$$

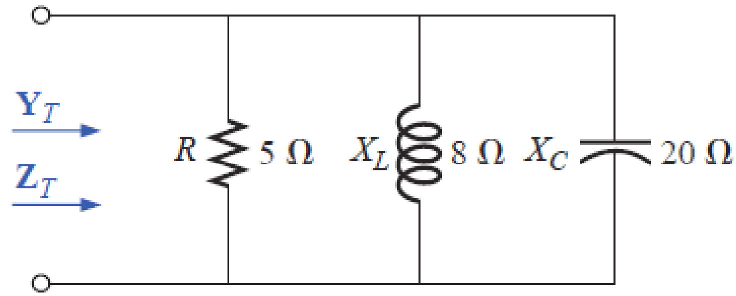


FIG. 15.58
Example 15.13.

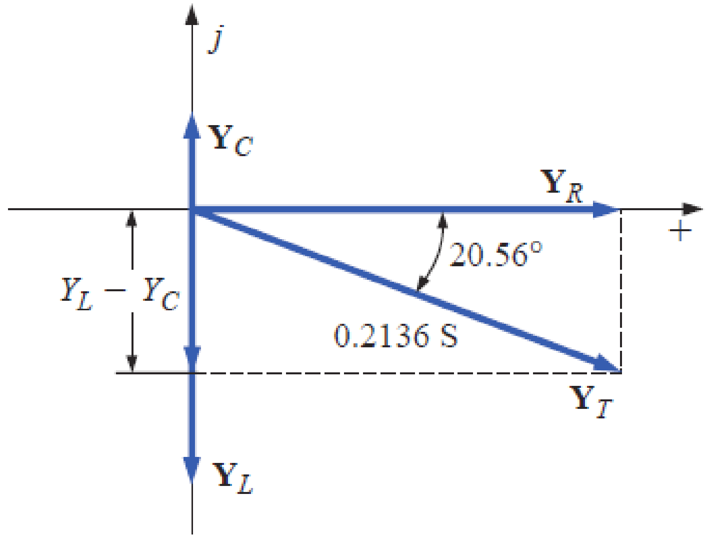


FIG. 15.59
Admittance diagram for the network of Fig. 15.58.

15.8 PARALLEL ac NETWORKS

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T$$

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \mathbf{E}\mathbf{Y}_1$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \mathbf{E}\mathbf{Y}_2$$

$$\mathbf{I} - \mathbf{I}_1 - \mathbf{I}_2 = 0 \Longrightarrow \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

The power:

$$P = EI \cos \theta_T$$

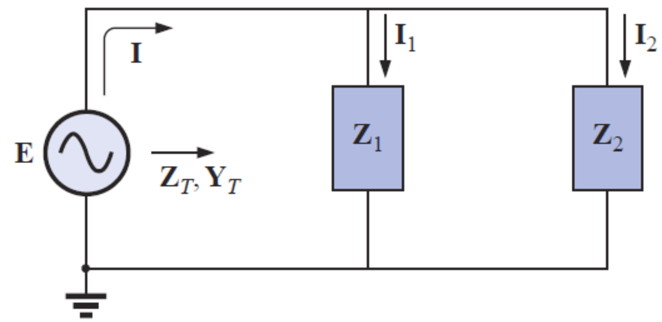


FIG. 15.61
Parallel ac network.

