Methods of Analysis and Selected Topics (ac)

17.1 INTRODUCTION

For networks with two or more sources that are not in series or parallel, methods such as ***mesh analysis*** or ***nodal analysis*** are employed. Only minor variations are required to the method already described for dc circuit.

17.2 INDEPENDENT VERSUS DEPENDENT SOURCES

The term* independent *specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.

dc: E and I, just a value (real number)

ac: $\boldsymbol{E}$ and $\boldsymbol{I}$, phasors (complex number)

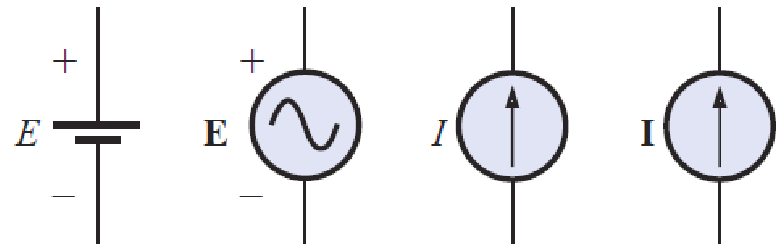


FIG. 17.1
Independent sources.

A* dependent *or* controlled source *is one whose magnitude is* determined *(or controlled) by a* current or voltage *of the system in which it appears.

dc: just magnitude real number

ac: phasors (complex number)

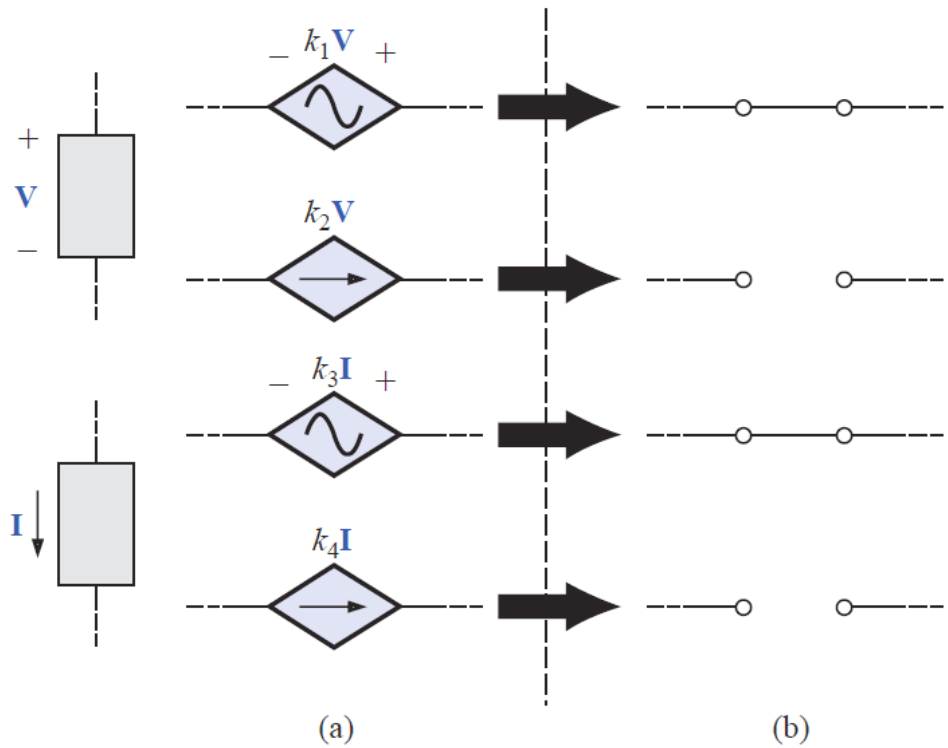
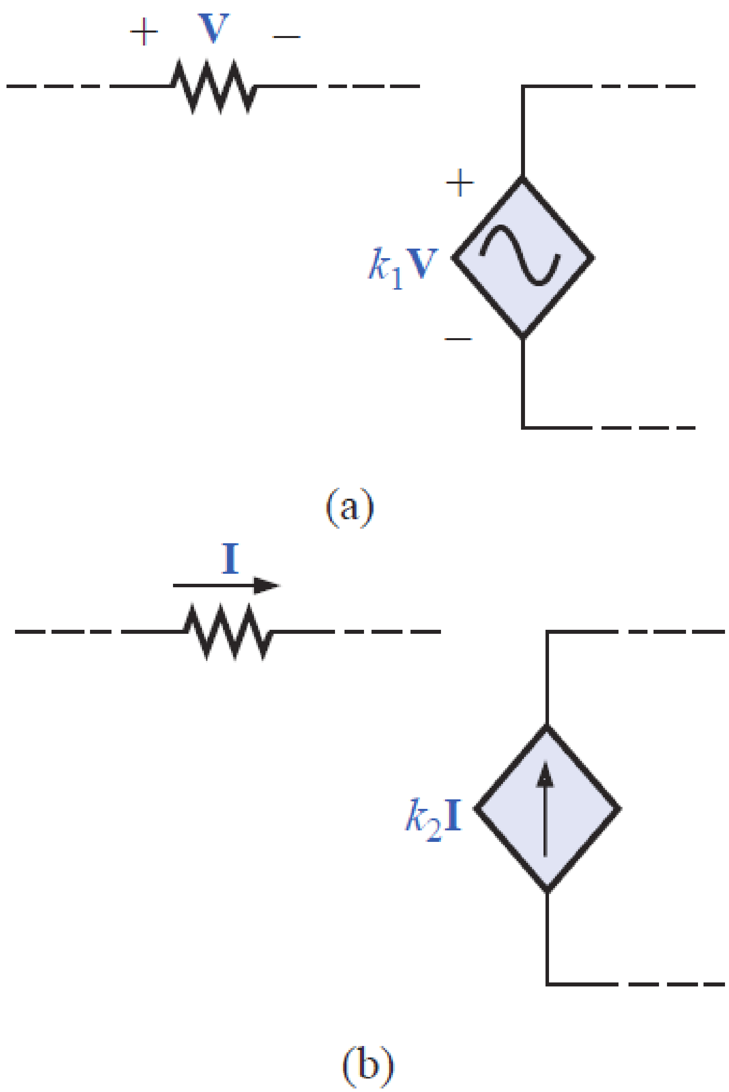


FIG. 17.4

Conditions of V = 0 V and I = 0 A for a controlled source.



Special notation for controlled or dependent sources

17.3 SOURCE CONVERSIONS

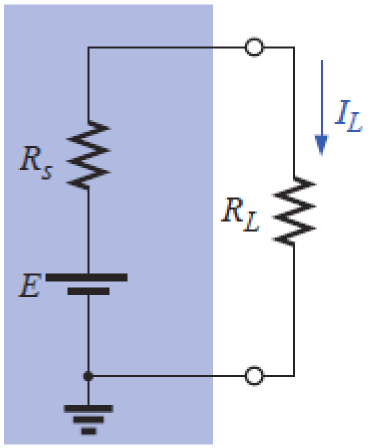


FIG. 8.6
Practical voltage source.

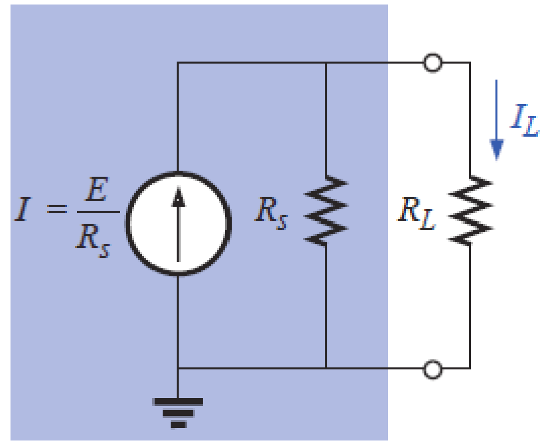
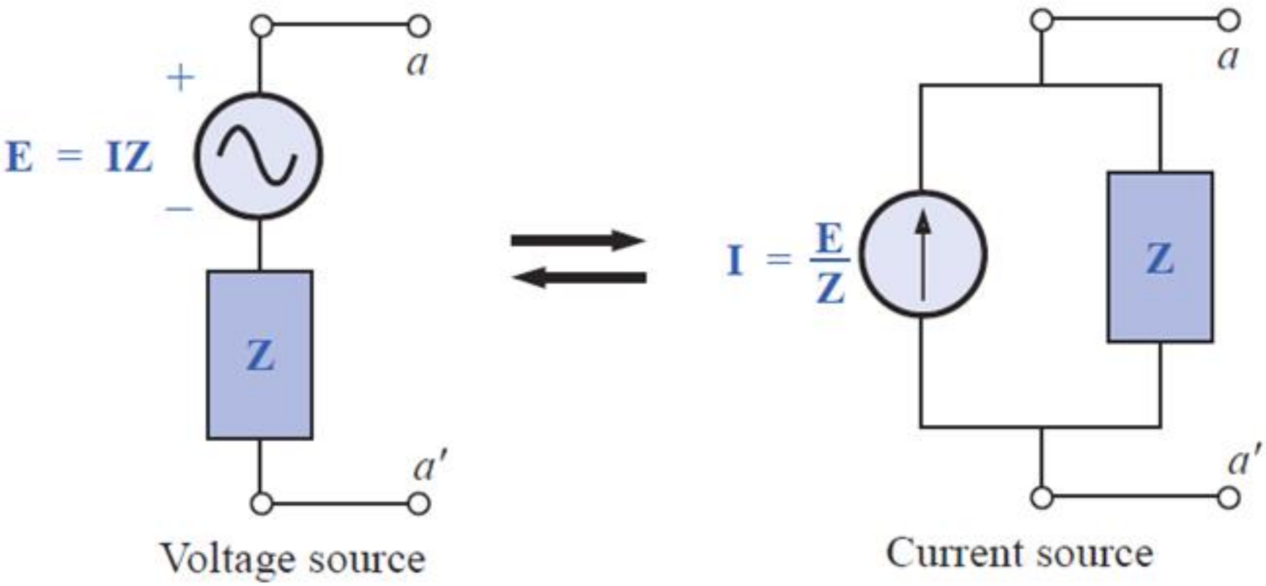


FIG. 8.7
Practical current source.

