Network Theorems (ac)

18.1 INTRODUCTION

The theorems studied earlier: Superposition theorem, Thevenin's Theorem, Norton's Theorem, and Maximum Power Transfer theorem have a very similar (almost identical) replica for ac circuit with the only change from just numbers and resistances to phasors and impedances.

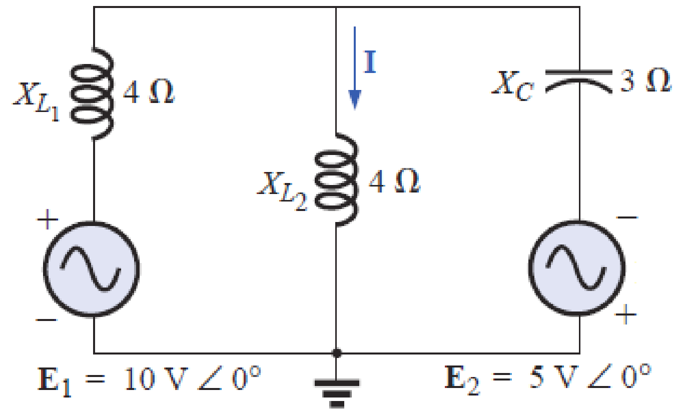
18.2 SUPERPOSITION THEOREM

The only variation in applying this theorem to ac networks with independent sources is that we will now be working with impedances and phasors instead of just resistors and real numbers.

EXAMPLE 18.1 Using the superposition theorem, find the current **I** through the 4-Ω reactance (X_{L_2}) of Fig. 18.1.

Solution: For the redrawn circuit (Fig. 18.2),

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\ \Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\ \Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\ \Omega$$



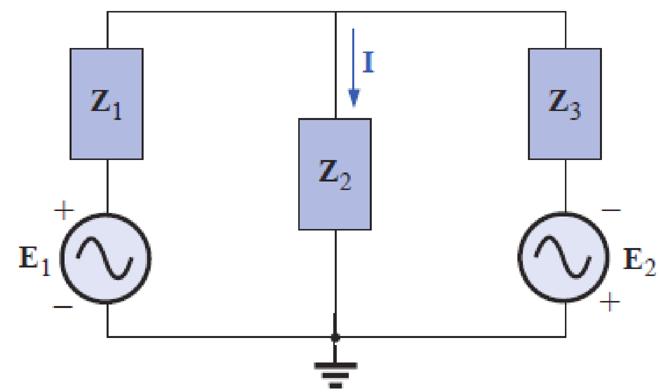


FIG. 18.2
Assigning the subscripted impedances to the network of Fig. 18.1.

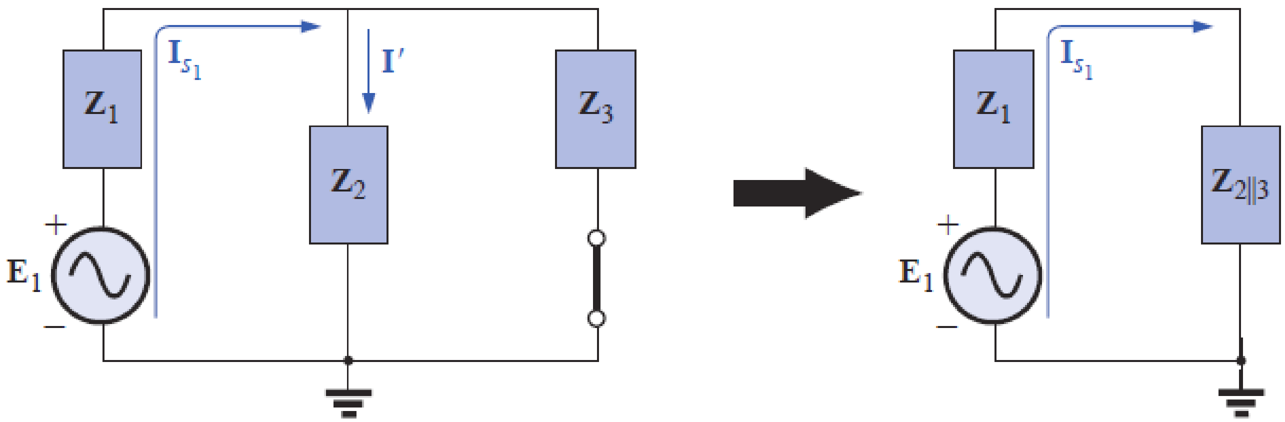


FIG. 18.3

Determining the effect of the voltage source $\mathbf{E}_1$ *on the current* **I** *of the network of Fig. 18.1.*

Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 18.3), we have

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega = 12\,\Omega\,\angle -90^\circ$$

$$I_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\,\text{V}\,\angle 0^\circ}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\,\text{V}\,\angle 0^\circ}{8\,\Omega\,\angle -90^\circ} = 1.25\,\text{A}\,\angle 90^\circ$$

and

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$

$$= \frac{(-j\,3\,\Omega\,)(j\,1.25\,\text{A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\,\text{A}}{j\,1} = 3.75\,\text{A}\,\angle -90^\circ$$

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 18.4), we have

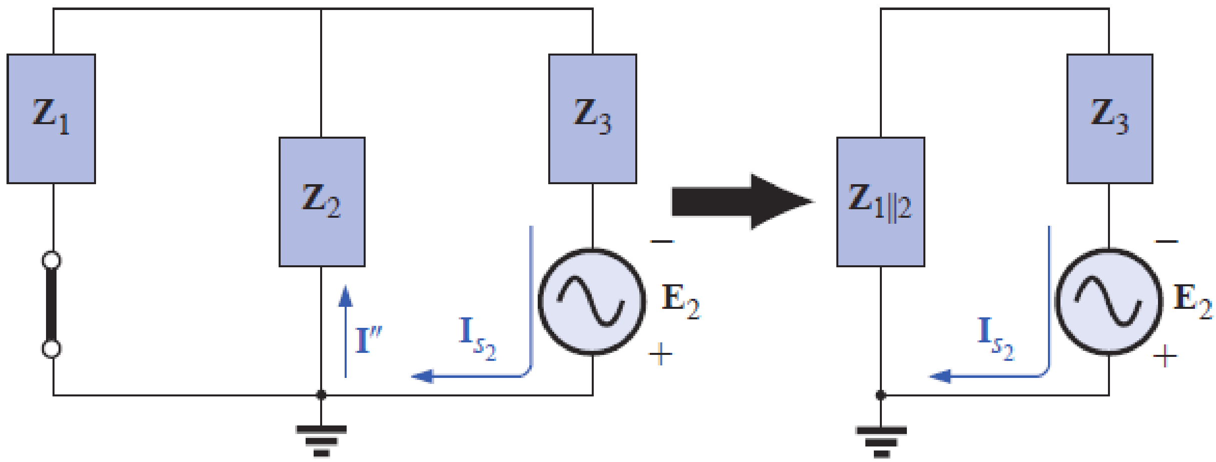


FIG. 18.4

Determining the effect of the voltage source $\mathbf{E}_2$ *on the current* **I** *of the network of Fig. 18.1.*

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\ \Omega}{2} = j\,2\ \Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\ \text{V}\ \angle 0^\circ}{j\,2\ \Omega - j\,3\ \Omega} = \frac{5\ \text{V}\ \angle 0^\circ}{1\ \Omega\ \angle -90^\circ} = 5\ \text{A}\ \angle 90^\circ$$

and
$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\ \text{A}\ \angle 90^\circ$$

The resultant current through the 4-Ω reactance X_{L_2} (Fig. 18.5) is

$$\mathbf{I} = \mathbf{I}' - \mathbf{I}''$$
$$= 3.75\ \text{A} \angle -90° - 2.50\ \text{A} \angle 90° = -j\,3.75\ \text{A} - j\,2.50\ \text{A}$$
$$= -j\,6.25\ \text{A}$$
$$\mathbf{I} = \mathbf{6.25\ A \angle -90°}$$

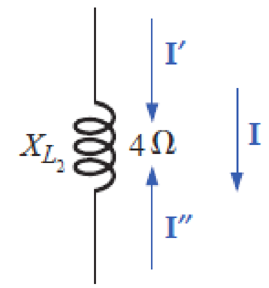


FIG. 18.5
Determining the resultant current for the network of Fig. 18.1.

EXAMPLE 18.2 Using superposition, find the current **I** through the 6-Ω resistor of Fig. 18.6.

Solution: For the redrawn circuit (Fig. 18.7),

$$\mathbf{Z}_1 = j\,6\ \Omega \qquad \mathbf{Z}_2 = 6 - j\,8\ \Omega$$

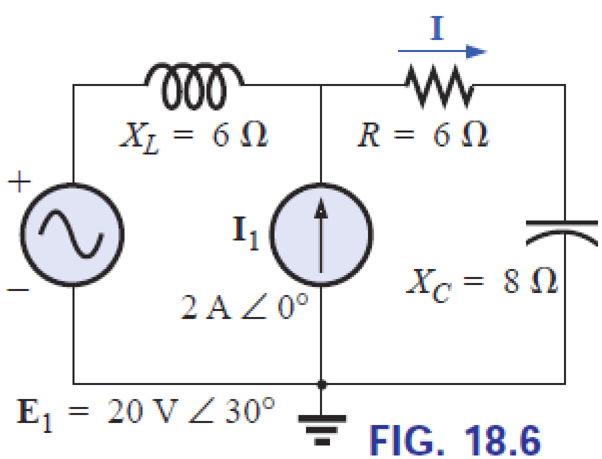


FIG. 18.6
Example 18.2.

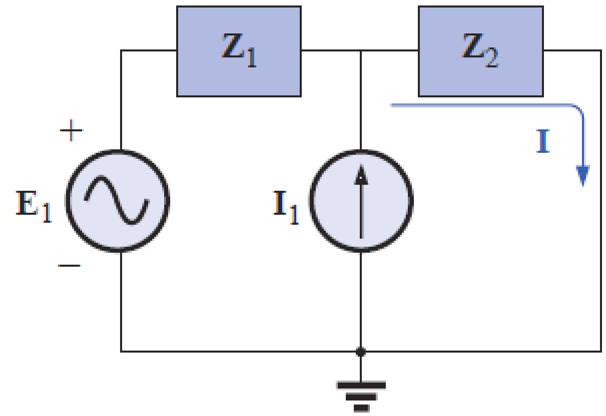


FIG. 18.7
Assigning the subscripted impedances to the network of Fig. 18.6.