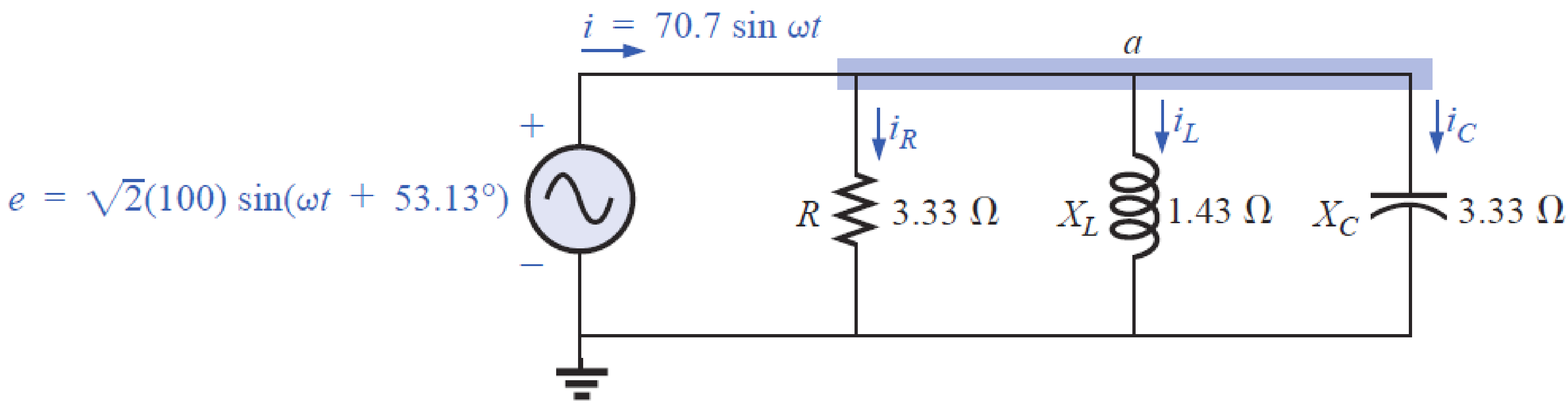


FIG. 15.71
Parallel R-L-C ac network.

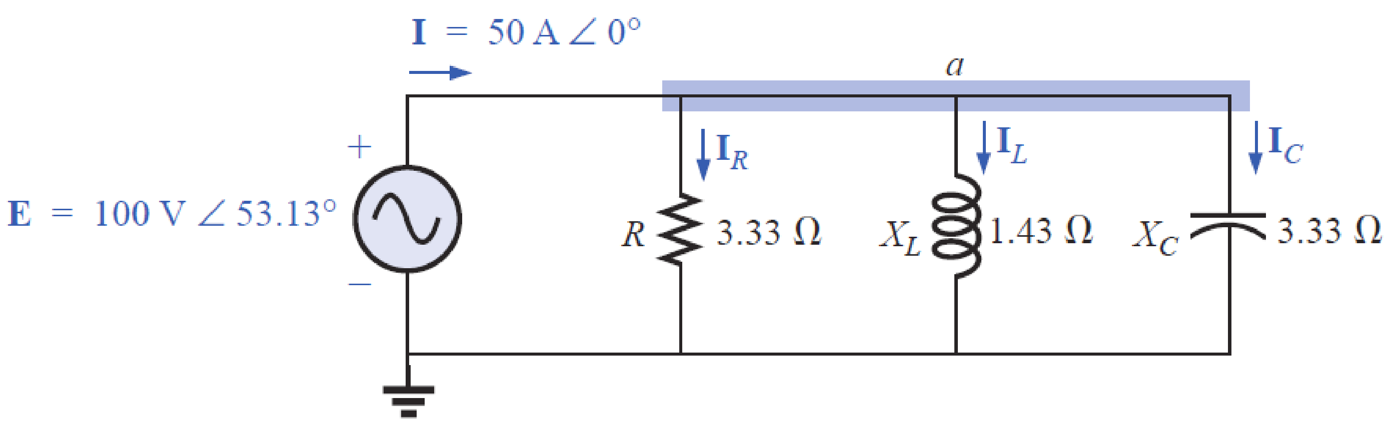


FIG. 15.72
Applying phasor notation to the network of Fig. 15.71.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G \angle 0^\circ + B_L \angle -90^\circ + B_C \angle 90^\circ$$

$$= \frac{1}{3.33\ \Omega} \angle 0^\circ + \frac{1}{1.43\ \Omega} \angle -90^\circ + \frac{1}{3.33\ \Omega} \angle 90^\circ$$

$$= 0.3\ \text{S} \angle 0^\circ + 0.7\ \text{S} \angle -90^\circ + 0.3\ \text{S} \angle 90^\circ$$

$$= 0.3\ \text{S} - j\,0.7\ \text{S} + j\,0.3\ \text{S}$$

$$= 0.3\ \text{S} - j\,0.4\ \text{S} = \mathbf{0.5\ S \angle -53.13^\circ}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S} \angle -53.13^\circ} = \mathbf{2\ \Omega \angle 53.13^\circ}$$