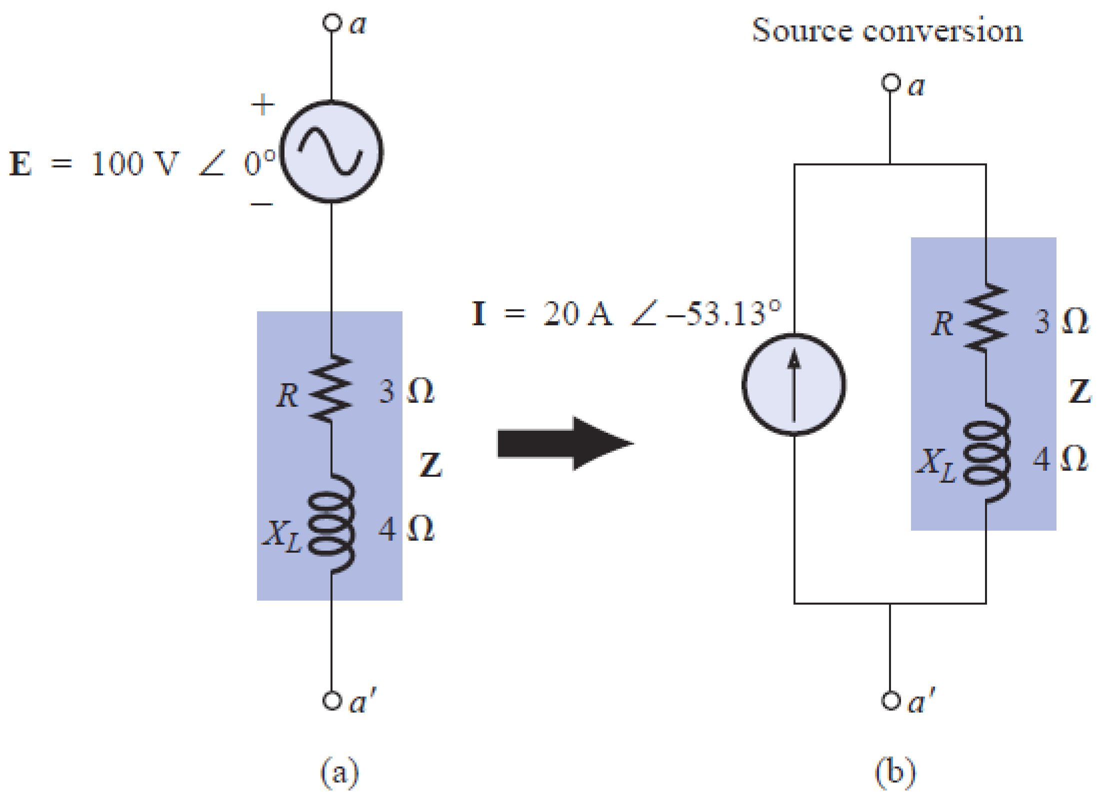


EXAMPLE 17.1 Convert the voltage source of Fig. 17.6(a) to a current source.

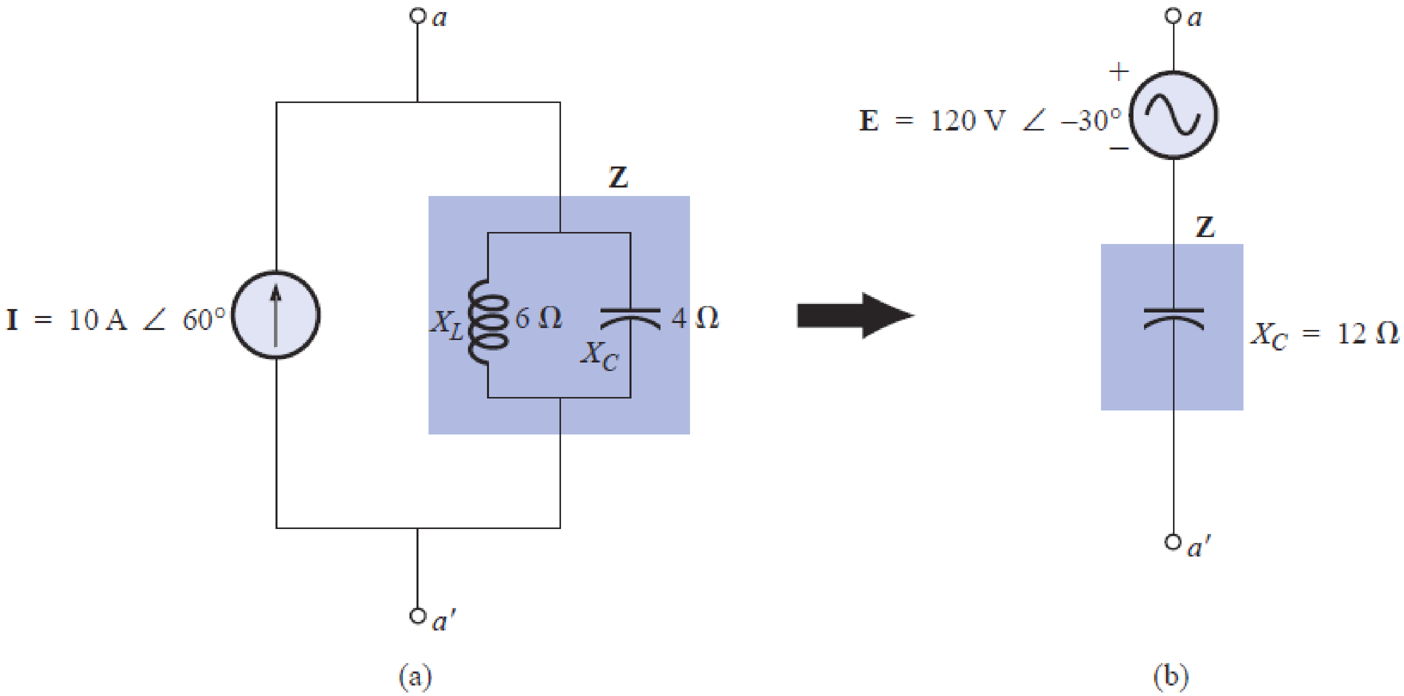
Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100\ \text{V} \angle 0^\circ}{5\ \Omega \angle 53.13^\circ}$$

$$= \mathbf{20\ A \angle -53.13^\circ} \qquad \text{[Fig. 17.6(b)]}$$



EXAMPLE 17.2 Convert the current source of Fig. 17.7(a) to a voltage source.



Solution:

$$\mathbf{Z} = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j\,X_C + j\,X_L}$$

$$= \frac{(4\ \Omega \angle -90°)(6\ \Omega \angle 90°)}{-j\,4\ \Omega + j\,6\ \Omega} = \frac{24\ \Omega \angle 0°}{2 \angle 90°}$$

$$= \mathbf{12\ \Omega \angle -90°} \qquad \text{[Fig. 17.7(b)]}$$

$$\mathbf{E} = \mathbf{IZ} = (10\text{ A} \angle 60°)(12\ \Omega \angle -90°)$$

$$= \mathbf{120\ V \angle -30°} \qquad \text{[Fig. 17.7(b)]}$$

Dependent Source

Case 1: Controlling variable is external to the network to be converted, procedure identical to the one used for independent source.

Case 2: Controlling variable is within the network to be converted, procedure will be seen later.

EXAMPLE 17.3 Convert the voltage source of Fig. 17.8(a) to a current source.

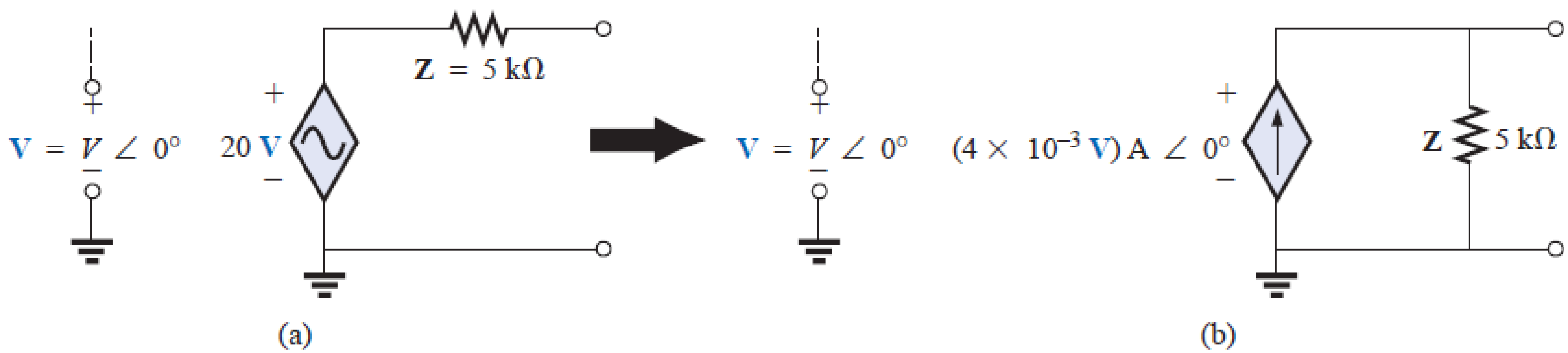


FIG. 17.8
Source conversion with a voltage-controlled voltage source.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{(20V)\ V\ \angle 0^\circ}{5\ \text{k}\Omega\ \angle 0^\circ}$$

$$= (4 \times 10^{-3}\ V)\ A\ \angle 0^\circ \qquad \text{[Fig. 17.8(b)]}$$

EXAMPLE 17.4 Convert the current source of Fig. 17.9(a) to a voltage source.

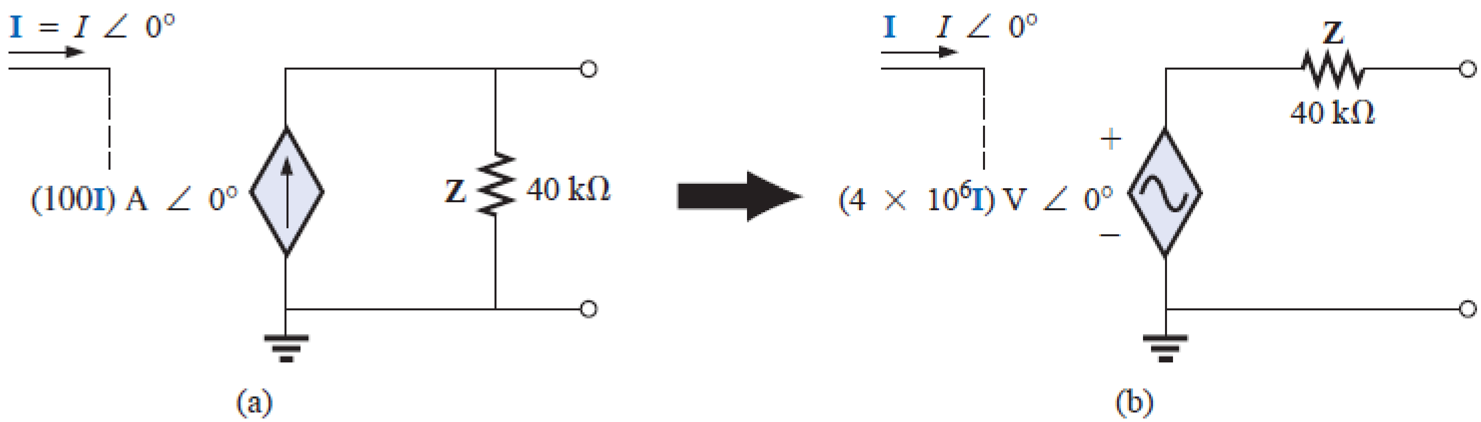


FIG. 17.9
Source conversion with a current-controlled current source.