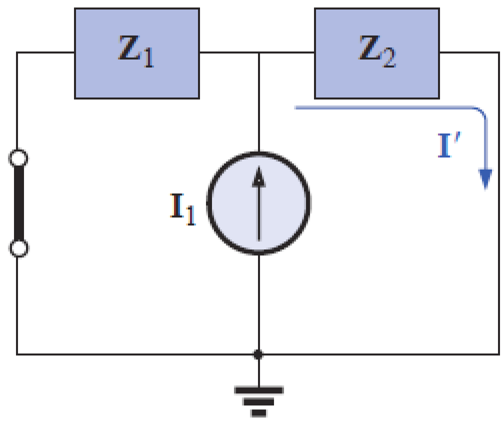


FIG. 18.8
Determining the effect of the current source $\mathbf{I}_1$ *on the current* $\mathbf{I}$ *of the network of Fig. 18.6.*

Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have

$$\mathbf{I}' = \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\,\Omega)(2\text{ A})}{j\,6\,\Omega + 6\,\Omega - j\,8\,\Omega} = \frac{j\,12\text{ A}}{6 - j\,2}$$

$$= \frac{12\text{ A}\,\angle 90^\circ}{6.32\,\angle -18.43^\circ}$$

$$\mathbf{I}' = 1.9\text{ A}\,\angle 108.43^\circ$$

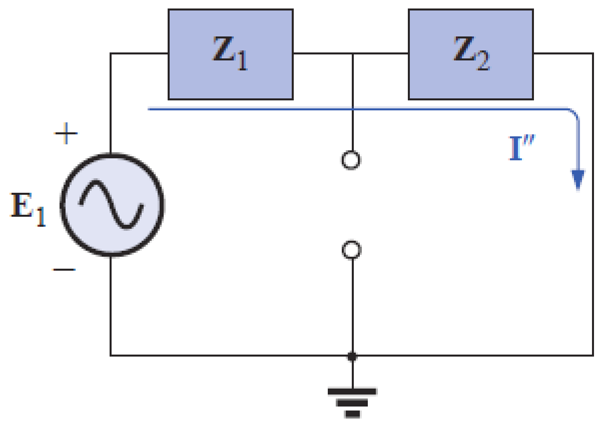


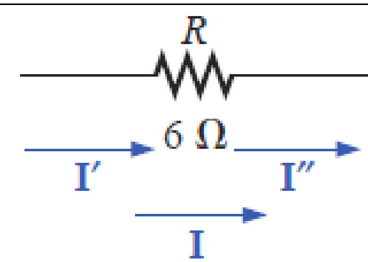
FIG. 18.9
Determining the effect of the voltage source $\mathbf{E}_1$ *on the current* $\mathbf{I}$ *of the network of Fig. 18.6.*

Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

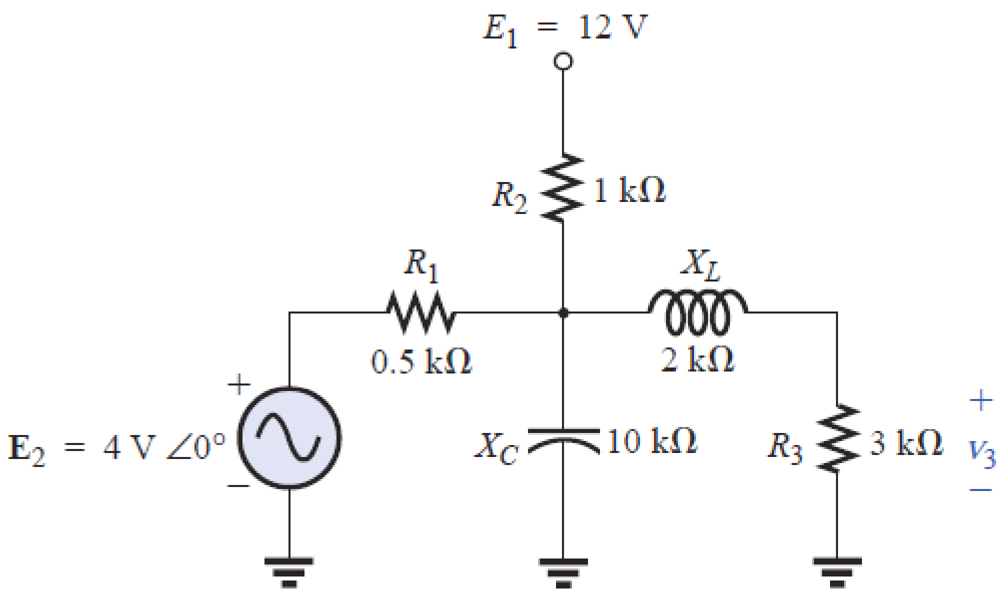
$$\mathbf{I}'' = \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V}\ \angle 30^\circ}{6.32\ \Omega\ \angle -18.43^\circ}$$
$$= 3.16\ \text{A}\ \angle 48.43^\circ$$

The total current through the 6-Ω resistor (Fig. 18.10) is

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}''$$
$$= 1.9\ \text{A}\ \angle 108.43^\circ + 3.16\ \text{A}\ \angle 48.43^\circ$$
$$= (-0.60\ \text{A} + j\,1.80\ \text{A}) + (2.10\ \text{A} + j\,2.36\ \text{A})$$
$$= 1.50\ \text{A} + j\,4.16\ \text{A}$$
$$\mathbf{I} = \mathbf{4.42\ A\ \angle 70.2^\circ}$$



EXAMPLE 18.4 For the network of Fig. 18.12, determine the sinusoidal expression for the voltage v_3 using superposition.



Solution: For the dc source, recall that for dc analysis, in the steady state the capacitor can be replaced by an open-circuit equivalent, and the inductor by a short-circuit equivalent. The result is the network of Fig. 18.13.

The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\ \text{k}\Omega \parallel 3\ \text{k}\Omega = 0.429\ \text{k}\Omega$$

and

$$V_3 = \frac{R'E_1}{R' + R_2}$$

$$= \frac{(0.429\ \text{k}\Omega)(12\ \text{V})}{0.429\ \text{k}\Omega + 1\ \text{k}\Omega} = \frac{5.148\ \text{V}}{1.429}$$

$$V_3 \cong \mathbf{3.6\ V}$$

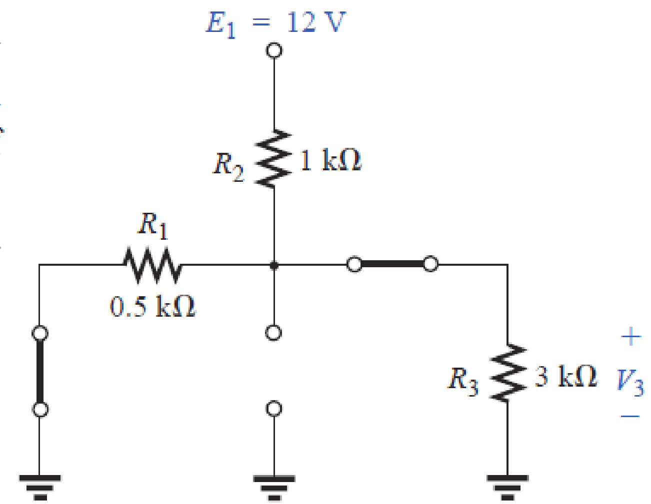


FIG. 18.13

Determining the effect of the dc voltage source E_1 on the voltage v_3 of the network of Fig. 18.12.

For ac analysis, the dc source is set to zero and the network is redrawn, as shown in Fig. 18.14.

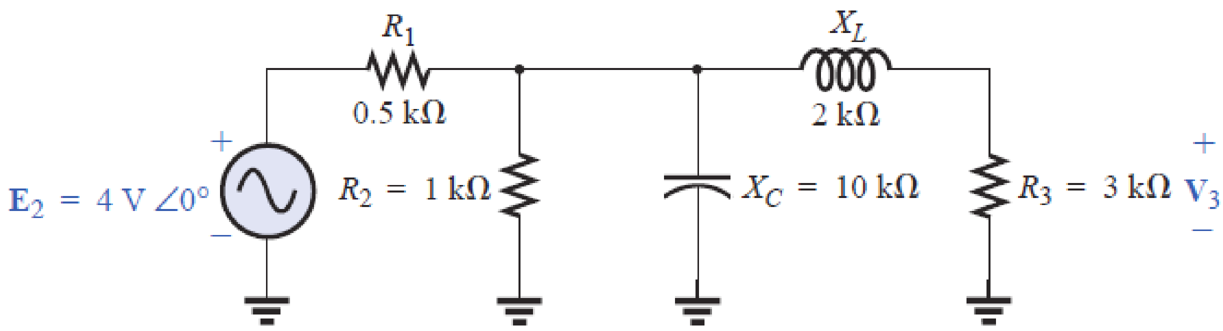


FIG. 18.14

Redrawing the network of Fig. 18.12 to determine the effect of the ac voltage source $\mathbf{E}_2$.

The block impedances are then defined as in Fig. 18.15, and series-parallel techniques are applied as follows:

$$\mathbf{Z}_1 = 0.5\ \text{k}\Omega \angle 0°$$

$$\mathbf{Z}_2 = (R_2 \angle 0° \parallel (X_C \angle -90°)$$

$$= \frac{(1\ \text{k}\Omega \angle 0°)(10\ \text{k}\Omega \angle -90°)}{1\ \text{k}\Omega - j\,10\ \text{k}\Omega} = \frac{10\ \text{k}\Omega \angle -90°}{10.05 \angle -84.29°}$$

$$= 0.995\ \text{k}\Omega \angle -5.71°$$

$$\mathbf{Z}_3 = R_3 + j\,X_L = 3\ \text{k}\Omega + j\,2\ \text{k}\Omega = 3.61\ \text{k}\Omega \angle 33.69°$$

and $\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3$

$$= 0.5\ \text{k}\Omega + (0.995\ \text{k}\Omega \angle -5.71°) \parallel (3.61\ \text{k}\Omega \angle 33.69°)$$

$$= 1.312\ \text{k}\Omega \angle 1.57°$$

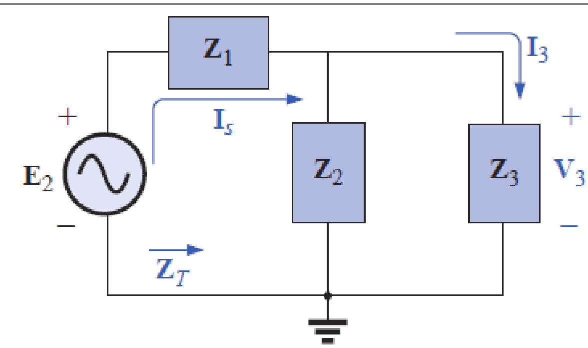


FIG. 18.15

Assigning the subscripted impedances to the network of Fig. 18.14.

$$\mathbf{I}_s = \frac{\mathbf{E}_2}{\mathbf{Z}_T} = \frac{4\text{ V}\angle 0^\circ}{1.312\text{ k}\Omega\angle 1.57^\circ} = 3.05\text{ mA}\angle -1.57^\circ$$

Current divider rule:

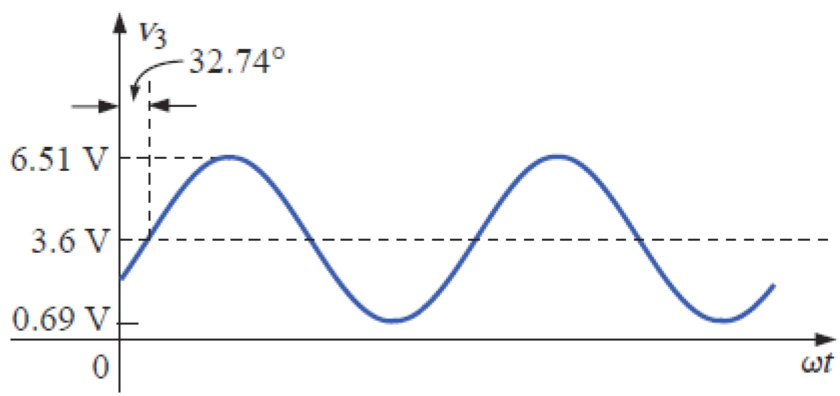
$$\mathbf{I}_3 = \frac{\mathbf{Z}_2\mathbf{I}_s}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(0.995\text{ k}\Omega\angle -5.71^\circ)(3.05\text{ mA}\angle -1.57^\circ)}{0.995\text{ k}\Omega\angle -5.71^\circ + 3.61\text{ k}\Omega\angle 33.69^\circ} = 0.686\text{ mA}\angle -32.74^\circ$$

with
$$\begin{aligned}\mathbf{V}_3 &= (I_3\angle\theta)(R_3\angle 0^\circ)\\ &= (0.686\text{ mA}\angle -32.74^\circ)(3\text{ k}\Omega\angle 0^\circ)\\ &= \mathbf{2.06\text{ V}\angle -32.74^\circ}\end{aligned}$$

The total solution:

$$\begin{aligned}v_3 &= v_3\text{ (dc)} + v_3\text{ (ac)}\\ &= 3.6\text{ V} + 2.06\text{ V}\angle -32.74^\circ\\ v_3 &= \mathbf{3.6 + 2.91\sin(\omega t - 32.74^\circ)}\end{aligned}$$

The result is a sinusoidal voltage having a peak value of 2.91 V riding on an average value of 3.6 V, as shown in Fig. 18.16.