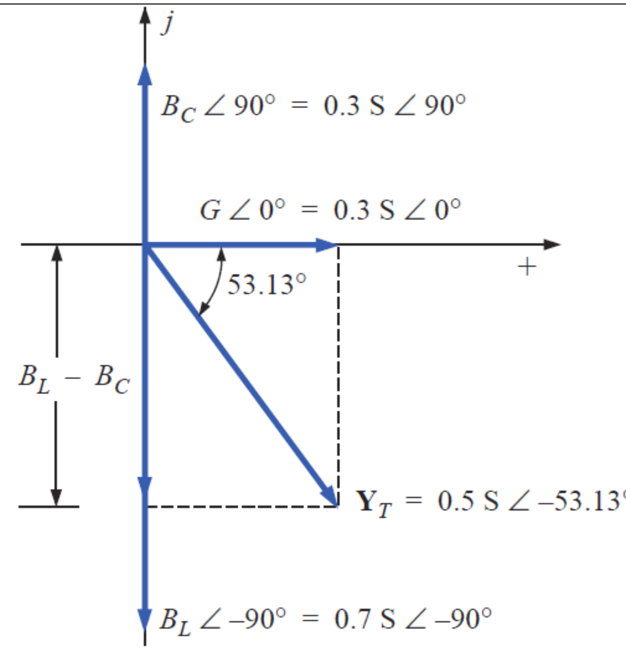


FIG. 15.73
Admittance diagram for the parallel R-L-C network of Fig. 15.71.

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100\text{ V}\angle 53.13^\circ)(0.5\text{ S}\angle -53.13^\circ)$$
$$= \mathbf{50\text{ A}\angle 0^\circ}$$

$\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$

$$\mathbf{I}_R = (E\angle\theta)(G\angle 0^\circ)$$
$$= (100\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle 0^\circ) = \mathbf{30\text{ A}\angle 53.13^\circ}$$

$$\mathbf{I}_L = (E\angle\theta)(B_L\angle -90^\circ)$$
$$= (100\text{ V}\angle 53.13^\circ)(0.7\text{ S}\angle -90^\circ) = \mathbf{70\text{ A}\angle -36.87^\circ}$$

$$\mathbf{I}_C = (E\angle\theta)(B_C\angle 90^\circ)$$
$$= (100\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle +90^\circ) = \mathbf{30\text{ A}\angle 143.13^\circ}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0 \quad\text{or}\quad \mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

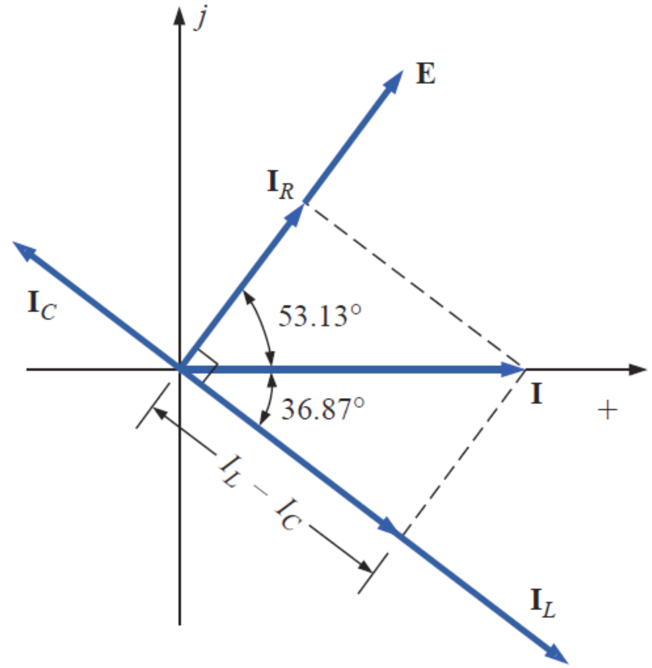


FIG. 15.74
Phasor diagram for the parallel R-L-C network of Fig. 15.71.

Time domain:

$$i = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\ sin\ \omega t}$$
$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13^\circ) = \mathbf{42.42\ sin(\omega t + 53.13°)}$$
$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87^\circ) = \mathbf{98.98\ sin(\omega t - 36.87°)}$$
$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13^\circ) = \mathbf{42.42\ sin(\omega t + 143.13°)}$$