Solution:

$$\begin{aligned}\mathbf{E} = \mathbf{IZ} &= [(100\mathbf{I})\text{ A } \angle 0°][40\text{ k}\Omega \angle 0°]\\ &= (4 \times 10^6\mathbf{I})\text{ V } \angle 0° \qquad \text{[Fig. 17.9(b)]}\end{aligned}$$

17.4 MESH ANALYSIS (FORMAT APPROACH)

In the formulation used for dc circuit: we replace

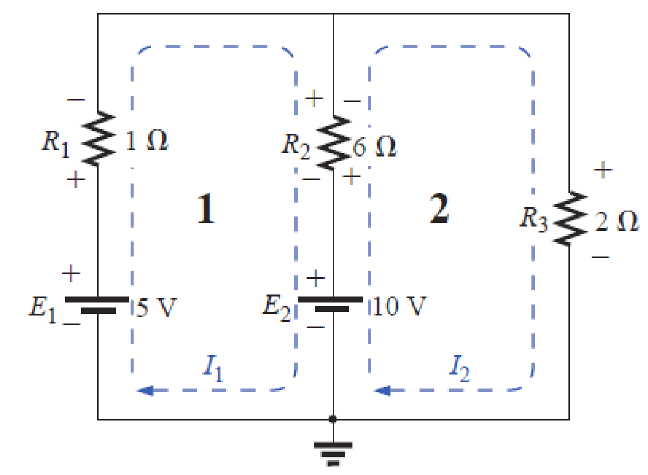
- The resistances by impedances
- The sources value by phasors
- **The equation become complex number equations**

The equations obtained are

$$-7I_1 + 6I_2 = 5 \qquad\qquad 7I_1 - 6I_2 = -5$$
$$6I_1 - 8I_2 = -10 \quad \longrightarrow \quad 8I_2 - 6I_1 = 10$$

and expanded as

Col. 1	Col. 2	Col. 3
$(1 + 6)I_1 -$	$6I_2$	$= (5 - 10)$
$(2 + 6)I_2 -$	$6I_1$	$= 10$



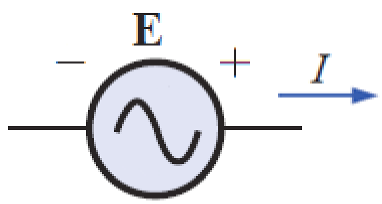
1. ***Assign a loop current to each independent, closed loop in a clockwise direction.***
2. ***The N^o of required equations = N^o of independent, closed loops.***

Column 1 ***of each equation is formed by summing the impedance values of those impedances through which the loop current of interest passes and multiplying the result by that loop current.***

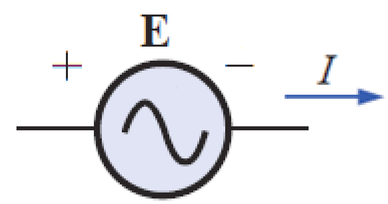
3. *the mutual terms are always subtracted from the first column. A* **mutual term** *is simply any impedance element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each term is the product of the mutual impedance and the other loop current passing through the same element.*

4. *The column to the right of the equality sign is the algebraic sum of the voltage sources in the loop considered.*

“ + ” *for*



“ - ” *for*



5. *Solve the resulting simultaneous equations for the desired loop currents*.

Any current source is first converted to a voltage source (or use the supermesh approach)

EXAMPLE 17.9 Using the format approach to mesh analysis, find the current $\mathbf{I}_2$ in Fig. 17.15.

Solution 1: The network is redrawn in Fig. 17.16:

$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 1\,\Omega + j\,2\,\Omega \qquad \mathbf{E}_1 = 8\text{ V}\angle 20^\circ$$
$$\mathbf{Z}_2 = R_2 - jX_C = 4\,\Omega - j\,8\,\Omega \qquad \mathbf{E}_2 = 10\text{ V}\angle 0^\circ$$
$$\mathbf{Z}_3 = +jX_{L_2} = +j\,6\,\Omega$$

Step 1 is as indicated in Fig. 17.16.

Steps 2 to 4:

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 + \mathbf{E}_2 \\ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 &= -\mathbf{E}_2 \end{aligned}$$

which are rewritten as

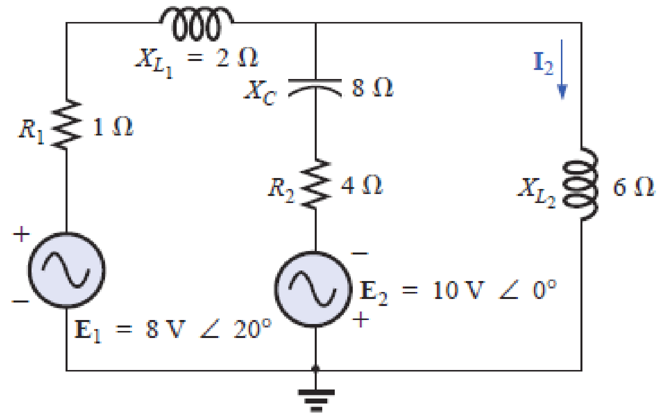
$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

Step 5: Using determinants, we have

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{Z}_2 & -\mathbf{E}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{-(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{E}_2 + \mathbf{Z}_2(\mathbf{E}_1 + \mathbf{E}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2^2}$$

$$= \frac{\mathbf{Z}_2\mathbf{E}_1 - \mathbf{Z}_1\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$



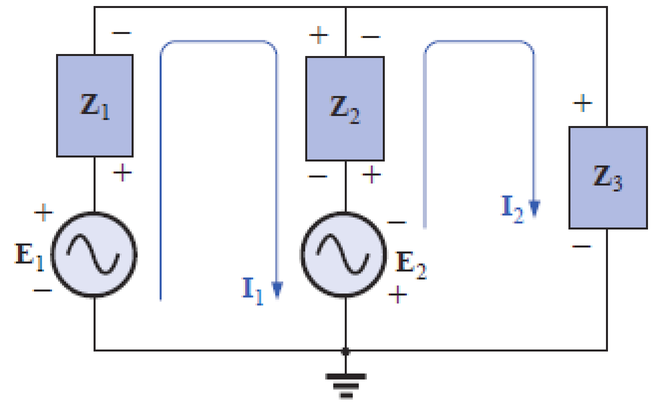


FIG. 17.16
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.15.

Substituting numerical values yields

$$\mathbf{I}_2 = \frac{(4\ \Omega - j\,8\ \Omega)(8\ \text{V} \angle 20^\circ) - (1\ \Omega + j\,2\ \Omega)(10\ \text{V} \angle 0^\circ)}{(1\ \Omega + j\,2\ \Omega)(4\ \Omega - j\,8\ \Omega) + (1\ \Omega + j\,2\ \Omega)(+j\,6\ \Omega) + (4\ \Omega - j\,8\ \Omega)(+j\,6\ \Omega)}$$

$$= \frac{(4 - j\,8)(7.52 + j\,2.74) - (10 + j\,20)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$

$$= \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\ \text{A} \angle -58.74^\circ}{63.53 \angle 28.18^\circ}$$

$$= \mathbf{1.27\ A} \angle \mathbf{-86.92^\circ}$$

Calculators are very helpful in solving these kinds of equations!!!!!!!!!!!!!!

EXAMPLE 17.10 Write the mesh equations for the network of Fig. 17.18. Do not solve.

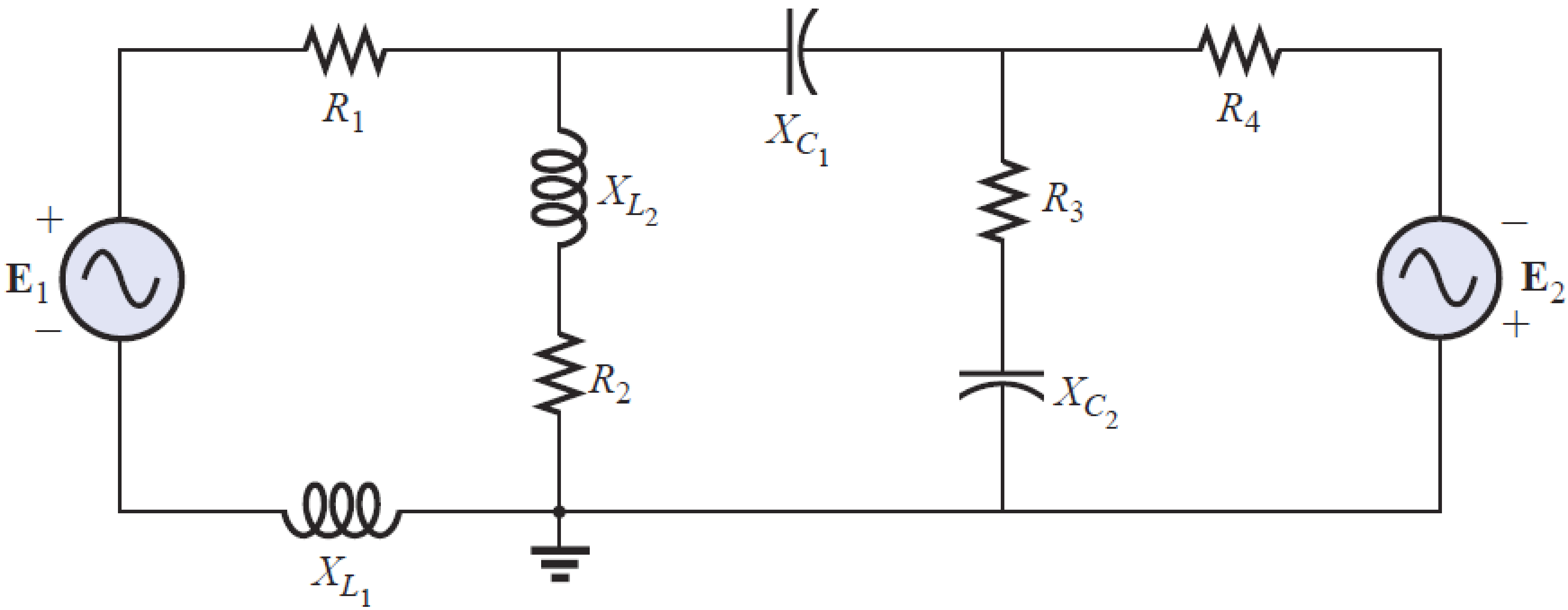


FIG. 17.18
Example 17.10.