Dependent Sources

To apply superposition theorem on circuits with dependent sources, there are two cases:

1. **Case 1**: if the ***controlling variables*** are ***outside*** the circuit to be analyzed => we proceed with superposition as usual.
2. **Case 2**: if the ***controlling variables*** are ***within*** the circuit to be analyzed => we set the dependent source to zero only when its controlling variable is zero

EXAMPLE 18.5 Using the superposition theorem, determine the current $\mathbf{I}_2$ for the network of Fig. 18.17. The quantities μ and h are constants.

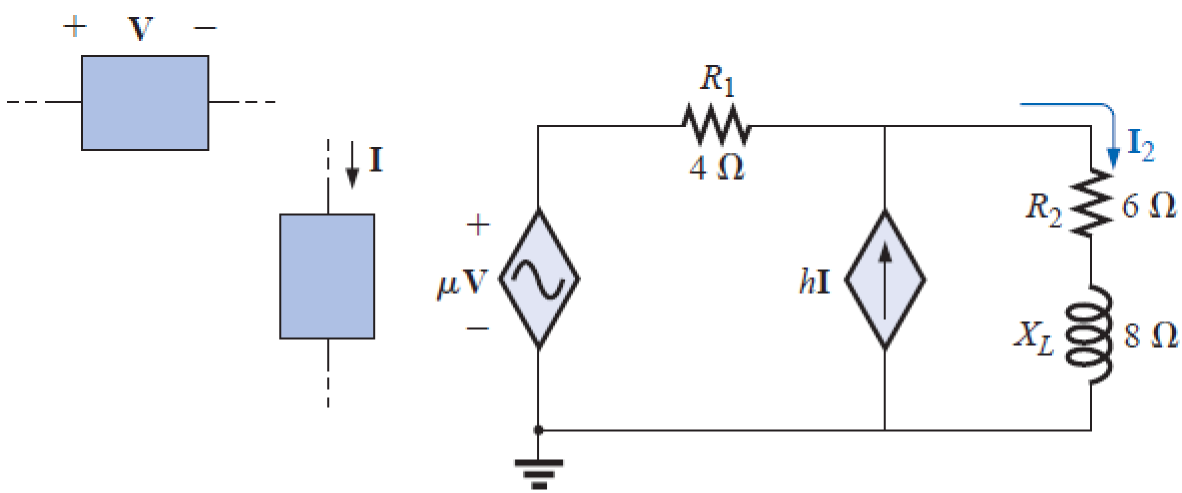


FIG. 18.17

Solution: With a portion of the system redrawn (Fig. 18.18),

$$\mathbf{Z}_1 = R_1 = 4\ \Omega \qquad \mathbf{Z}_2 = R_2 + j X_L = 6 + j\,8\ \Omega$$

For the voltage source (Fig. 18.19),

$$\mathbf{I}' = \frac{\mu\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mu\mathbf{V}}{4\ \Omega + 6\ \Omega + j\,8\ \Omega} = \frac{\mu\mathbf{V}}{10\ \Omega + j\,8\ \Omega}$$
$$= \frac{\mu\mathbf{V}}{12.8\ \Omega \angle 38.66^\circ} = 0.078\ \mu\mathbf{V}/\Omega \angle -38.66^\circ$$

For the current source (Fig. 18.20),

$$\mathbf{I}'' = \frac{\mathbf{Z}_1(h\mathbf{I})}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(4\ \Omega)(h\mathbf{I})}{12.8\ \Omega \angle 38.66^\circ} = 4(0.078)h\mathbf{I} \angle -38.66^\circ$$
$$= 0.312h\mathbf{I} \angle -38.66^\circ$$

The current $\mathbf{I}_2$ is

$$\mathbf{I}_2 = \mathbf{I}' + \mathbf{I}''$$
$$= 0.078\ \mu\mathbf{V}/\Omega \angle -38.66^\circ + 0.312h\mathbf{I} \angle -38.66^\circ$$

For $\mathbf{V} = 10\ \text{V} \angle 0^\circ$, $\mathbf{I} = 20\ \text{mA} \angle 0^\circ$, $\mu = 20$, and $h = 100$,

$$\mathbf{I}_2 = 0.078(20)(10\ \text{V} \angle 0^\circ)/\Omega \angle -38.66^\circ + 0.312(100)(20\ \text{mA} \angle 0^\circ) \angle -38.66^\circ$$
$$= 15.60\ \text{A} \angle -38.66^\circ + 0.62\ \text{A} \angle -38.66^\circ$$
$$\mathbf{I}_2 = \mathbf{16.22\ A \angle -38.66^\circ}$$

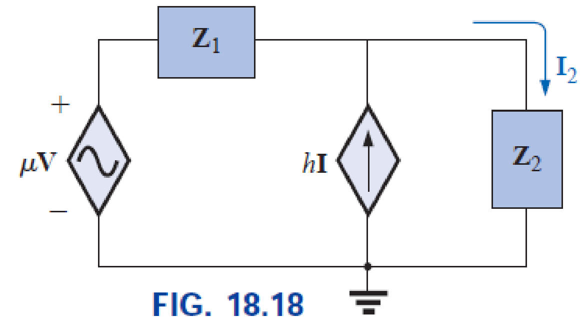


FIG. 18.18

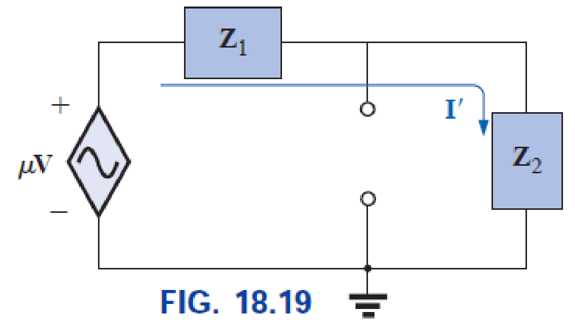


FIG. 18.19

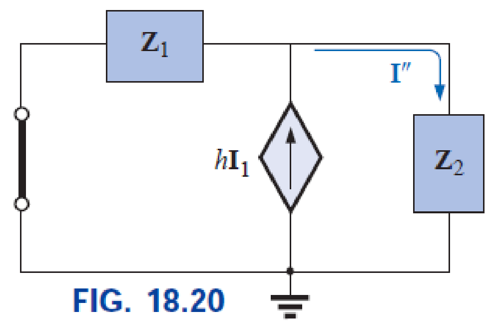


FIG. 18.20

EXAMPLE 18.6 Determine the current $\mathbf{I}_L$ through the resistor R_L of Fig. 18.21.

Solution: Note that the controlling variable $\mathbf{V}$ is determined by the network to be analyzed. From the above discussions, it is understood that the dependent source cannot be set to zero unless $\mathbf{V}$ is zero. If we set $\mathbf{I}$ to zero, the network lacks a source of voltage, and $\mathbf{V} = 0$ with $\mu\mathbf{V} = 0$. The resulting $\mathbf{I}_L$ under this condition is zero. Obviously, therefore, the network must be analyzed as it appears in Fig. 18.21, with the result that neither source can be eliminated, as is normally done using the superposition theorem.

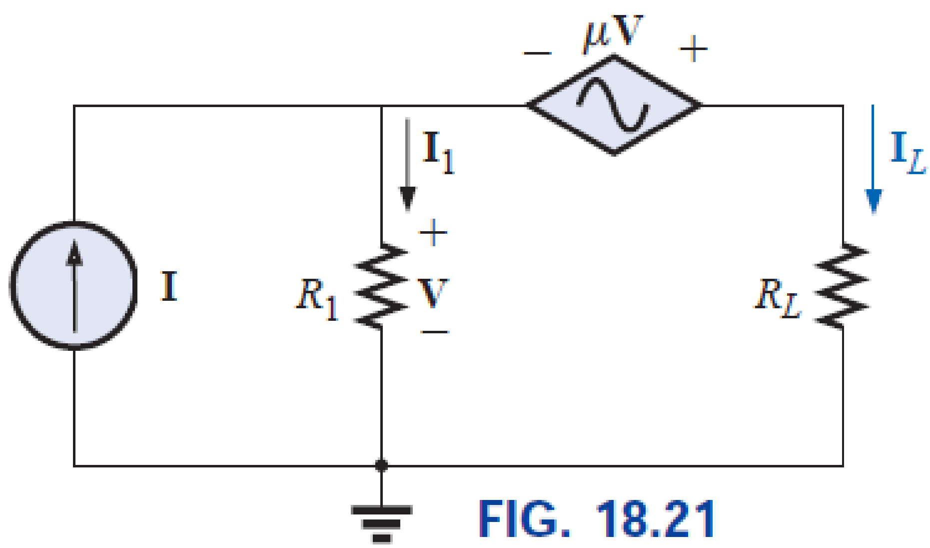


FIG. 18.21

Applying Kirchhoff's voltage law, we have

$$\mathbf{V}_L = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

and
$$\mathbf{I}_L = \frac{\mathbf{V}_L}{R_L} = \frac{(1 + \mu)\mathbf{V}}{R_L}$$

The result, however, must be found in terms of **I** since **V** and μ**V** are only dependent variables.

Applying Kirchhoff's current law gives us

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_L = \frac{\mathbf{V}}{R_1} + \frac{(1 + \mu)\mathbf{V}}{R_L}$$

and
$$\mathbf{I} = \mathbf{V}\left(\frac{1}{R_1} + \frac{1 + \mu}{R_L}\right)$$

or
$$\mathbf{V} = \frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}$$

Substituting into the above yields

$$\mathbf{I}_L = \frac{(1 + \mu)\mathbf{V}}{R_L} = \frac{(1 + \mu)}{R_L}\left(\frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}\right)$$

Therefore,
$$\mathbf{I}_L = \frac{\mathbf{(1 + \mu)}\boldsymbol{R_1}\mathbf{I}}{\boldsymbol{R_L} + \mathbf{(1 + \mu)}\boldsymbol{R_1}}$$

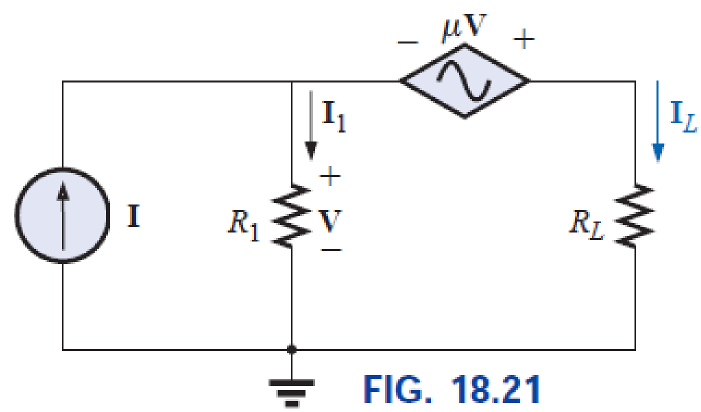


FIG. 18.21

18.3 THEVENIN'S THEOREM

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source (Phasor) and an impedance in series, as shown in Fig. 18.22.