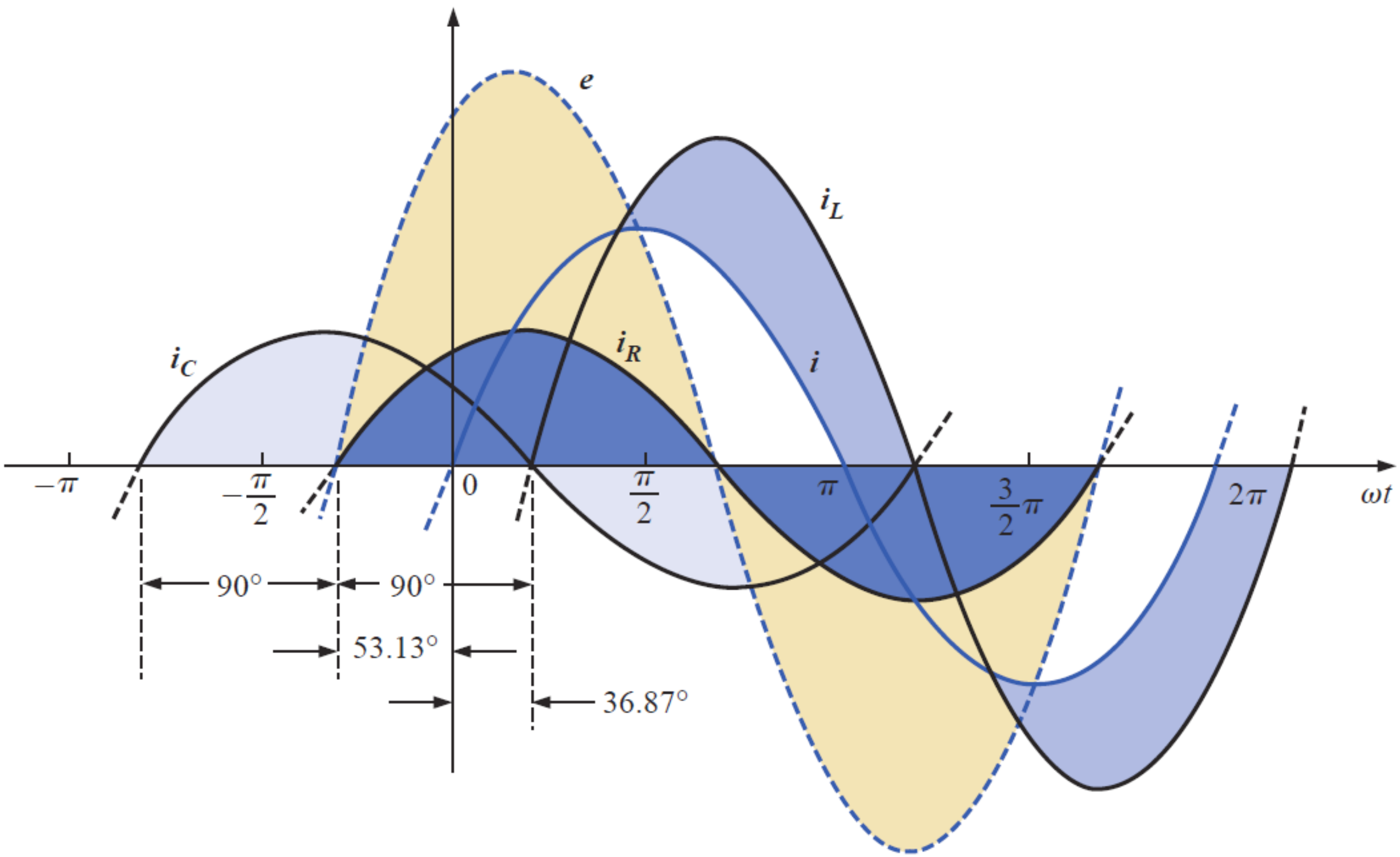


FIG. 15.75
Waveforms for the parallel R-L-C network of Fig. 15.71.

Power: The total power in watts delivered to the circuit is

$$P_T = EI \cos\theta = (100\text{ V})(50\text{ A}) \cos 53.13^\circ = (5000)(0.6)$$
$$= \mathbf{3000\ W}$$

or

$$P_T = E^2G = (100\text{ V})^2(0.3\text{ S}) = \mathbf{3000\ W}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= EI_R \cos\theta_R + EI_L \cos\theta_L + EL_C \cos\theta_C \\ &= (100\text{ V})(30\text{ A}) \cos 0^\circ + (100\text{ V})(70\text{ A}) \cos 90^\circ \\ &\qquad + (100\text{ V})(30\text{ A}) \cos 90^\circ \\ &= 3000\text{ W} + 0 + 0 = \mathbf{3000\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13^\circ = \mathbf{0.6\ lagging}$$

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.3\text{ S}}{0.5\text{ S}} = \mathbf{0.6\ lagging}$$

15.9 CURRENT DIVIDER RULE

Just similar to dc current divider rule:

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

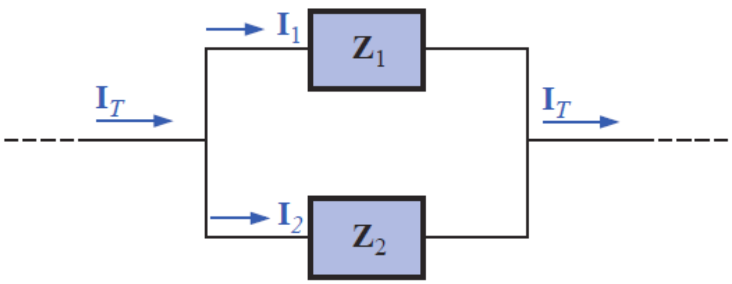


FIG. 15.76
Applying the current divider rule.

EXAMPLE 15.16 Using the current divider rule, find the current through each parallel branch of Fig. 15.78.

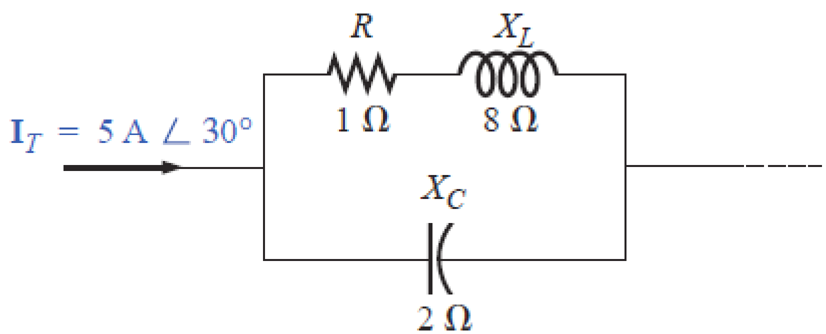


FIG. 15.78
Example 15.16.

Solution:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C\mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega\ \angle -90^\circ)(5\ \text{A}\ \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A}\angle -60^\circ}{1 + j\,6}$$

$$= \frac{10\ \text{A}\angle -60^\circ}{6.083\ \angle 80.54^\circ} \cong \mathbf{1.644\ A\ \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L}\mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A}\ \angle 30^\circ)}{6.08\ \Omega\ \angle 80.54^\circ}$$

$$= \frac{(8.06\ \angle 82.87^\circ)(5\ \text{A}\angle 30^\circ)}{6.08\ \angle 80.54^\circ} = \frac{40.30\ \text{A}\angle 112.87^\circ}{6.083\ \angle 80.54^\circ}$$

$$= \mathbf{6.625\ A\ \angle 32.33^\circ}$$

15.11 SUMMARY: PARALLEL ac NETWORKS

For series ac circuits with reactive elements:

1. ***The total admittance (impedance) will be frequency dependent.***
2. ***The impedance of any one element can be less than the total impedance (recall that for dc the total resistance must always be less than the smallest parallel resistance).***
3. ***The inductive and capacitive susceptances are always in direct opposition on an admittance diagram.***
4. ***Depending on the frequency applied, the same circuit can be either predominantly inductive or predominantly capacitive.***
5. ***At high frequencies the capacitive elements will usually have the most impact on the total impedance, while at lower frequencies the inductive elements will usually have the most impact.***
6. ***The magnitude of the current through any one branch can be greater than the source current.***

7. ***The magnitude of the current through an element, compared to the other elements of the circuit, is directly related to the magnitude of its impedance; that is, the smaller the impedance of an element, the larger the magnitude of the current through the element.***

8. ***The current through a coil or capacitor are always in direct opposition on a phasor diagram.***

9. ***The applied voltage is always in phase with the current through the resistive elements, leads the current through all the inductive elements by 90°, and lags the current through all the capacitive elements by 90°.***

10. ***The smaller the resistive element of a circuit compared to the net reactive susceptance, the closer the power factor is to unity.***

15.12 EQUIVALENT CIRCUITS

In series or parallel ac circuits:

- The total impedance of two or more elements is equivalent to an impedance that can be achieved with fewer elements of different values.