Solution: The network is redrawn in Fig. 17.19. Again note the reduced complexity and increased clarity provided by the use of subscripted impedances:

$$\mathbf{Z}_1 = R_1 + jX_{L_1} \qquad \mathbf{Z}_4 = R_3 - jX_{C_2}$$
$$\mathbf{Z}_2 = R_2 + jX_{L_2} \qquad \mathbf{Z}_5 = R_4$$

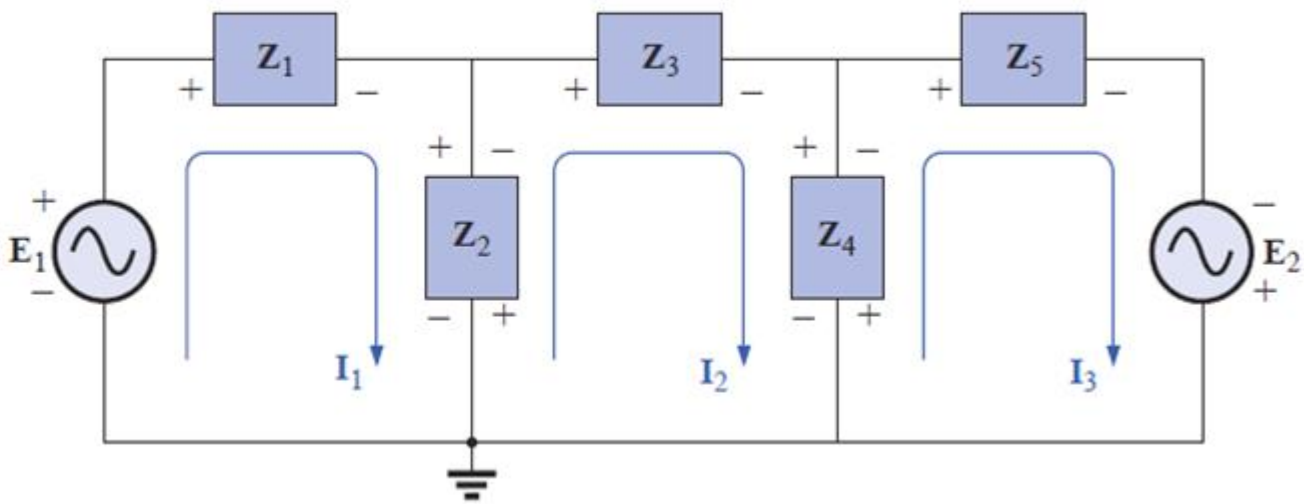
~~$\mathbf{Z}_3 = jX_{C_1}$~~ $\mathbf{Z}_3 = -jX_{C_1}$

and

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 \\ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 &= 0 \\ \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_4 &= \mathbf{E}_2 \end{aligned}$$

or

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2(\mathbf{Z}_2) \qquad\qquad + 0 \qquad &= \mathbf{E}_1 \\ \mathbf{I}_1\mathbf{Z}_2 \qquad - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) + \mathbf{I}_3(\mathbf{Z}_4) &= 0 \\ 0 \qquad - \mathbf{I}_2(\mathbf{Z}_4) \qquad + \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) &= \mathbf{E}_2 \end{aligned}$$



EXAMPLE 17.11 Using the format approach, write the mesh equations for the network of Fig. 17.20.

Solution: The network is redrawn as shown in Fig. 17.21, where

$$\mathbf{Z}_1 = R_1 + jX_{L_1} \qquad \mathbf{Z}_3 = jX_{L_2}$$
$$\mathbf{Z}_2 = R_2 \qquad \mathbf{Z}_4 = jX_{L_3}$$

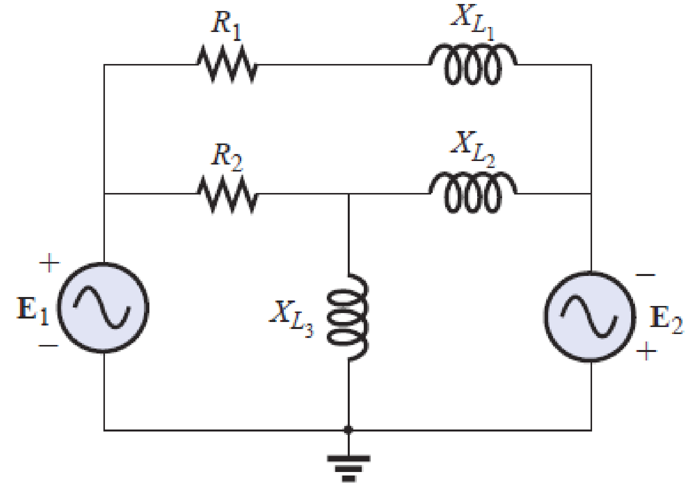
and

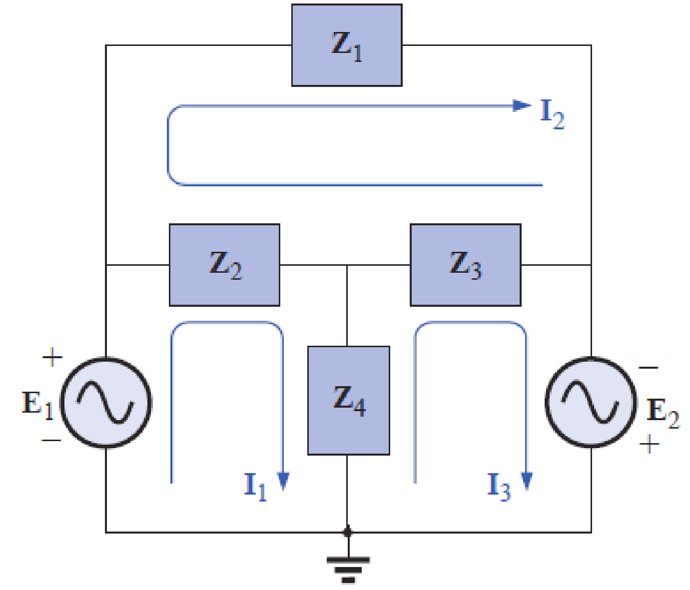
$$\mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_3 = 0$$
$$\mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_1\mathbf{Z}_4 = \mathbf{E}_2$$

or

$$\begin{array}{rrrl} \mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) & - \mathbf{I}_2\mathbf{Z}_2 & - \mathbf{I}_3\mathbf{Z}_4 & = \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 & + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) & - \mathbf{I}_3\mathbf{Z}_3 & = 0 \\ -\mathbf{I}_1\mathbf{Z}_4 & - \mathbf{I}_2\mathbf{Z}_3 & + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) & = \mathbf{E}_2 \end{array}$$

Note the symmetry *about* the diagonal axis; that is, note the location of $-\mathbf{Z}_2$, $-\mathbf{Z}_4$, and $-\mathbf{Z}_3$ off the diagonal.

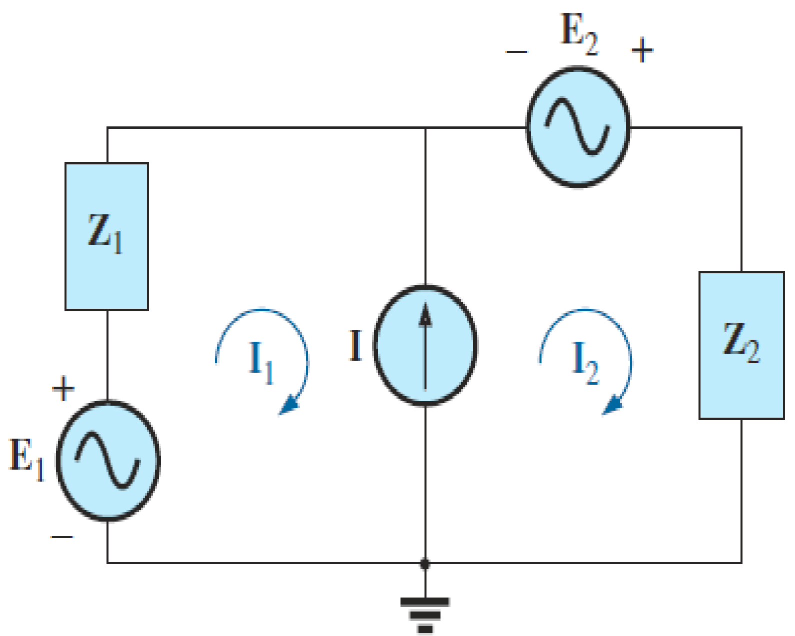




Independent Current Sources For independent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each current source as an open circuit (recall the *supermesh* designation in Chapter 8), and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited to the mesh currents.
3. Step 4 is as before.

EXAMPLE 17.7 Write the mesh currents for the network in Fig. 17.13 having an independent current source.



Solution:

Steps 1 and 2 are defined in Fig. 17.13.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 + \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2 = 0$ (only remaining independent path)

with $\mathbf{I}_1 + \mathbf{I} = \mathbf{I}_2$

The result is two equations and two unknowns.

For Dependent Current Sources

The procedure for mesh analysis is modified as follows:

1. ***Treat dependent source as independent source when applying Kirchhoff's Voltage Law and Kirchhoff's Current Law except substitute their current with the controlled quantity***
2. ***Everything else is the same as before***

EXAMPLE 17.8 Write the mesh currents for the network of Fig. 17.14 having a dependent current source.

Solution:

Steps 1 and 2 are defined on Fig. 17.14.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0$

and $k\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$

Now $\mathbf{I} = \mathbf{I}_1$ so that

$k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$ or $\mathbf{I}_2 = \mathbf{I}_1(1 - k)$

The result is two equations and two unknowns.

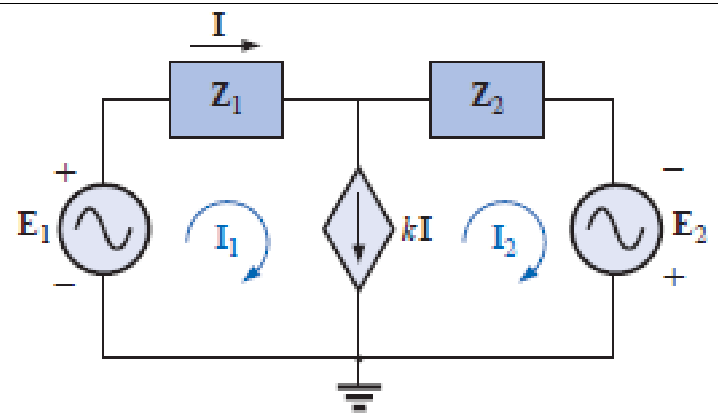


FIG. 17.14
Applying mesh analysis to a network with a current-controlled current source.

For Dependent Voltage Sources

The procedure for mesh analysis is modified as follows:

1. ***Treat dependent source as independent source when applying Kirchhoff's voltage law except substitute their voltage with the controlled quantity***
2. ***Everything else is the same as before***

EXAMPLE 17.6 Write the mesh currents for the network of Fig. 17.12 having a dependent voltage source.

Solution:

Steps 1 and 2 are defined on Fig. 17.12.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$

$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu \mathbf{V}_x - \mathbf{I}_2 R_3 = 0$

Then substitute $\mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$

The result is two equations and two unknowns.

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I} - \mathbf{I}_2) = 0$$
$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2 R_3 = 0$$

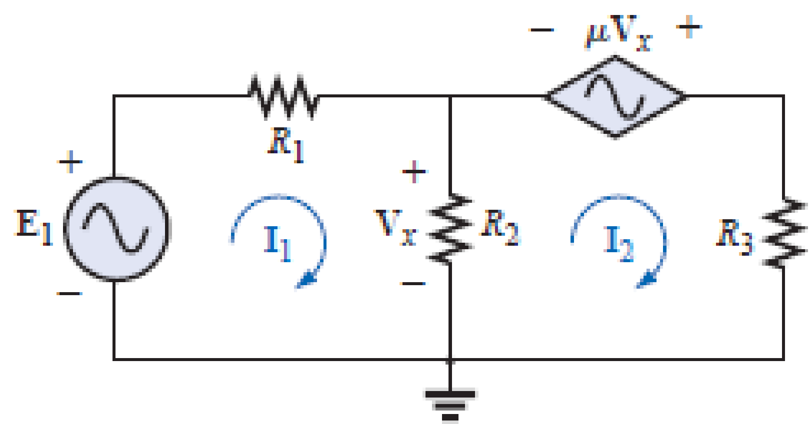


FIG. 17.12
Applying mesh analysis to a network with a voltage-controlled voltage source.