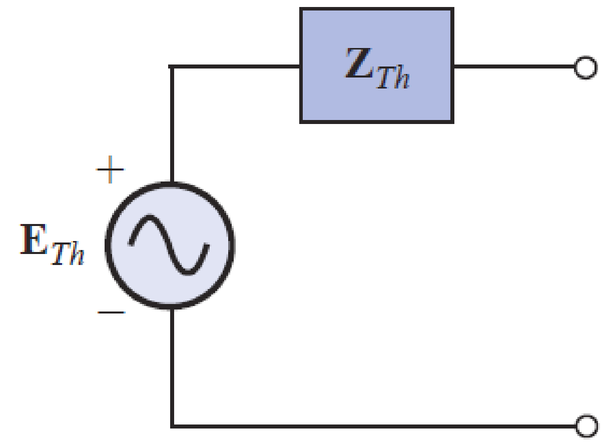


FIG. 18.22
Thévenin equivalent circuit for ac networks.

Since the reactances of a circuit are frequency dependent, the Thévenin circuit found for a particular network is applicable only at *one* frequency.

1. ***Remove that portion of the network across which the Thévenin equivalent circuit is to be found.***
2. ***Mark (○, ●, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate Z_{Th} by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate E_{Th} by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.***
5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.***

EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.23.

Solution:

Steps 1 and 2 (Fig. 18.24):

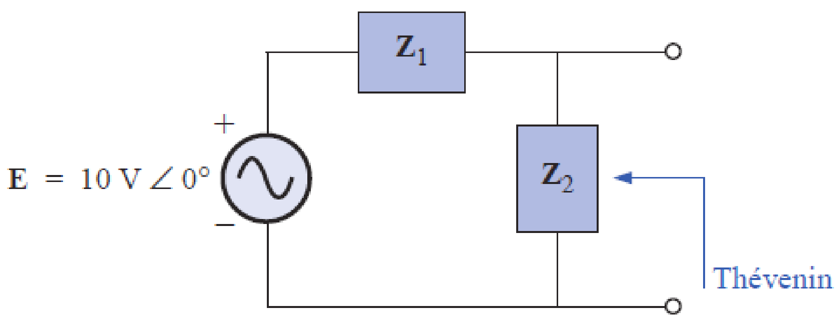


FIG. 18.24
Assigning the subscripted impedances to the network of Fig. 18.23.

$$\mathbf{Z}_1 = j X_L = j\,8\,\Omega \qquad \mathbf{Z}_2 = -j X_C = -j\,2\,\Omega$$

Step 3 (Fig. 18.25):

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2 16\,\Omega}{j\,6} = \frac{16\,\Omega}{6\,\angle 90^\circ}$$
$$= \mathbf{2.67\,\Omega\,\angle -90^\circ}$$

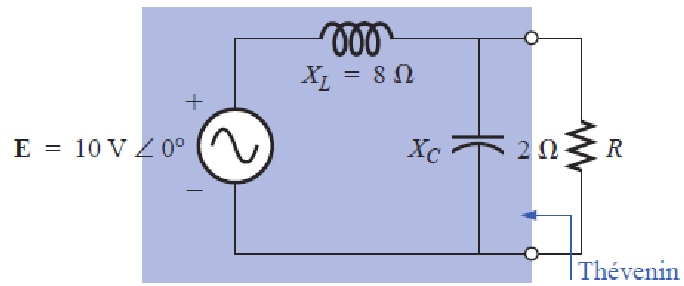


FIG. 18.23
Example 18.7.

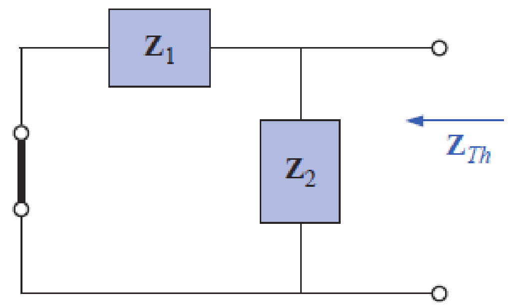


FIG. 18.25
Determining the Thévenin impedance for the network of Fig. 18.23.

Step 4 (Fig. 18.26):

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$

$$= \frac{(-j\,2\,\Omega)(10\text{ V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\text{ V}}{j\,6} = \mathbf{3.33\ V\ \angle -180^\circ}$$

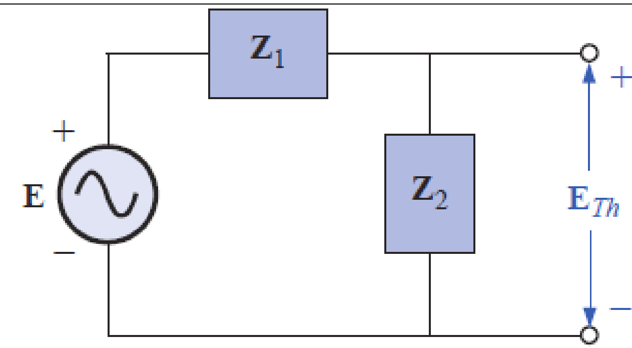


FIG. 18.26
Determining the open-circuit Thévenin voltage for the network of Fig. 18.23.

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.27.

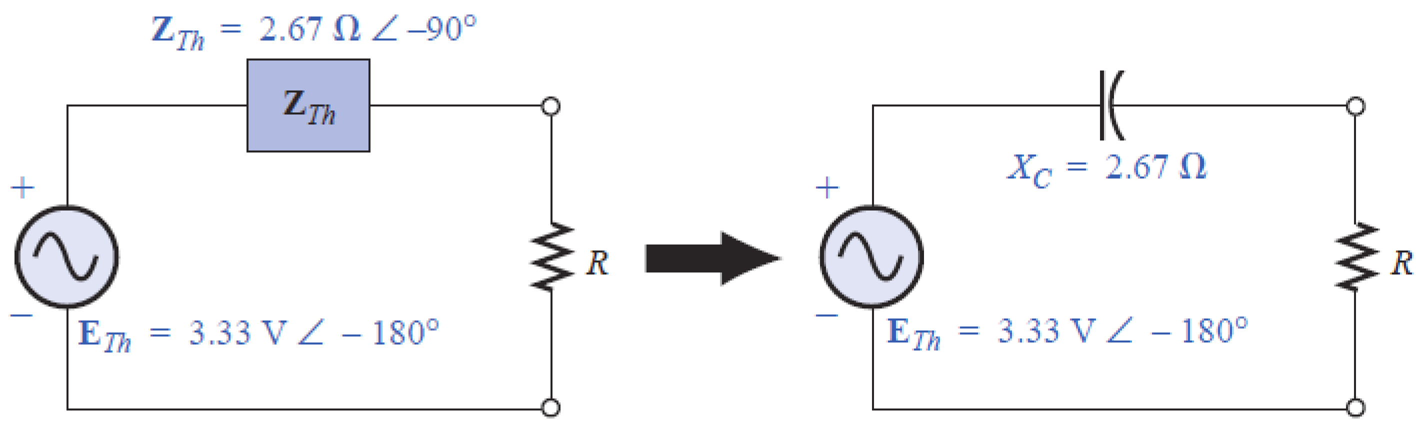


FIG. 18.27
The Thévenin equivalent circuit for the network of Fig. 18.23.

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch a-a' in Fig. 18.28.

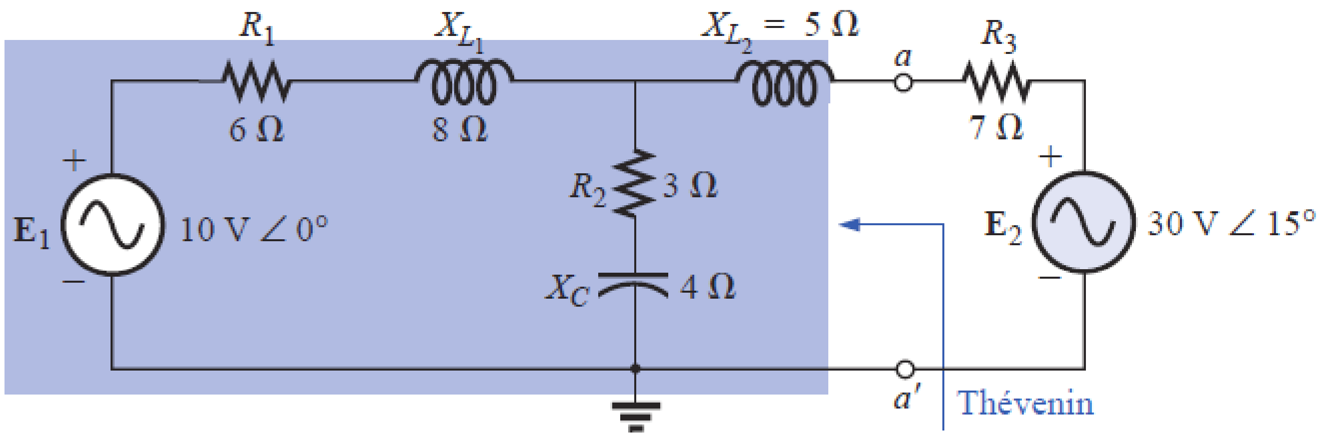


FIG. 18.28
Example 18.8.

Solution:

Steps 1 and 2 (Fig. 18.29): Note the reduced complexity with subscripted impedances:

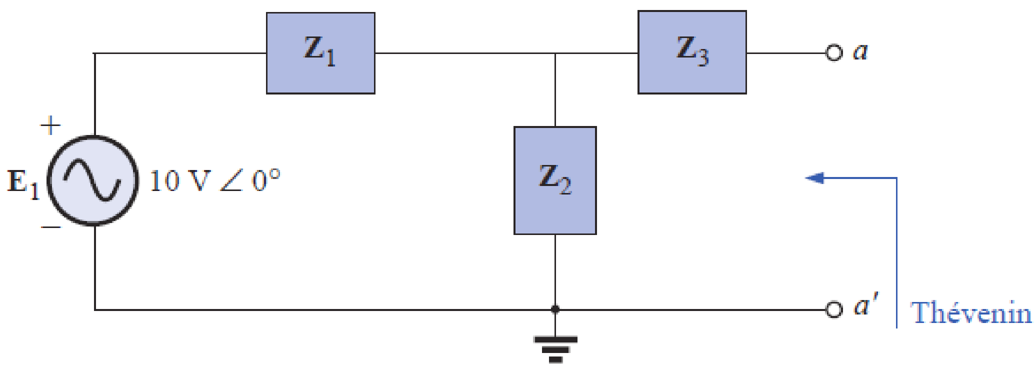


FIG. 18.29

Assigning the subscripted impedances to the network of Fig. 18.28.

$$\mathbf{Z}_1 = R_1 + j\,X_{L_1} = 6\,\Omega + j\,8\,\Omega$$

$$\mathbf{Z}_2 = R_2 - j\,X_C = 3\,\Omega - j\,4\,\Omega$$

$$\mathbf{Z}_3 = +j\,X_{L_2} = j\,5\,\Omega$$

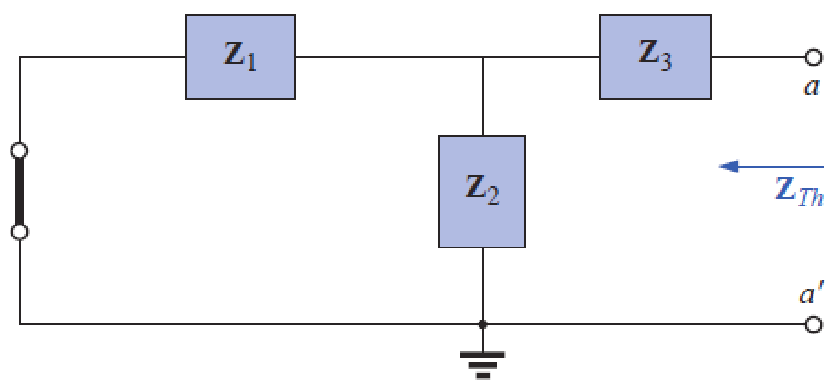
Step 3 (Fig. 18.30):