$$\mathbf{Z}_T = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(5\ \Omega \angle -90^\circ)(10\ \Omega \angle 90^\circ)}{5\ \Omega \angle -90^\circ + 10\ \Omega \angle 90^\circ} = \frac{50 \angle 0^\circ}{5 \angle 90^\circ}$$

$$= 10\ \Omega \angle -90^\circ$$

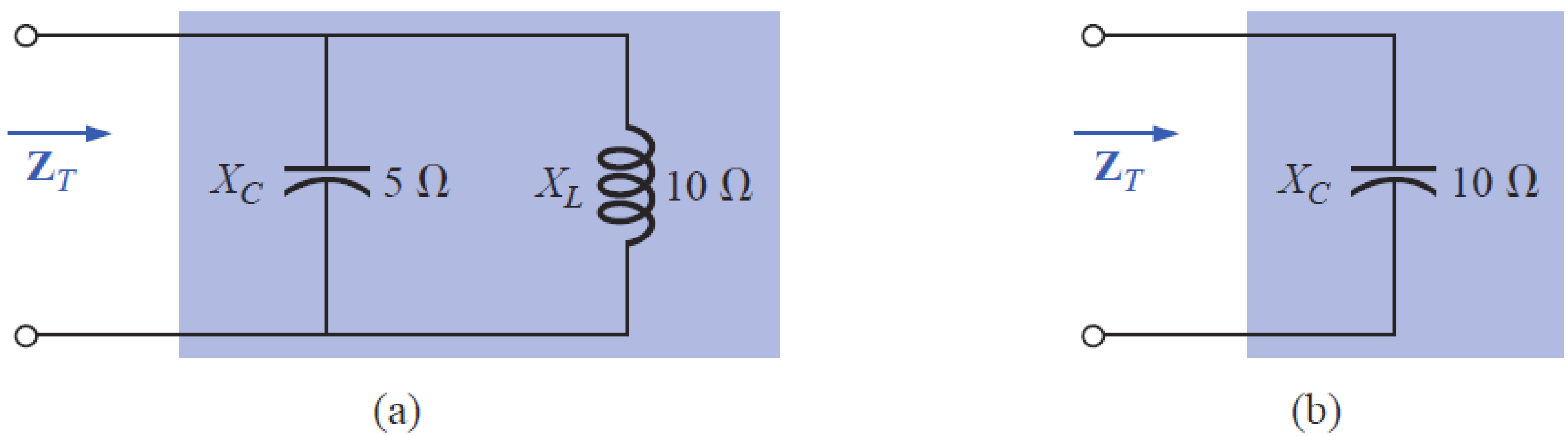


FIG. 15.87

Defining the equivalence between two networks at a specific frequency.

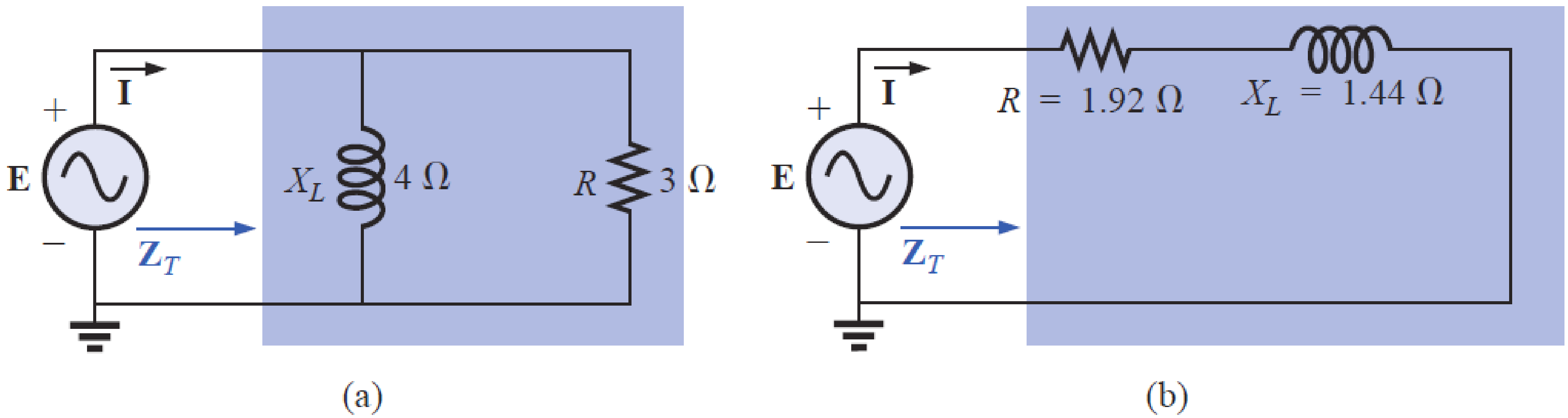


FIG. 15.88

Finding the series equivalent circuit for a parallel R-L network.

$$\mathbf{Z}_T = \frac{\mathbf{Z}_L \mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R} = \frac{(4\ \Omega \angle 90^\circ)(3\ \Omega \angle 0^\circ)}{4\ \Omega \angle 90^\circ + 3\ \Omega \angle 0^\circ}$$

$$= \frac{12 \angle 90^\circ}{5 \angle 53.13^\circ} = 2.40\ \Omega \angle 36.87^\circ$$

$$= 1.920\ \Omega + j\,1.440\ \Omega$$

the term* equivalent *refers only to the fact that for the same applied potential, the same impedance and input current will result.