17.5 NODAL ANALYSIS (FORMAT APPROACH)

1. *Choose a reference node and assign a subscripted voltage label to the (N -1) remaining nodes of the network.*

2. *The number of equations required for a complete solution is equal to the number of subscripted voltages (N-1).*

   a. *Column 1 of each equation is formed by summing the admittances tied to the node of interest and multiplying the result by that subscripted nodal voltage.*

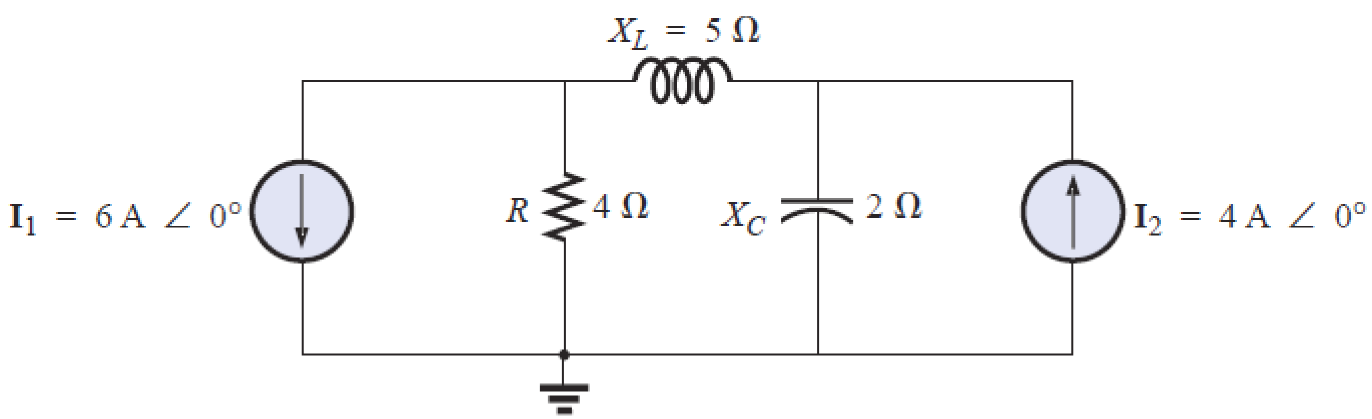
3. *the mutual terms, (tying two nodes), are subtracted from the first column. It is possible to have more than one mutual term. Each mutual term is the product of the mutual admittance and the other nodal voltage tied to that admittance.*

4. *The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.*

5. *Solve the resulting simultaneous equations for the desired voltages.*

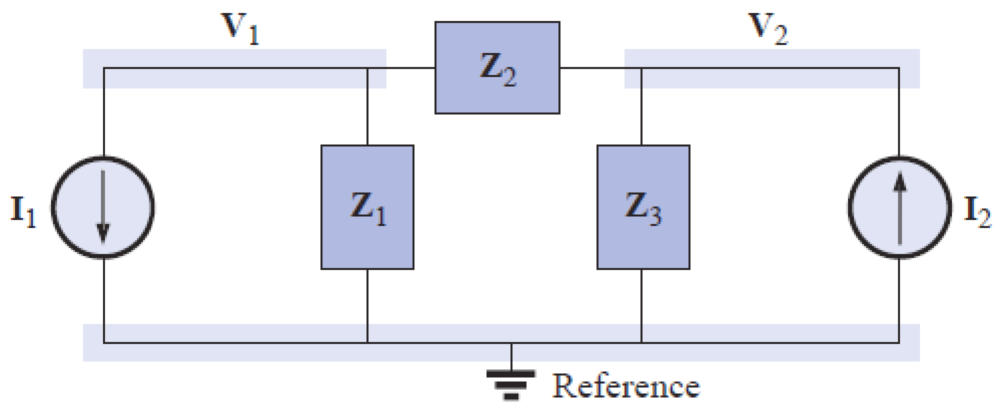
Any Voltage Source is first converted to a Current Source (or use the supernode approach)

EXAMPLE 17.16 Using the format approach to nodal analysis, find the voltage across the 4-Ω resistor in Fig. 17.30.



Solution 1: Choosing nodes (Fig. 17.31) and writing the nodal equations, we have

$$\mathbf{Z}_1 = R = 4\ \Omega \qquad \mathbf{Z}_2 = j X_L = j\,5\ \Omega \qquad \mathbf{Z}_3 = -j X_C = -j\,2\ \Omega$$



$$\begin{aligned} \mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) &= -\mathbf{I}_1 \\ \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{V}_1(\mathbf{Y}_2) &= +\mathbf{I}_2 \end{aligned}$$

or

$$\begin{aligned} \mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) &= -\mathbf{I}_1 \\ -\mathbf{V}_1(\mathbf{Y}_2) + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) &= +\mathbf{I}_2 \end{aligned}$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3}$$

Using determinants yields

$$\mathbf{V}_1 = \frac{\begin{vmatrix} -\mathbf{I}_1 & -\mathbf{Y}_2 \\ +\mathbf{I}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Y}_1 + \mathbf{Y}_2 & -\mathbf{Y}_2 \\ -\mathbf{Y}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}}$$

$$= \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{(\mathbf{Y}_1 + \mathbf{Y}_2)(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{Y}_2^2}$$

$$= \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_2\mathbf{Y}_3 + \mathbf{Y}_1\mathbf{Y}_2}$$

Substituting numerical values, we have

$$\mathbf{V}_1 = \frac{-[(1/-j\,2\ \Omega) + (1/j\,5\ \Omega)]6\ \text{A}\ \angle 0^\circ + 4\ \text{A}\ \angle 0^\circ(1/j\,5\ \Omega)}{(1/4\ \Omega)(1/-j\,2\ \Omega) + (1/j\,5\ \Omega)(1/-j\,2\ \Omega) + (1/4\ \Omega)(1/j\,5\ \Omega)}$$

$$= \frac{-(+j\,0.5 - j\,0.2)6\ \angle 0^\circ + 4\ \angle 0^\circ(-j\,0.2)}{(1/-j\,8) + (1/10) + (1/j\,20)}$$

$$= \frac{(-0.3\ \angle 90^\circ)(6\ \angle 0^\circ) + (4\ \angle 0^\circ)(0.2\ \angle -90^\circ)}{j\,0.125 + 0.1 - j\,0.05}$$

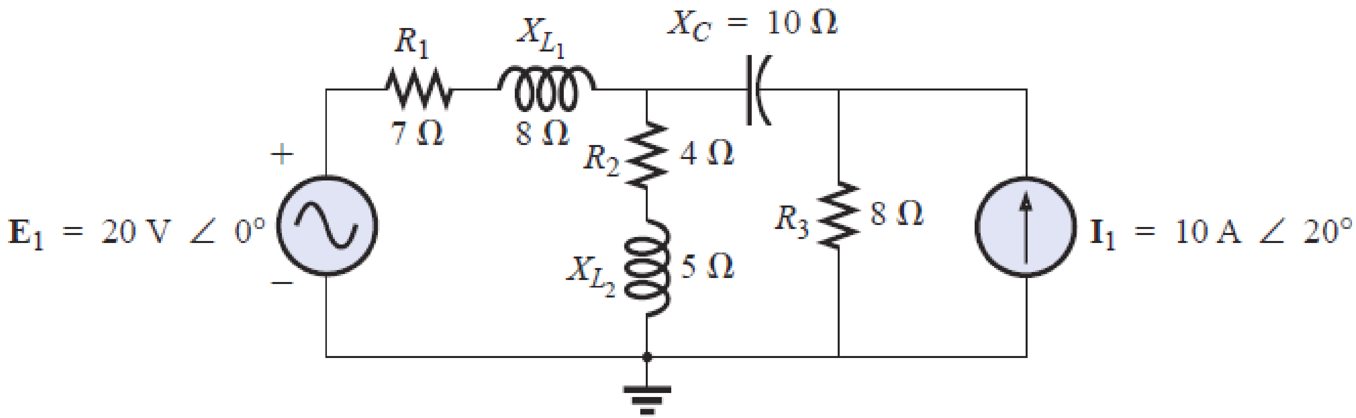
$$= \frac{-1.8\ \angle 90^\circ + 0.8\ \angle -90^\circ}{0.1 + j\,0.075}$$

$$= \frac{2.6\ \text{V}\ \angle -90^\circ}{0.125\ \angle 36.87^\circ}$$

$$\mathbf{V}_1 = \mathbf{20.80\ V\angle -126.87^\circ}$$

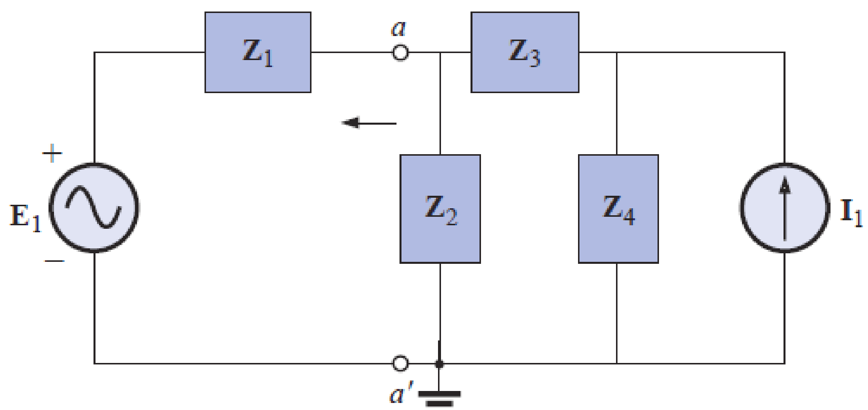
Calculators are very helpful in solving these kinds of equations!!!!!!!!!!!!!!

EXAMPLE 17.17 Using the format approach, write the nodal equations for the network of Fig. 17.33.