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\,5\,\Omega + \frac{(10\,\Omega\,\angle 53.13^\circ)(5\,\Omega\,\angle -53.13^\circ)}{(6\,\Omega + j\,8\,\Omega) + (3\,\Omega - j\,4\,\Omega)}$$

$$= j\,5 + \frac{50\,\angle 0^\circ}{9 + j\,4} = j\,5 + \frac{50\,\angle 0^\circ}{9.85\,\angle 23.96^\circ}$$

$$= j\,5 + 5.08\,\angle -23.96^\circ = j\,5 + 4.64 - j\,2.06$$

$$\mathbf{Z}_{Th} = \mathbf{4.64\,\Omega + j\,2.94\,\Omega = 5.49\,\Omega\,\angle 32.36^\circ}$$

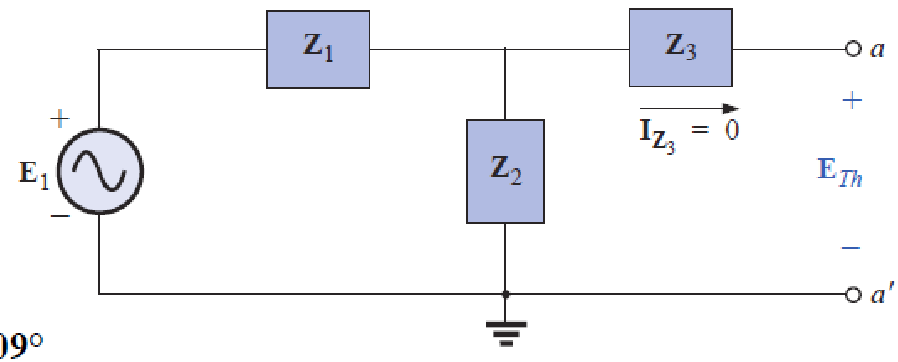


Step 4 (Fig. 18.31): Since *a-a′* is an open circuit, $\mathbf{I}_{Z_3} = 0$. Then $\mathbf{E}_{Th}$ is the voltage drop across $\mathbf{Z}_2$:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(5\ \Omega \angle -53.13^\circ)(10\ \text{V} \angle 0^\circ)}{9.85\ \Omega \angle 23.96^\circ}$$

$$\mathbf{E}_{Th} = \frac{50\ \text{V} \angle -53.13^\circ}{9.85 \angle 23.96^\circ} = \mathbf{5.08\ V \angle -77.09^\circ}$$



Step 5: The Thévenin equivalent circuit is shown in Fig. 18.32.

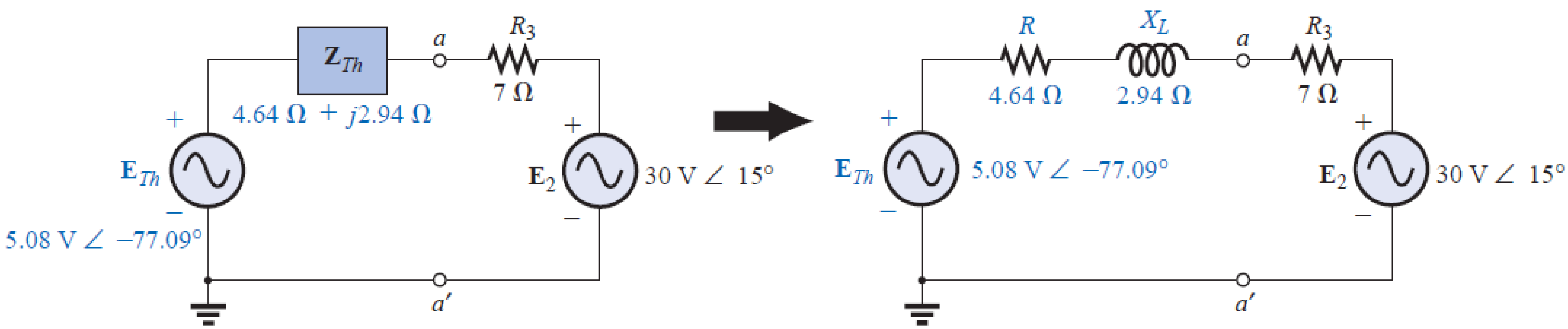


FIG. 18.32
The Thévenin equivalent circuit for the network of Fig. 18.28.

In electronic circuits using superposition permits separation of the DC and AC analyses.

EXAMPLE 18.9 Determine the Thévenin equivalent circuit for the transistor network external to the resistor R_L in the network of Fig. 18.33. Then determine $\mathbf{V}_L$.

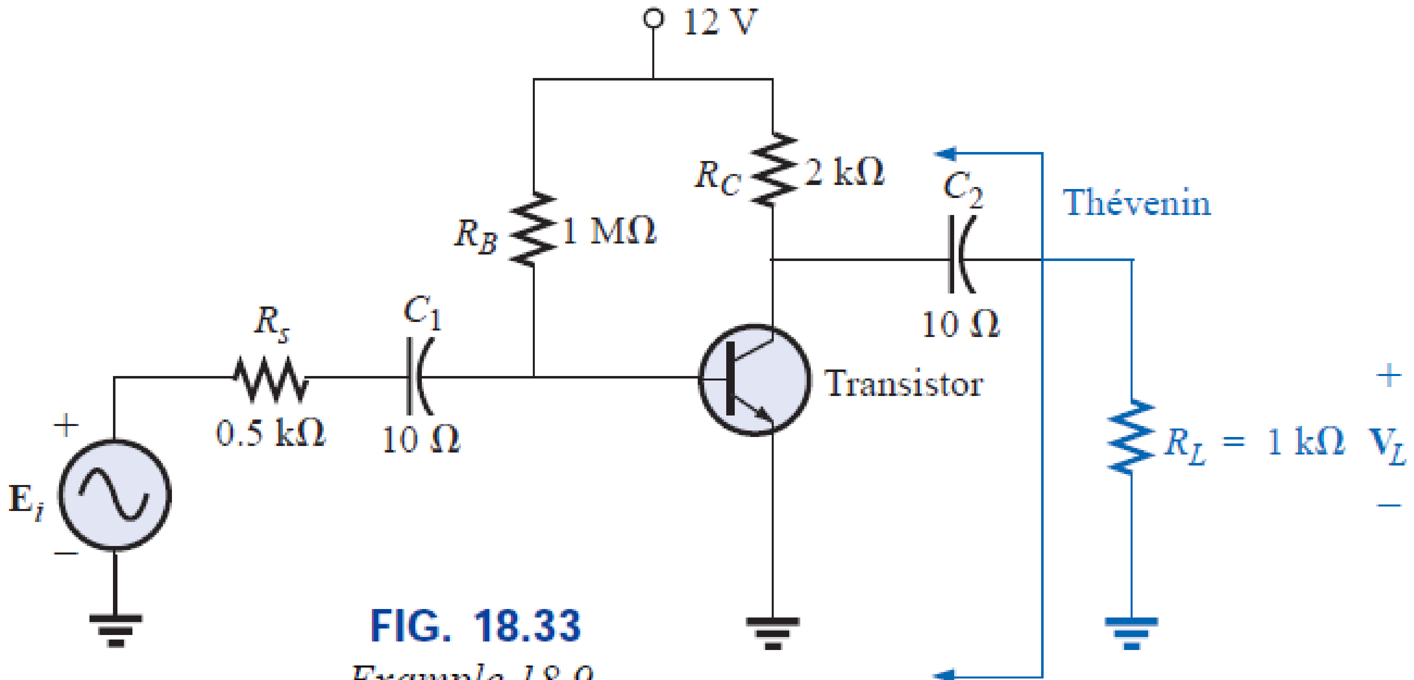


FIG. 18.33
Example 18.9.

Solution: Applying superposition.

dc Conditions Substituting the open-circuit equivalent for the coupling capacitor C_2 will isolate the dc source and the resulting currents from the load resistor. The result is that for dc conditions, $V_L = 0$ V. Although the output dc voltage is zero, the application of the dc voltage is important to the basic operation of the transistor in a number of important ways, one of which is to determine the parameters of the "equivalent circuit" to appear in the ac analysis to follow.

ac Conditions For the ac analysis, an equivalent circuit is substituted for the transistor, as established by the dc conditions above, that will behave like the actual transistor. Fig. 18.34⇨ the equivalent circuit. The equivalent circuit includes a resistor of 2.3 kΩ and a controlled current source whose magnitude is determined by the product of a factor of 100 and the current I_1 in another part of the network.

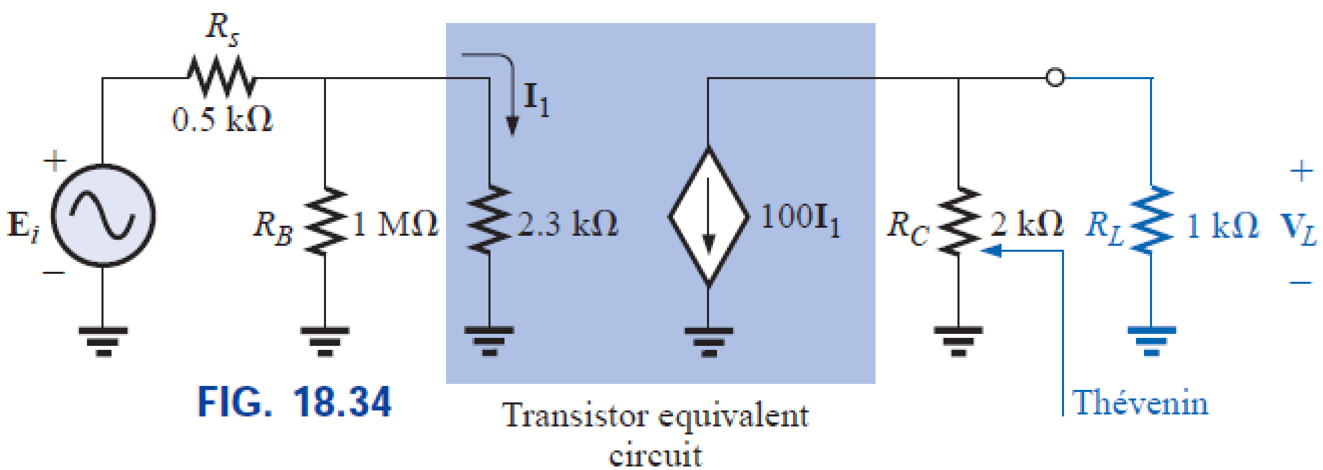


FIG. 18.34

The ac equivalent network for the transistor amplifier of Fig. 18.33.

For the analysis to follow, the effect of the resistor R_B will be ignored since it is so much larger than the parallel 2.3-kΩ resistor.

$\mathbf{Z}_{Th}$ When $\mathbf{E}_i$ is set to zero volts, the current $\mathbf{I}_1$ will be zero amperes, and the controlled source $100\mathbf{I}_1$ will be zero amperes also. The result is an open-circuit equivalent for the source, as appearing in Fig. 18.35.