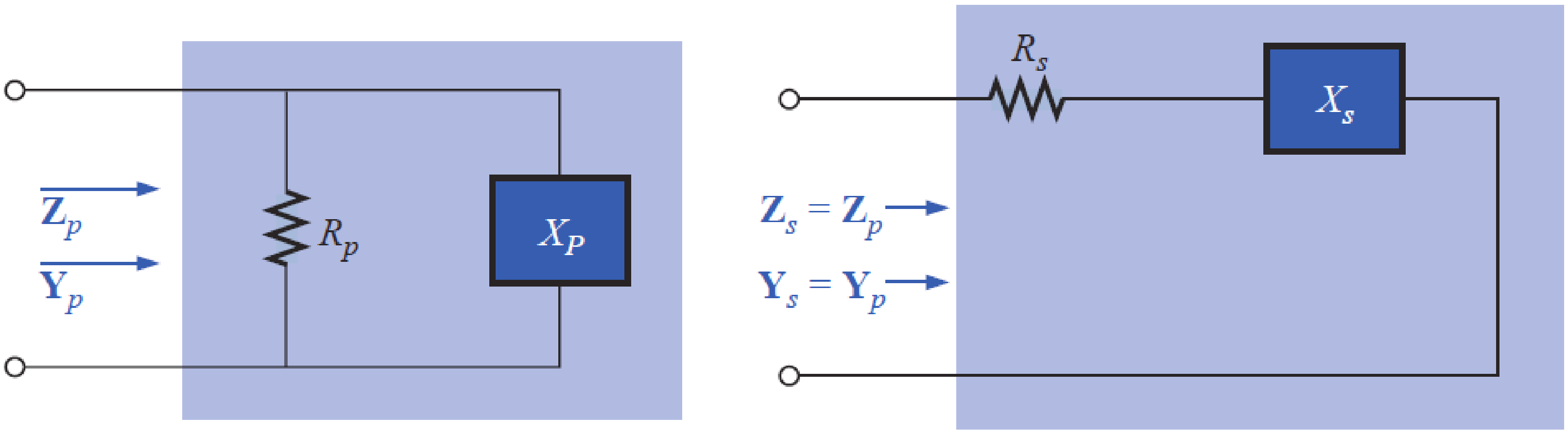


FIG. 15.89
Defining the parameters of equivalent series and parallel networks.

$$\mathbf{Y}_p = \frac{1}{R_p} + \frac{1}{\pm j\,X_p} = \frac{1}{R_P} \mp j\frac{1}{X_p}$$

$$\mathbf{Z}_p = \frac{1}{\mathbf{Y}_p} = \frac{1}{(1/R_p) \mp j\,(1/X_p)} = \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2} \pm j\frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2}$$

$$\mathbf{Z}_p = \frac{R_p X_p^2}{X_p^2 + R_p^2} \pm j\frac{R_p^2 X_p}{X_p^2 + R_p^2} = R_s \pm j\,X_s$$