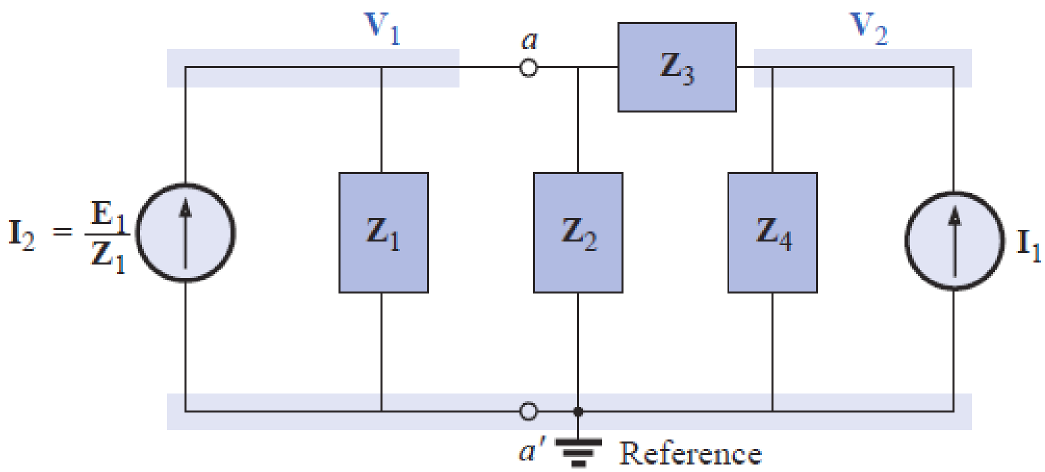


Solution: The circuit is redrawn in Fig. 17.34, where

$$\mathbf{Z}_1 = R_1 + j\,X_{L_1} = 7\ \Omega + j\,8\ \Omega \qquad \mathbf{E}_1 = 20\ \text{V}\ \angle 0°$$
$$\mathbf{Z}_2 = R_2 + j\,X_{L_2} = 4\ \Omega + j\,5\ \Omega \qquad \mathbf{I}_1 = 10\ \text{A}\ \angle 20°$$
$$\mathbf{Z}_3 = -j\,X_C = -j\ 10\ \Omega$$
$$\mathbf{Z}_4 = R_3 = 8\ \Omega$$



Converting the voltage source to a current source and choosing nodes, we obtain Fig. 17.35. Note the "neat" appearance of the network using the subscripted impedances. Working directly with Fig. 17.33 would be more difficult and could produce errors.



Write the nodal equations:

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) = +\mathbf{I}_2$$
$$\mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_3) = +\mathbf{I}_1$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4}$$

which are rewritten as

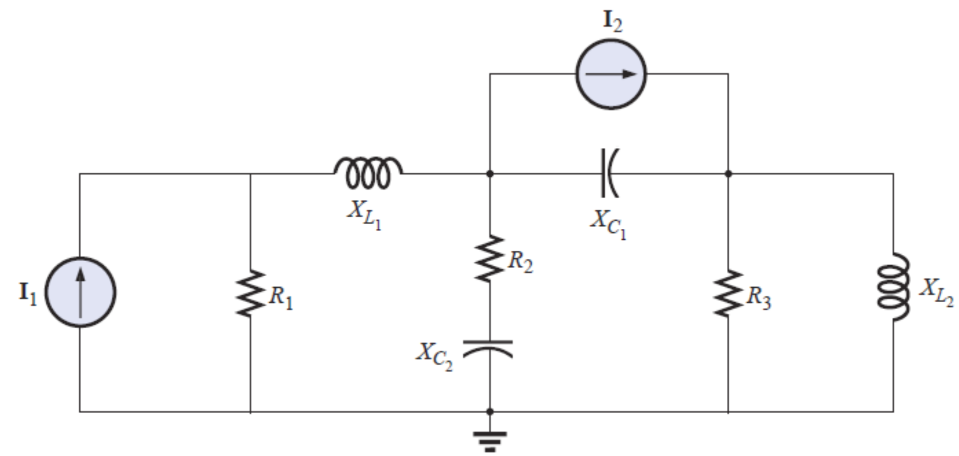
$$\begin{aligned} \mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) \qquad &= +\mathbf{I}_2 \\ -\mathbf{V}_1(\mathbf{Y}_3) \qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) &= +\mathbf{I}_1 \end{aligned}$$

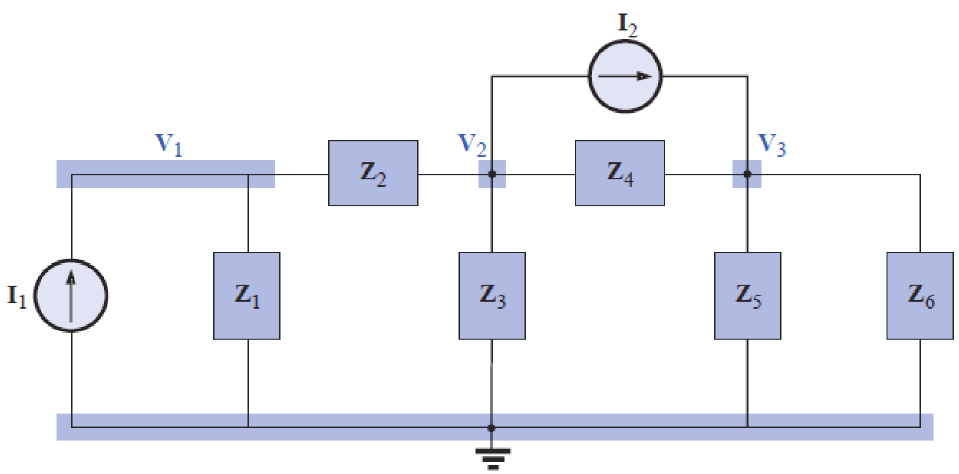
EXAMPLE 17.18 Write the nodal equations for the network of Fig. 17.36. Do not solve.

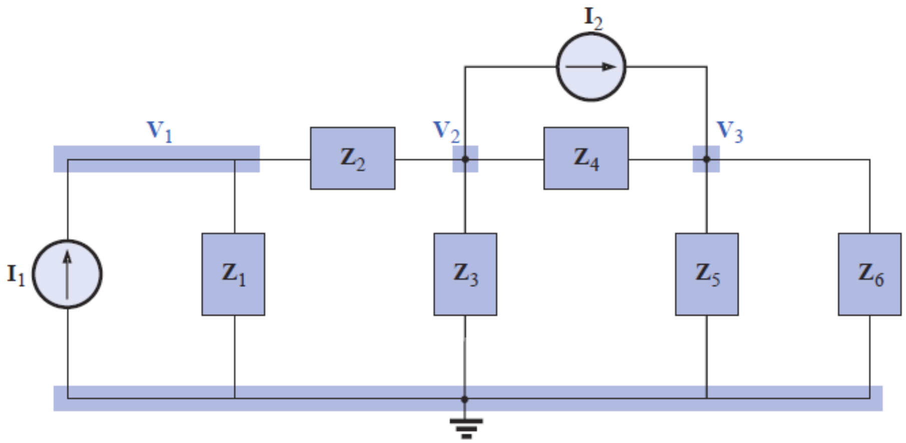
Solution: Choose nodes (Fig. 17.37):

$$\mathbf{Z}_1 = R_1 \qquad \mathbf{Z}_2 = j X_{L_1} \qquad \mathbf{Z}_3 = R_2 - j X_{C_2}$$

$$\mathbf{Z}_4 = -j X_{C_1} \qquad \mathbf{Z}_5 = R_3 \qquad \mathbf{Z}_6 = j X_{L_2}$$







and write the nodal equations:

$$\begin{aligned}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) &= +\mathbf{I}_1 \\
\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_2) - \mathbf{V}_3(\mathbf{Y}_4) &= -\mathbf{I}_2 \\
\mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) - \mathbf{V}_2(\mathbf{Y}_4) &= +\mathbf{I}_2
\end{aligned}$$

which are rewritten as

$$\begin{array}{llll}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) & - \mathbf{V}_2(\mathbf{Y}_2) & + 0 & = +\mathbf{I}_1 \\
-\mathbf{V}_1(\mathbf{Y}_2) & + \mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) & - \mathbf{V}_3(\mathbf{Y}_4) & = -\mathbf{I}_2 \\
0 & - \mathbf{V}_2(\mathbf{Y}_4) & + \mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) & = +\mathbf{I}_2
\end{array}$$

$$\mathbf{Y}_1 = \frac{1}{R_1} \qquad \mathbf{Y}_2 = \frac{1}{j X_{L_1}} \qquad \mathbf{Y}_3 = \frac{1}{R_2 - j X_{C_2}}$$

$$\mathbf{Y}_4 = \frac{1}{-j X_{C_1}} \qquad \mathbf{Y}_5 = \frac{1}{R_3} \qquad \mathbf{Y}_6 = \frac{1}{j X_{L_2}}$$

Calculators are very helpful in solving these kinds of equations!!!!!!!!!!!!!!

Dependent Voltage Sources between Defined Nodes

The procedure for nodal analysis is modified as follows:

1. ***Treat dependent source as independent source when applying Kirchhoff's Current Law except substitute their voltage with the controlled quantity***
2. ***Everything else is the same as before***

EXAMPLE 17.15 Write the nodal equations for the network of Fig. 17.29 having a dependent voltage source between two defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.29.

Step 3: Replacing the dependent source $\mu\mathbf{V}_x$ with a short-circuit equivalent will result in the following equation when Kirchhoff's current law is applied at node $\mathbf{V}_1$:

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and
$$\mathbf{V}_2 = \mu\mathbf{V}_x = \mu[\mathbf{V}_1 - \mathbf{V}_2]$$

or
$$\mathbf{V}_2 = \frac{\mu}{1+\mu}\mathbf{V}_1$$

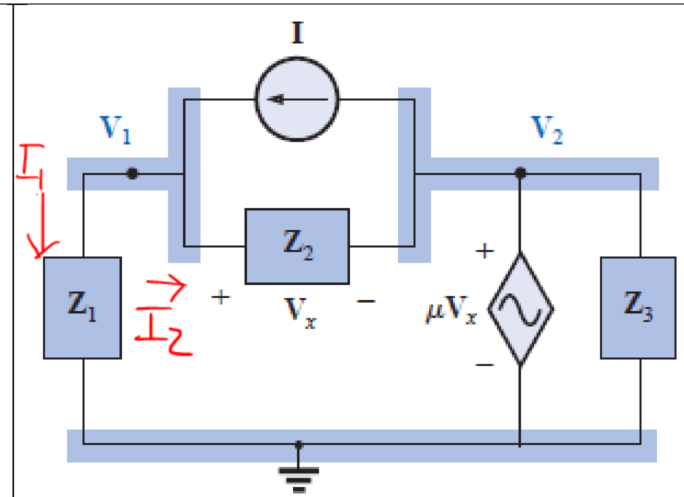


FIG. 17.29
Applying nodal analysis to a network with a voltage-controlled voltage source.

Dependent Current Sources

The procedure for nodal analysis is modified as follows:

1. ***Treat dependent source as independent source when applying Kirchhoff's Current Law except substitute their current with the controlled quantity***
2. ***Everything else is the same as before***

EXAMPLE 17.13 Write the nodal equations for the network of Fig. 17.27 having a dependent current source.

Solution:

Steps 1 and 2 are as defined in Fig. 17.27.

Step 3: At node $\mathbf{V}_1$,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}$$

At node $\mathbf{V}_2$,

$$\mathbf{I}_2 + \mathbf{I}_3 + k\mathbf{I}' = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\left[\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}\right] = 0$$

and
$$\mathbf{V}_1\left[\frac{1 - k}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1 - k}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] = 0$$

resulting in two equations and two unknowns.