It is fairly obvious from Fig. 18.35 that

$$\mathbf{Z}_{Th} = \mathbf{2\ k\Omega}$$

$\mathbf{E}_{Th}$ For $\mathbf{E}_{Th}$, the current $\mathbf{I}_1$ of Fig. 18.34 will be

$$\mathbf{I}_1 = \frac{\mathbf{E}_i}{R_s + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{0.5\ \text{k}\Omega + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}$$

and $$100\mathbf{I}_1 = (100)\left(\frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}\right) = 35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i$$

Referring to Fig. 18.36, we find that

$$\begin{aligned}\mathbf{E}_{Th} &= -(100\mathbf{I}_1)R_C \\ &= -(35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i)(2 \times 10^3\ \Omega) \\ \mathbf{E}_{Th} &= \mathbf{-71.42E}_i\end{aligned}$$

The Thévenin equivalent circuit appears in Fig. 18.37 with the original load R_L.

Output Voltage $\mathbf{V}_L$

$$\mathbf{V}_L = \frac{-R_L\mathbf{E}_{Th}}{R_L + \mathbf{Z}_{Th}} = \frac{-(1\ \text{k}\Omega)(71.42\mathbf{E}_i)}{1\ \text{k}\Omega + 2\ \text{k}\Omega}$$

and $$\mathbf{V}_L = \mathbf{-23.81E}_i$$

revealing that the output voltage is 23.81 times the applied voltage with a phase shift of 180° due to the minus sign.

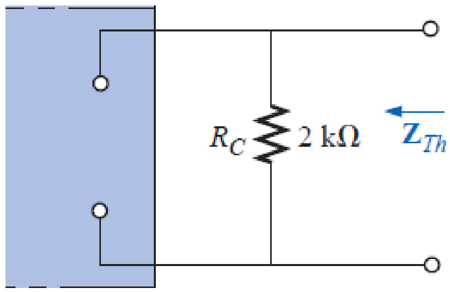


FIG. 18.35
Determining the Thévenin impedance for the network of Fig. 18.34.

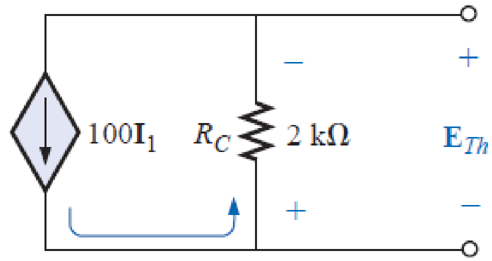


FIG. 18.36
Determining the Thévenin voltage for the network of Fig. 18.34.

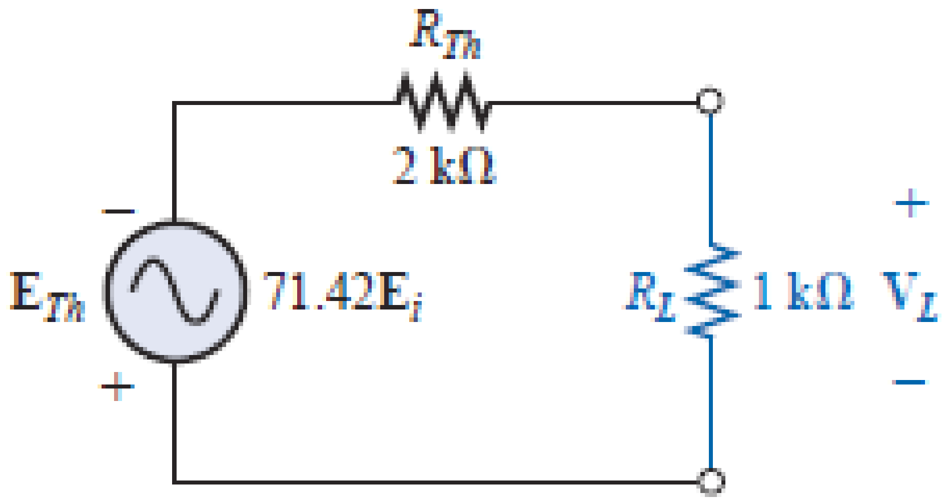


FIG. 18.37
The Thévenin equivalent circuit for the network of Fig. 18.34.

Dependent Sources:

Case 1: Controlling variable external to the network under investigation: $\Longrightarrow$ Can use the method shown above.

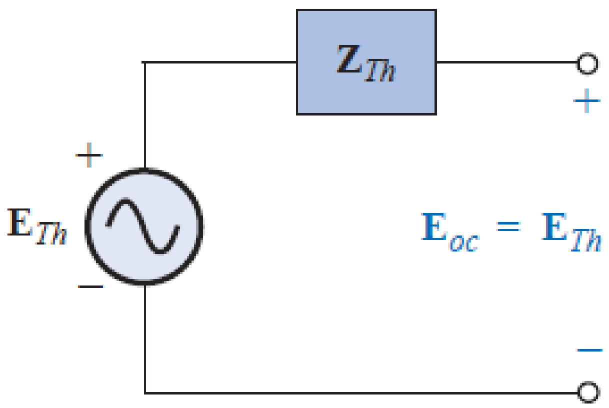
Case 2: Controlling variable is part of the network under investigation: $\Longrightarrow$ Use New Approach.

New Approach for Thévenin's theorem:

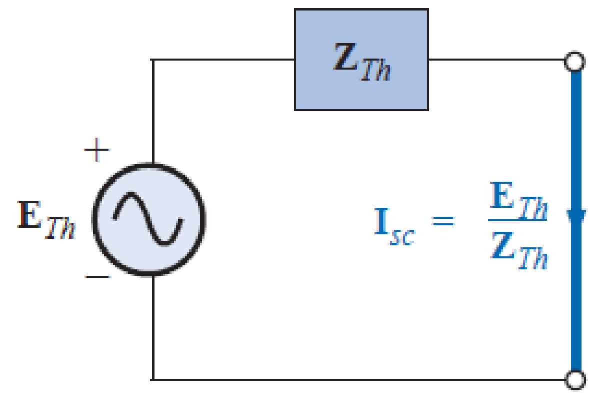
This new approach can be used for any circuit; however, it is especially useful for circuits with dependent sources controlled by variables within the circuit to be analyzed.

1. ***Step 1***: find the open circuit voltage. The Thévenin's equivalent voltage will be equal to the open circuit voltage.

2. ***Step 2***: find the short circuit current. The Thévenin's equivalent impedance will be equal to the ratio between the open circuit voltage and short circuit current.



$$\mathbf{E}_{oc} = \mathbf{E}_{Th}$$



$$\mathbf{I}_{sc} = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} \Longrightarrow \mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}$$

EXAMPLE 18.10 Using each of the three techniques described in this section, determine the Thévenin equivalent circuit for the network of Fig. 18.40.

Solution: Since for each approach the Thévenin voltage is found in exactly the same manner, it will be determined first. From Fig. 18.40, where $\mathbf{I}_{X_C} = 0$,

Due to the polarity for **V** and defined terminal polarities

$$\mathbf{V}_{R_2} = \mathbf{E}_{Th} = \mathbf{E}_{oc} = -\frac{R_2(\mu\mathbf{V})}{R_1 + R_2} = -\frac{\mu R_2 \mathbf{V}}{R_1 + R_2}$$

The following three methods for determining the Thévenin impedance appear in the order in which they were introduced in this section.

Method 1: See Fig. 18.41.

$$\mathbf{Z}_{Th} = R_1 \parallel R_2 - j X_C$$

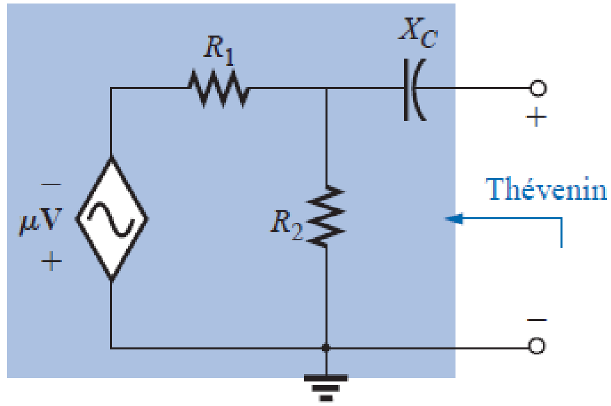


FIG. 18.40

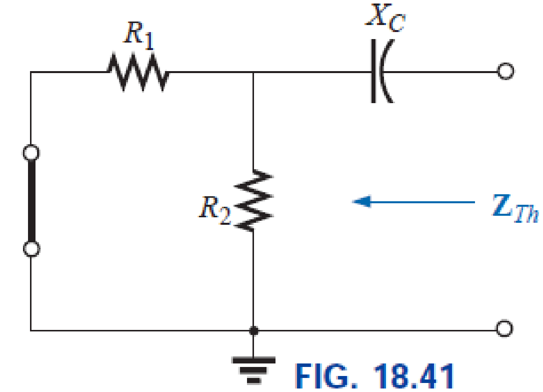


FIG. 18.41

Method 2: See Fig. 18.42. Converting the voltage source to a current source (Fig. 18.43), we have (current divider rule)

$$\mathbf{I}_{sc} = \frac{-(R_1 \parallel R_2)\dfrac{\mu\mathbf{V}}{R_1}}{(R_1 \parallel R_2) - jX_C} = \frac{-\dfrac{R_1R_2}{R_1+R_2}\left(\dfrac{\mu\mathbf{V}}{R_1}\right)}{(R_1 \parallel R_2) - jX_C}$$

$$= \frac{\dfrac{-\mu R_2\mathbf{V}}{R_1+R_2}}{(R_1 \parallel R_2) - jX_C}$$

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-\mu R_2\mathbf{V}}{R_1+R_2}}{\dfrac{\dfrac{-\mu R_2\mathbf{V}}{R_1+R_2}}{(R_1 \parallel R_2) - jX_C}} = \frac{1}{\dfrac{1}{(R_1 \parallel R_2) - jX_C}}$$

$$\mathbf{Z}_{Th} = \boldsymbol{R_1 \parallel R_2 - jX_C}$$

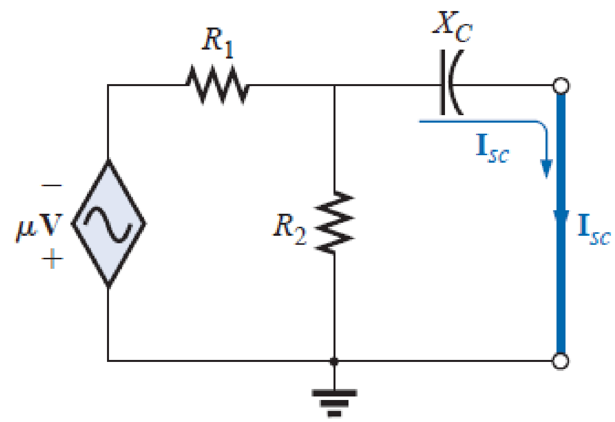


FIG. 18.42

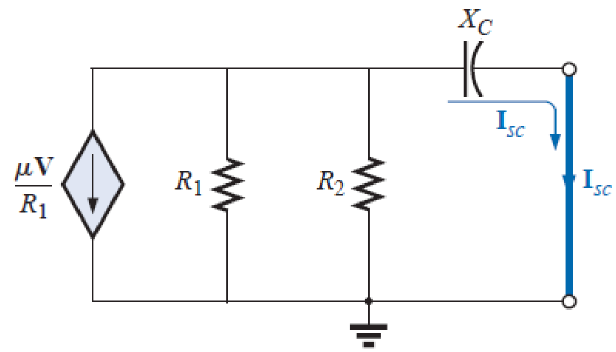


FIG. 18.43

In each case, the Thévenin impedance is the same. The resulting Thévenin equivalent circuit is shown in Fig. 18.45.