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2}$$

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2}$$

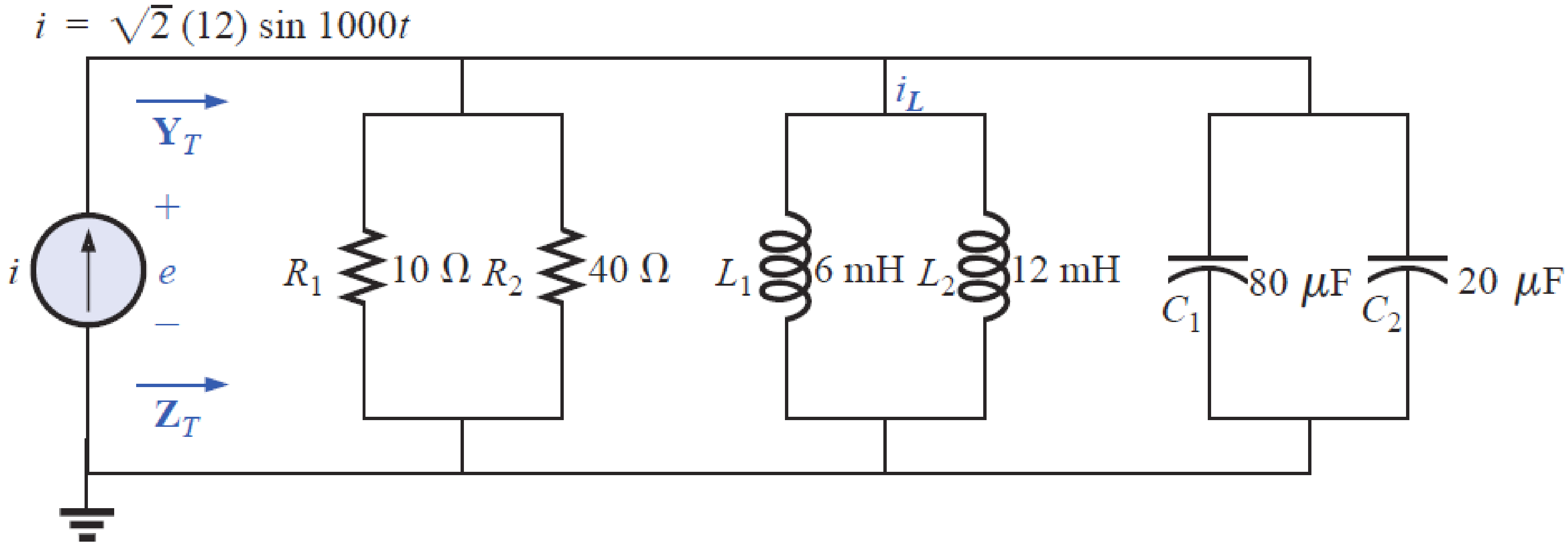
$$\mathbf{Z}_s = R_s \pm j X_s$$

$$\mathbf{Y}_s = \frac{1}{\mathbf{Z}_s} = \frac{1}{R_s \pm j X_s} = \frac{R_s}{R_s^2 + X_s^2} \mp j \frac{X_s}{R_s^2 + X_s^2}$$

$$= G_p \mp j B_p = \frac{1}{R_p} \mp j \frac{1}{X_p}$$

$$R_p = \frac{R_s^2 + X_s^2}{R_s}$$

$$X_p = \frac{R_s^2 + X_s^2}{X_s}$$



$R_T = 10\ \Omega \parallel 40\ \Omega = 8\ \Omega$

$L_T = 6\ \text{mH} \parallel 12\ \text{mH} = 4\ \text{mH}$

$C_T = 80\ \mu\text{F} + 20\ \mu\text{F} = 100\ \mu\text{F}$

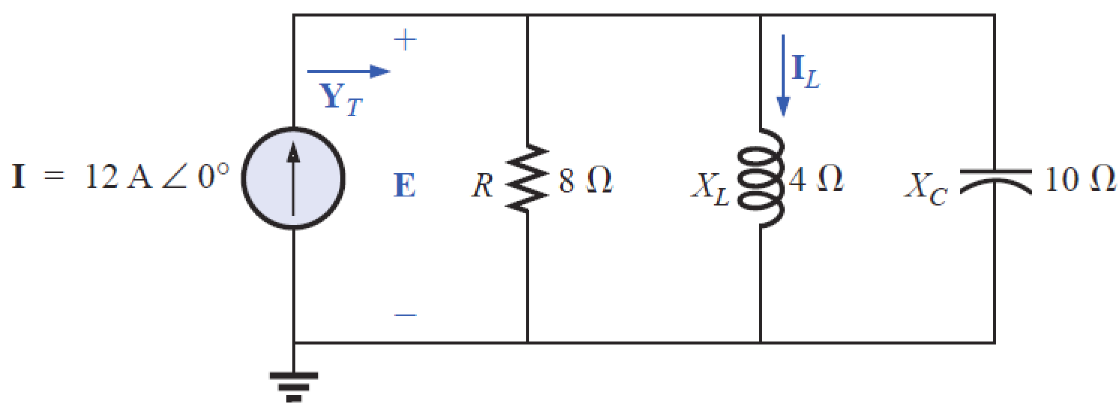


FIG. 15.93

Applying phasor notation to the network of Fig. 15.92.