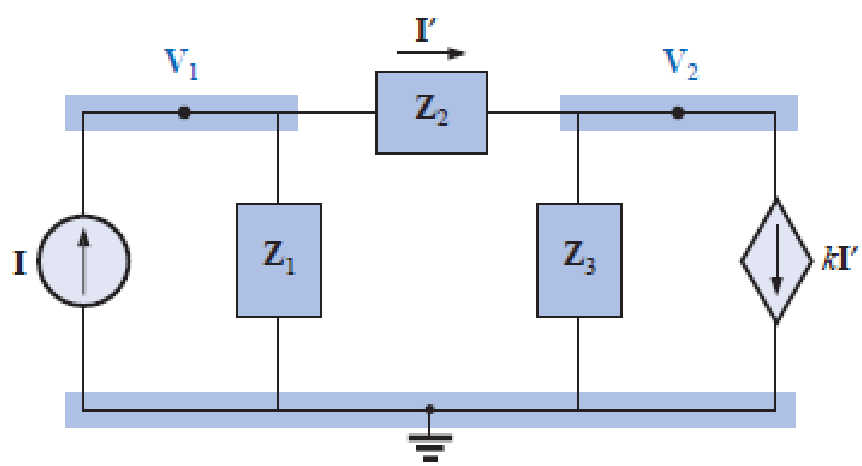


FIG. 17.27

EXAMPLE 17.19 Apply nodal analysis to the network of Fig. 17.38. Determine the voltage $\mathbf{V}_L$.

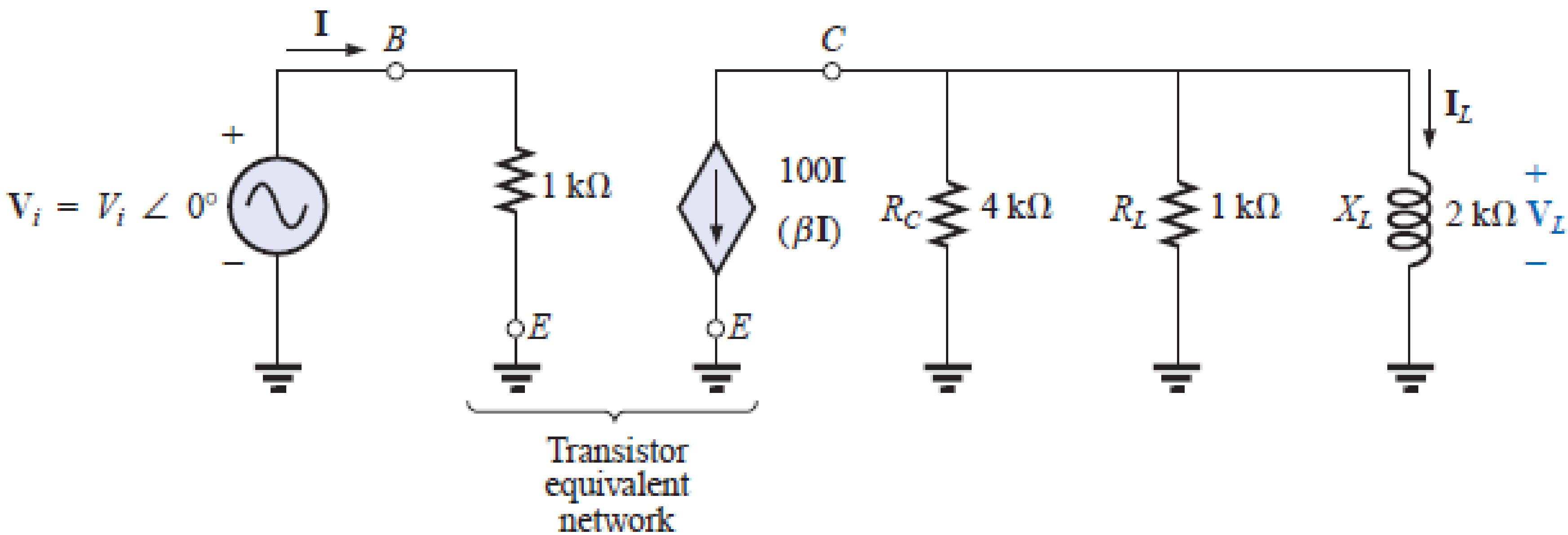


FIG. 17.38
Example 17.19.

Solution: In this case there is no need for a source conversion. The network is redrawn in Fig. 17.39 with the chosen nodal voltage and subscripted impedances.

Apply the format approach:

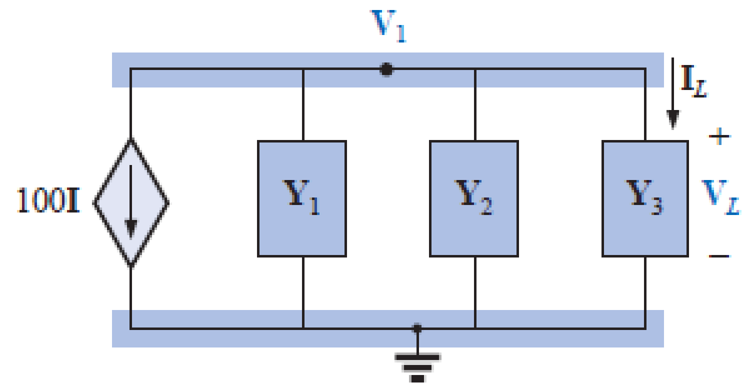


FIG. 17.39
Assigning the nodal voltage and subscripted impedances for the network of Fig. 17.38.

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{4\text{ k}\Omega} = 0.25\text{ mS}\angle 0° = G_1\angle 0°$$

$$\mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{1\text{ k}\Omega} = 1\text{ mS}\angle 0° = G_2\angle 0°$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{2\text{ k}\Omega\angle 90°} = 0.5\text{ mS}\angle -90°$$

$$= -j\,0.5\text{ mS} = -j\,B_L$$

$$\mathbf{V}_1:\quad (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3)\mathbf{V}_1 = -100\mathbf{I}$$

and
$$\mathbf{V}_1 = \frac{-100\mathbf{I}}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3}$$

$$= \frac{-100\mathbf{I}}{0.25\text{ mS} + 1\text{ mS} - j\,0.5\text{ mS}}$$

$$= \frac{-100\times 10^3\mathbf{I}}{1.25 - j\,0.5} = \frac{-100\times 10^3\mathbf{I}}{1.3463\angle -21.80°}$$

$$= -74.28\times 10^3\mathbf{I}\angle 21.80°$$

$$= -74.28\times 10^3\left(\frac{\mathbf{V}_i}{1\text{ k}\Omega}\right)\angle 21.80°$$

$$\mathbf{V}_1 = \mathbf{V}_L = \mathbf{-(74.28V}_i\mathbf{)\ V\angle 21.80°}$$

17.6 BRIDGE NETWORKS (ac)

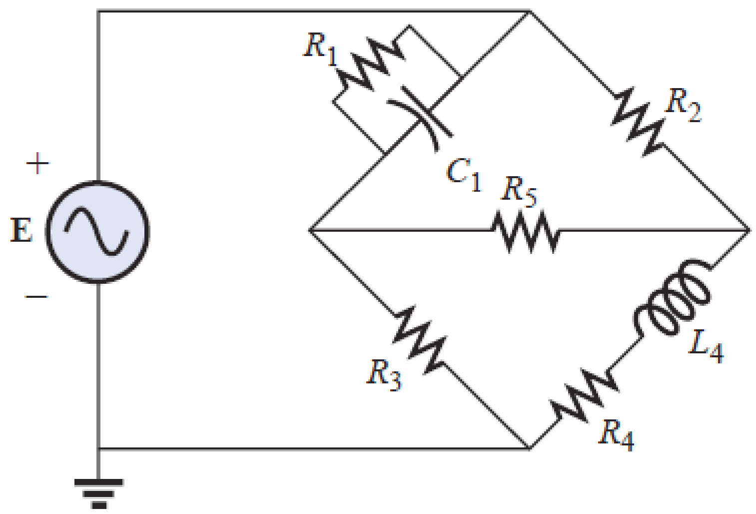


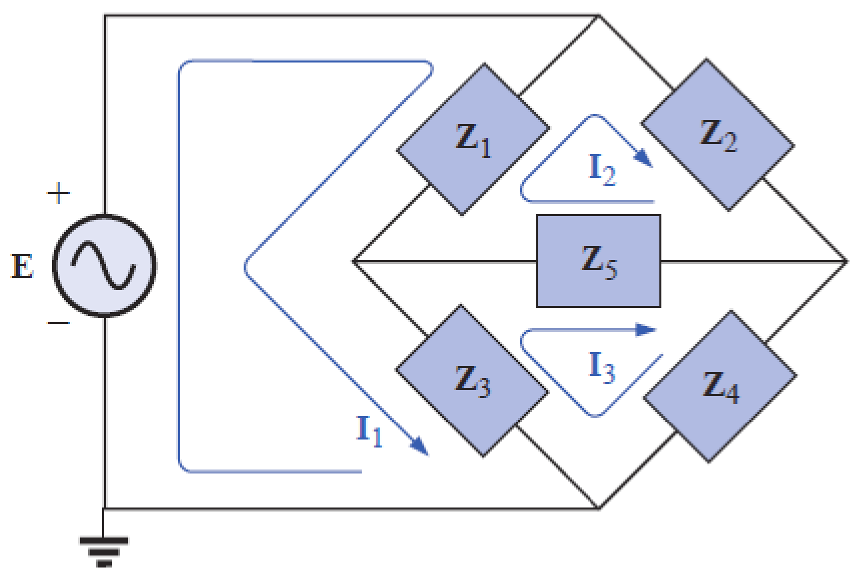
FIG. 17.40
Maxwell bridge.

MESH ANALYSIS

Apply **mesh analysis** to the network of Fig. 17.40. The network is redrawn in Fig. 17.41, where

$$\mathbf{Z}_1 = \frac{1}{\mathbf{Y}_1} = \frac{1}{G_1 + j B_C} = \frac{G_1}{G_1^2 + B_C^2} - j \frac{B_C}{G_1^2 + B_C^2}$$

$$\mathbf{Z}_2 = R_2 \qquad \mathbf{Z}_3 = R_3 \qquad \mathbf{Z}_4 = R_4 + j X_L \qquad \mathbf{Z}_5 = R_5$$



Applying the format approach:

$$\begin{aligned}
(\mathbf{Z}_1 + \mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_1)\mathbf{I}_2 - (\mathbf{Z}_3)\mathbf{I}_3 &= \mathbf{E}\\
(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)\mathbf{I}_2 - (\mathbf{Z}_1)\mathbf{I}_1 - (\mathbf{Z}_5)\mathbf{I}_3 &= 0\\
(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5)\mathbf{I}_3 - (\mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_5)\mathbf{I}_2 &= 0
\end{aligned}$$

which are rewritten as

$$\begin{aligned}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) - \mathbf{I}_2\mathbf{Z}_1 \qquad\qquad - \mathbf{I}_3\mathbf{Z}_3 &= \mathbf{E}\\
-\mathbf{I}_1\mathbf{Z}_1 + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) - \mathbf{I}_3\mathbf{Z}_5 &= 0\\
-\mathbf{I}_1\mathbf{Z}_3 \qquad - \mathbf{I}_2\mathbf{Z}_5 + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) &= 0
\end{aligned}$$