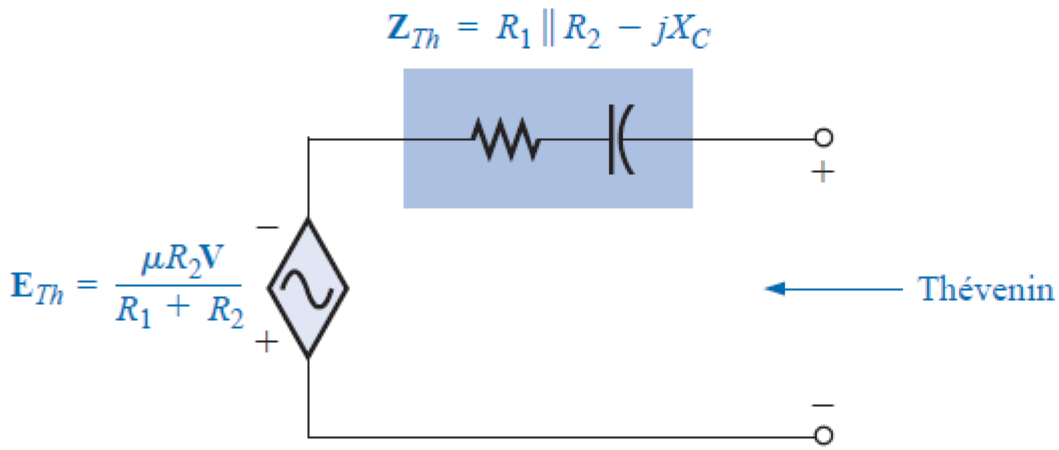


FIG. 18.45

The Thévenin equivalent circuit for the network of Fig. 18.40.

EXAMPLE 18.12 For the network of Fig. 18.50 (introduced in Example 18.6), determine the Thévenin equivalent circuit between the indicated terminals using each method described in this section. Compare your results.

Solution: First, using Kirchhoff's voltage law, $\mathbf{E}_{Th}$ (which is the same for each method) is written

$$\mathbf{E}_{Th} = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

However, $$\mathbf{V} = \mathbf{I}R_1$$

so $$\mathbf{E}_{Th} = (1 + \mu)\mathbf{I}R_1$$

$\mathbf{Z}_{Th}$

Method 1: See Fig. 18.51. Since $\mathbf{I} = 0$, $\mathbf{V}$ and $\mu\mathbf{V} = 0$, and

$$\mathbf{Z}_{Th} = R_1 \qquad \text{(incorrect)}$$

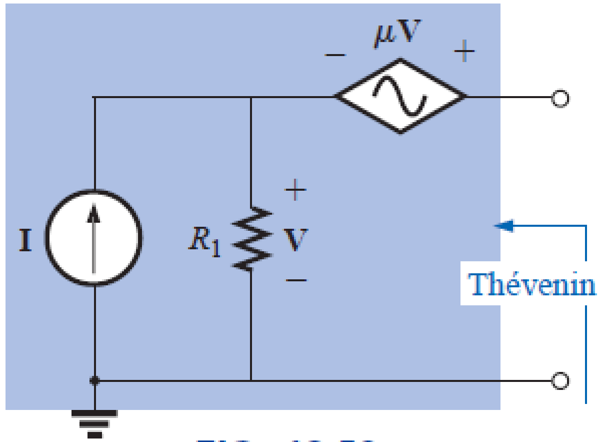


FIG. 18.50

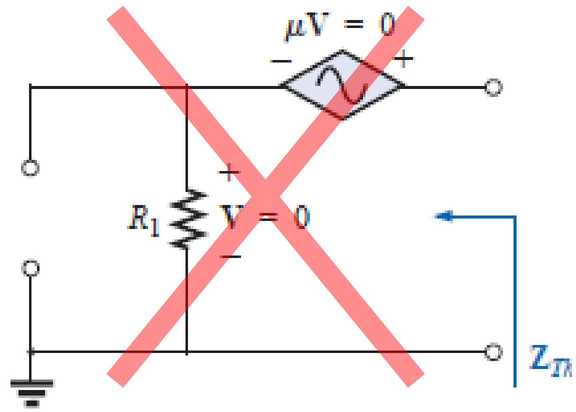


FIG. 18.51
Determining $\mathbf{Z}_{Th}$ incorrectly.

Method 2: See Fig. 18.52. Kirchhoff's voltage law around the indicated loop gives us

$$\mathbf{V} + \mu\mathbf{V} = 0$$

and

$$\mathbf{V}(1 + \mu) = 0$$

Since μ is a positive constant, the above equation can be satisfied only when $\mathbf{V} = 0$. Substitution of this result into Fig. 18.52 will yield the configuration of Fig. 18.53, and

$$\mathbf{I}_{sc} = \mathbf{I}$$

with $$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{(1 + \mu)\mathbf{I}R_1}{\mathbf{I}} = \mathbf{(1 + \mu)}\boldsymbol{R_1} \qquad \text{(correct)}$$

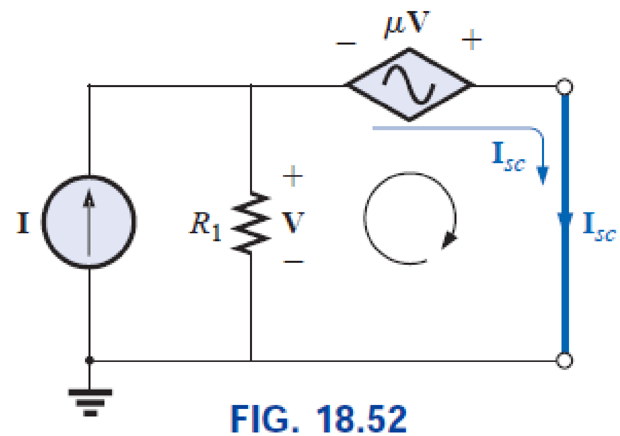


FIG. 18.52

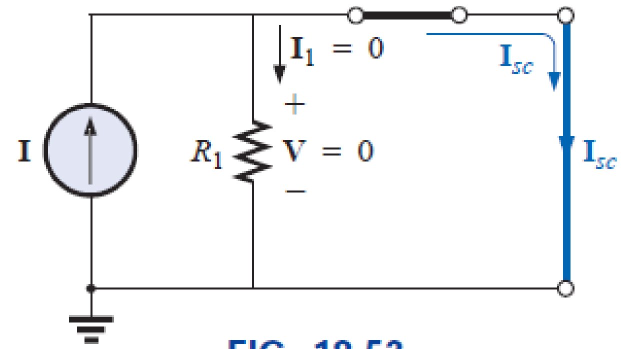


FIG. 18.53

Method 3: See Fig. 18.54.

$$\mathbf{E}_g = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

or

$$\mathbf{V} = \frac{\mathbf{E}_g}{1 + \mu}$$

and

$$\mathbf{I}_g = \frac{\mathbf{V}}{R_1} = \frac{\mathbf{E}_g}{(1 + \mu)R_1}$$

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = (1 + \mu)R_1 \qquad \text{(correct)}$$

The Thévenin equivalent circuit appears in Fig. 18.55, and

$$\mathbf{I}_L = \frac{(1 + \mu)R_1\mathbf{I}}{R_L + (1 + \mu)R_1}$$

which compares with the result of Example 18.6.

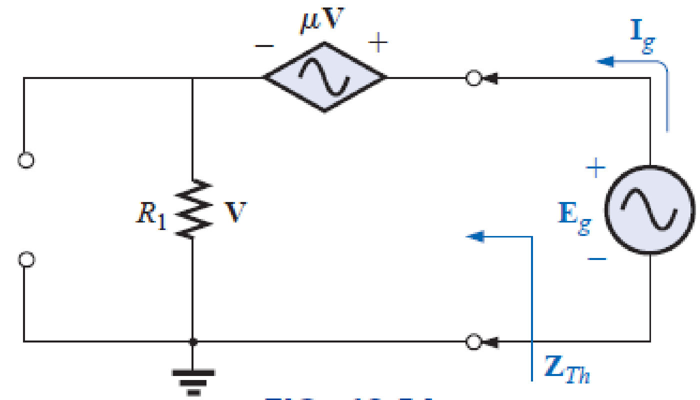


FIG. 18.54

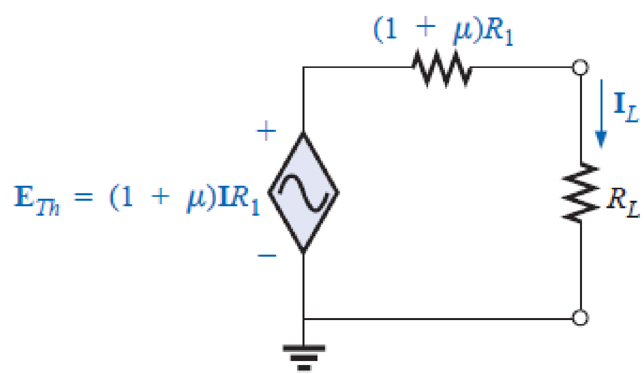


FIG. 18.55

EXAMPLE 18.13 Determine the Thévenin equivalent circuit for the indicated terminals of the network of Fig. 18.56.

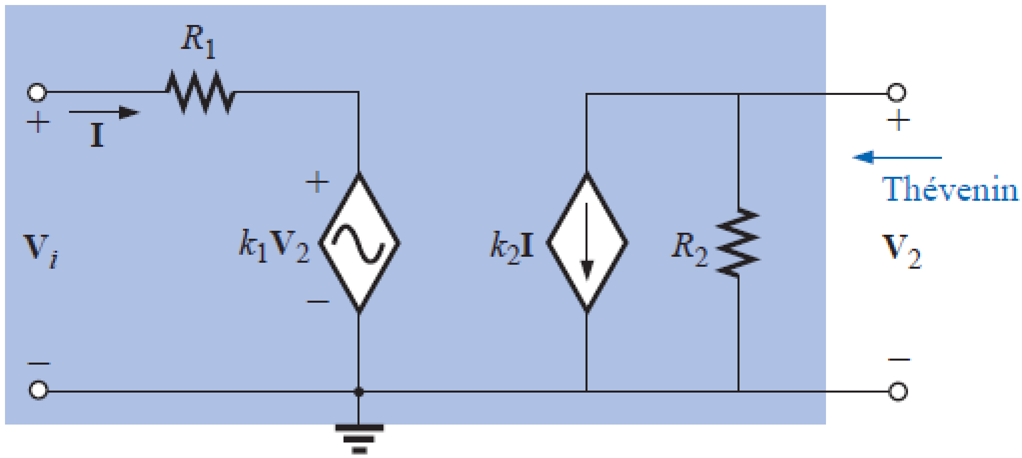


FIG. 18.56

Example 18.13: Transistor equivalent network.

Solution: Apply the second method introduced in this section.