Note the symmetry about the diagonal of the above equations. For balance, $\mathbf{I}_{\mathbf{Z}_5} = 0$ A, and

$$\mathbf{I}_{\mathbf{Z}_5} = \mathbf{I}_2 - \mathbf{I}_3 = 0$$

From the above equations,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & \mathbf{E} & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & 0 & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & 0 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & -\mathbf{Z}_1 & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & -\mathbf{Z}_5 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}$$

$$= \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_4 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\boldsymbol{\Delta}}$$

where Δ signifies the determinant of the denominator (or coefficients). Similarly,

$$\mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

and

$$\mathbf{I}_{\mathbf{Z}_5} = \mathbf{I}_2 - \mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_4 - \mathbf{Z}_3\mathbf{Z}_2)}{\Delta}$$

For $\mathbf{I}_{\mathbf{Z}_5} = 0$, the following must be satisfied (for a finite Δ not equal to zero):

$$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{I}_{\mathbf{Z}_5} = 0 \tag{17.3}$$

NODAL ANALYSIS

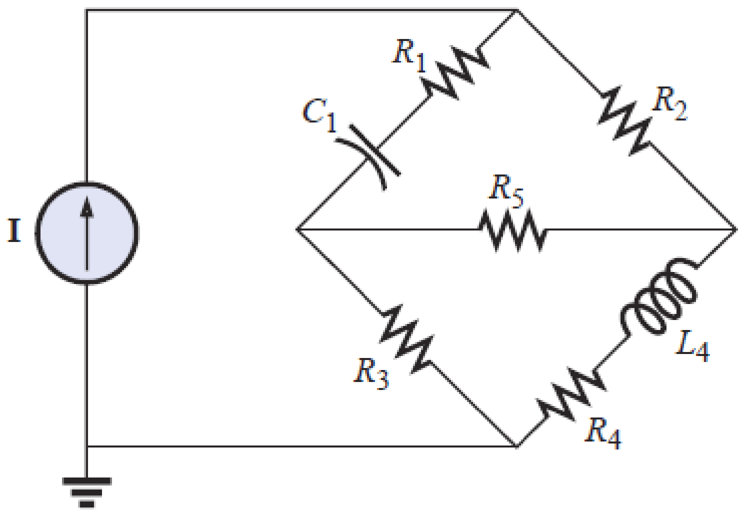


FIG. 17.42
Hay bridge.

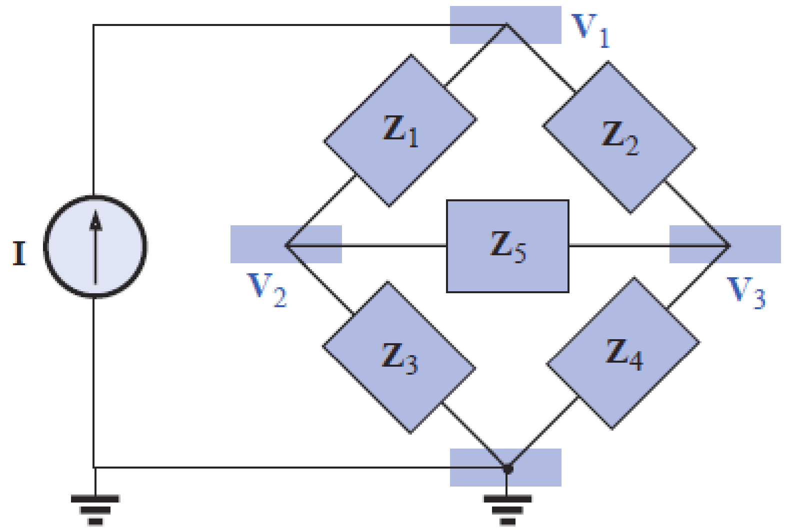


FIG. 17.43
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.42.

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{R_1 - j\,X_C} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{R_2}$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{R_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4} = \frac{1}{R_4 + j\,X_L} \qquad \mathbf{Y}_5 = \frac{1}{R_5}$$

and

$$(\mathbf{Y}_1 + \mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_1)\mathbf{V}_2 - (\mathbf{Y}_2)\mathbf{V}_3 = \mathbf{I}$$
$$(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)\mathbf{V}_2 - (\mathbf{Y}_1)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_3 = 0$$
$$(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\mathbf{V}_3 - (\mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_2 = 0$$

which are rewritten as

$$\begin{array}{lllll} \mathbf{V}_1(\mathbf{Y}_1+\mathbf{Y}_2) & -\mathbf{V}_2\mathbf{Y}_1 & -\mathbf{V}_3\mathbf{Y}_2 & = \mathbf{I} \\ -\mathbf{V}_1\mathbf{Y}_1 & +\mathbf{V}_2(\mathbf{Y}_1+\mathbf{Y}_3+\mathbf{Y}_5) & -\mathbf{V}_3\mathbf{Y}_5 & = 0 \\ -\mathbf{V}_1\mathbf{Y}_2 & -\mathbf{V}_2\mathbf{Y}_5 & +\mathbf{V}_3(\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5) & = 0 \end{array}$$

Again, note the symmetry about the diagonal axis. For balance, $\mathbf{V}_{\mathbf{Z}_5} = 0$ V, and

$$\mathbf{V}_{\mathbf{Z}_5} = \mathbf{V}_2 - \mathbf{V}_3 = 0$$

From the above equations,

$$\mathbf{V}_2 = \frac{\begin{vmatrix} \mathbf{Y}_1+\mathbf{Y}_2 & \mathbf{I} & -\mathbf{Y}_2 \\ -\mathbf{Y}_1 & 0 & -\mathbf{Y}_5 \\ -\mathbf{Y}_2 & 0 & (\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5) \end{vmatrix}}{\begin{vmatrix} \mathbf{Y}_1+\mathbf{Y}_2 & -\mathbf{Y}_1 & -\mathbf{Y}_2 \\ -\mathbf{Y}_1 & (\mathbf{Y}_1+\mathbf{Y}_3+\mathbf{Y}_5) & -\mathbf{Y}_5 \\ -\mathbf{Y}_2 & -\mathbf{Y}_5 & (\mathbf{Y}_2+\mathbf{Y}_4+\mathbf{Y}_5 \end{vmatrix}}$$

$$= \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_3+\mathbf{Y}_1\mathbf{Y}_4+\mathbf{Y}_1\mathbf{Y}_5+\mathbf{Y}_3\mathbf{Y}_5)}{\Delta}$$

Similarly,

$$\mathbf{V}_3 = \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_3+\mathbf{Y}_3\mathbf{Y}_2+\mathbf{Y}_1\mathbf{Y}_5+\mathbf{Y}_3\mathbf{Y}_5)}{\Delta}$$

Note the similarities between the above equations and those obtained for mesh analysis. Then

$$\mathbf{V_{Z_5}} = \mathbf{V}_2 - \mathbf{V}_3 = \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_4 - \mathbf{Y}_3\mathbf{Y}_2)}{\Delta}$$

For $\mathbf{V_{Z_5}} = 0$, the following must be satisfied for a finite Δ not equal to zero:

$$\boxed{\mathbf{Y}_1\mathbf{Y}_4 = \mathbf{Y}_3\mathbf{Y}_2} \qquad \mathbf{V_{Z_5}} = 0 \qquad \textbf{(17.4)}$$

However, substituting $\mathbf{Y}_1 = 1/\mathbf{Z}_1$, $\mathbf{Y}_2 = 1/\mathbf{Z}_2$, $\mathbf{Y}_3 = 1/\mathbf{Z}_3$, and $\mathbf{Y}_4 = 1/\mathbf{Z}_4$, we have

$$\frac{1}{\mathbf{Z}_1\mathbf{Z}_4} = \frac{1}{\mathbf{Z}_3\mathbf{Z}_2}$$

or

$$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{V_{Z_5}} = 0$$

BALANCE CRITERIA

$\boldsymbol{I} = 0$ and $\boldsymbol{V} = 0$

Since $\mathbf{I} = 0$,

$$\mathbf{I}_1 = \mathbf{I}_3 \quad \text{and} \quad \mathbf{I}_2 = \mathbf{I}_4$$

In addition, for $\mathbf{V} = 0$,

$$\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2 \quad \text{and} \quad \mathbf{I}_3\mathbf{Z}_3 = \mathbf{I}_4\mathbf{Z}_4$$

Thus,

$$\mathbf{I}_1\mathbf{Z}_3 = \mathbf{I}_2\mathbf{Z}_4 \quad \text{and} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1$$

$$\mathbf{I}_1\mathbf{Z}_1 = \left(\frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1\right)\mathbf{Z}_2 \quad \text{and} \quad \mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$$

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4} \qquad\qquad \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

ac bridge ***dc bridge***

Identical to the case for dc bridge:

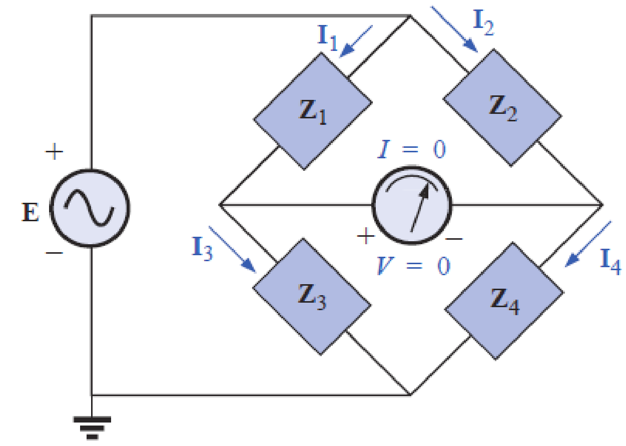


FIG. 17.44
Investigating the balance criteria for an ac bridge configuration.

17.7 Δ-Y, Y-Δ CONVERSIONS

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

Δ - Y Conversion

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2}$$

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3}$$

Y - Δ Conversion

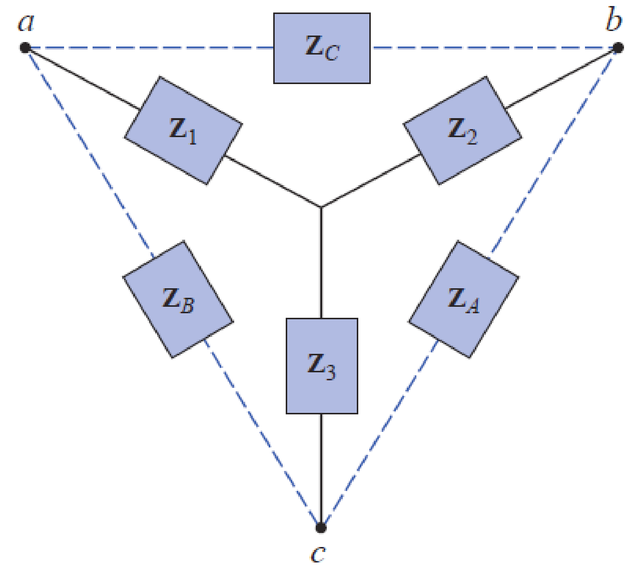


FIG. 17.46
Δ-Y configuration.

For equal impedances

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

Identical to the case of dc circuits