$\mathbf{E}_{Th}$

$$\mathbf{E}_{oc} = \mathbf{V}_2$$

$$\mathbf{I} = \frac{\mathbf{V}_i - k_1\mathbf{V}_2}{R_1} = \frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}$$

and

$$\mathbf{E}_{oc} = -k_2\mathbf{I}R_2 = -k_2R_2\left(\frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}\right)$$

$$= \frac{-k_2R_2\mathbf{V}_i}{R_1} + \frac{k_1k_2R_2\mathbf{E}_{oc}}{R_1}$$

or

$$\mathbf{E}_{oc}\left(1 - \frac{k_1k_2R_2}{R_1}\right) = \frac{-k_2R_2\mathbf{V}_i}{R_1}$$

and

$$\mathbf{E}_{oc}\left(\frac{R_1 - k_1k_2R_2}{\cancel{R_1}}\right) = \frac{-k_2R_2\mathbf{V}_i}{\cancel{R_1}}$$

so

$$\mathbf{E}_{oc} = \frac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2} = \mathbf{E}_{Th}$$

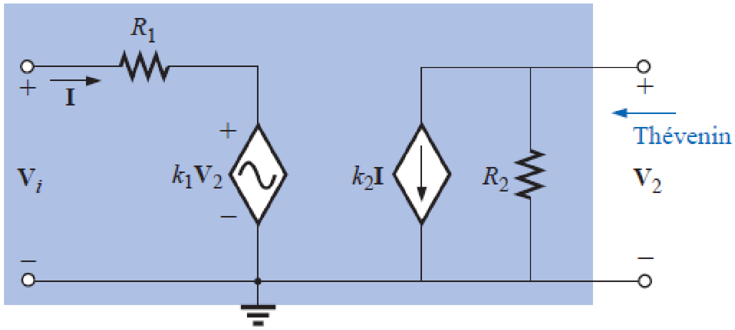


FIG. 18.56
Example 18.13: Transistor equivalent network.

$\mathbf{I}_{sc}$ For the network of Fig. 18.57, where

$$\mathbf{V}_2 = 0 \qquad k_1\mathbf{V}_2 = 0 \qquad \mathbf{I} = \frac{\mathbf{V}_i}{R_1}$$

and
$$\mathbf{I}_{sc} = -k_2\mathbf{I} = \frac{-k_2\mathbf{V}_i}{R_1}$$

so
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{-k_2\mathbf{V}_i}{R_1}} = \frac{R_1R_2}{R_1 - k_1k_2R_2}$$

and
$$\mathbf{Z}_{Th} = \frac{R_2}{1 - \dfrac{k_1k_2R_2}{R_1}}$$

Frequently, the approximation $k_1 \cong 0$ is applied. Then the Thévenin voltage and impedance are

$$\mathbf{E}_{Th} = \frac{-k_2R_2\mathbf{V}_i}{R_1} \qquad k_1 = 0$$

$$\mathbf{Z}_{Th} = R_2 \qquad k_1 = 0$$

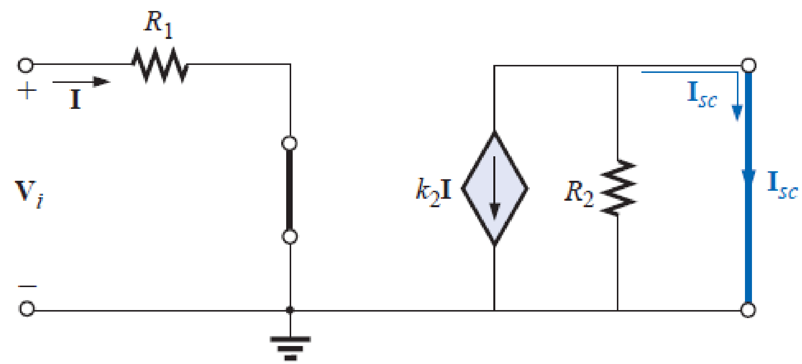
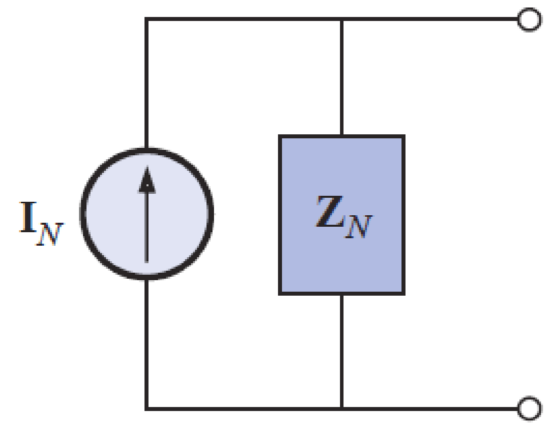


FIG. 18.57
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.56.

18.4 NORTON'S THEOREM

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a current source (Phasor) and an impedance in parallel, as shown in Fig. 18.59.	 **FIG. 18.59** *The Norton equivalent circuit for ac networks.*

Since the reactances of a circuit are frequency dependent, the Norton's circuit found for a particular network is applicable only at *one* frequency.

1. *Remove that portion of the network across which the Norton equivalent circuit is to be found.*
2. *Mark (○, ●, and so on) the terminals of the remaining two-terminal network.*
3. *Calculate* $\mathbf{Z}_N$ *by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting* ***impedance*** *between the two marked terminals.*
4. *Calculate* $\mathbf{I}_N$ *by first replacing the voltage and current sources and then finding the* ***short-circuit current*** *between the marked terminals.*
5. *Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.*

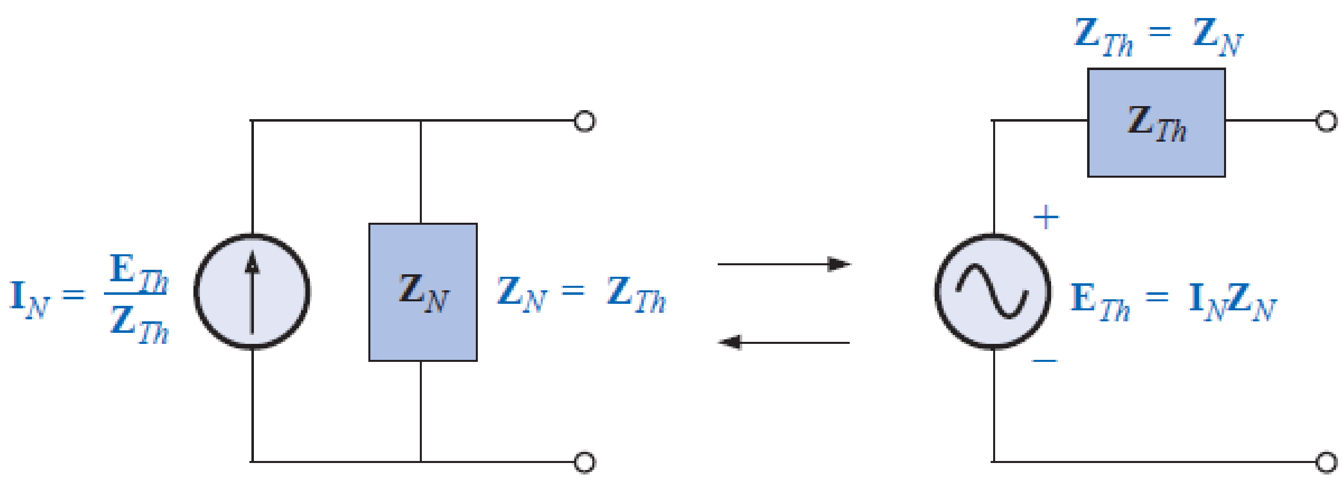


FIG. 18.60

Conversion between the Thévenin and Norton equivalent circuits.

EXAMPLE 18.15 Find the Norton equivalent circuit for the network external to the 7-Ω capacitive reactance in Fig. 18.66.

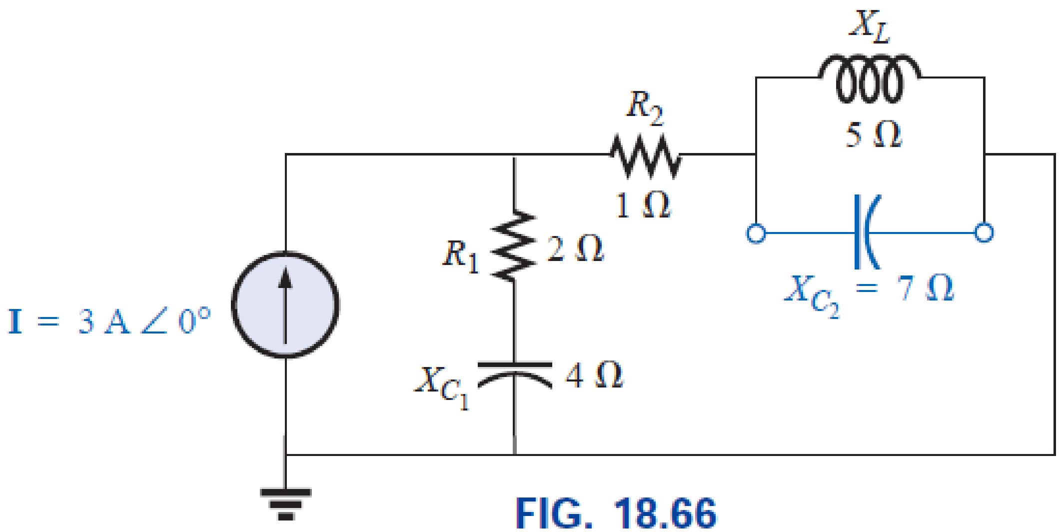


FIG. 18.66

Example 18.15.

Solution:

Steps 1 and 2 (Fig. 18.67):

$$\mathbf{Z}_1 = R_1 - j\,X_{C_1} = 2\ \Omega - j\,4\ \Omega$$

$$\mathbf{Z}_2 = R_2 = 1\ \Omega$$

$$\mathbf{Z}_3 = +j\,X_L = j\,5\ \Omega$$

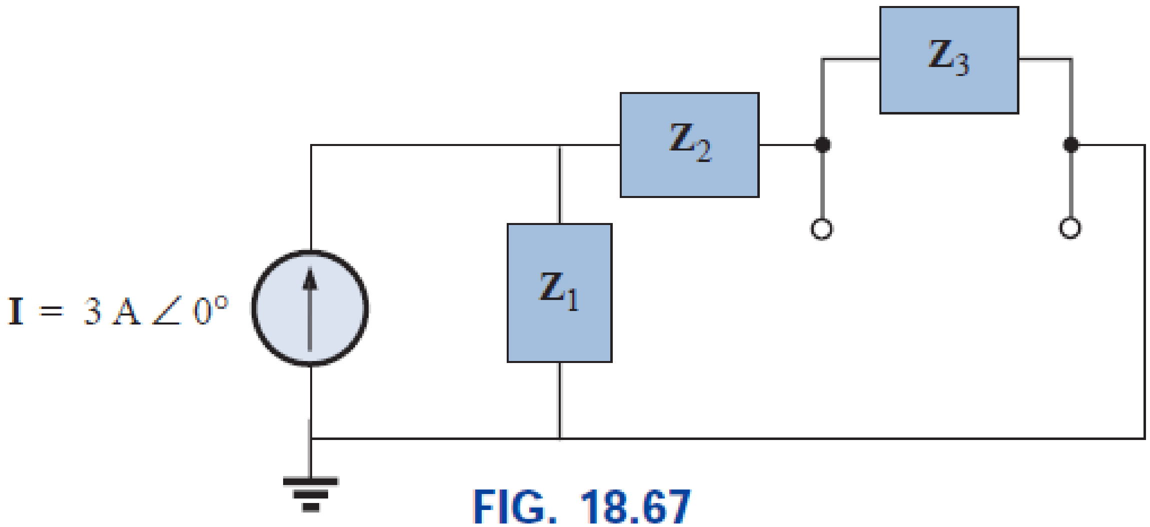


FIG. 18.67

Assigning the subscripted impedances to the network of Fig. 18.66.

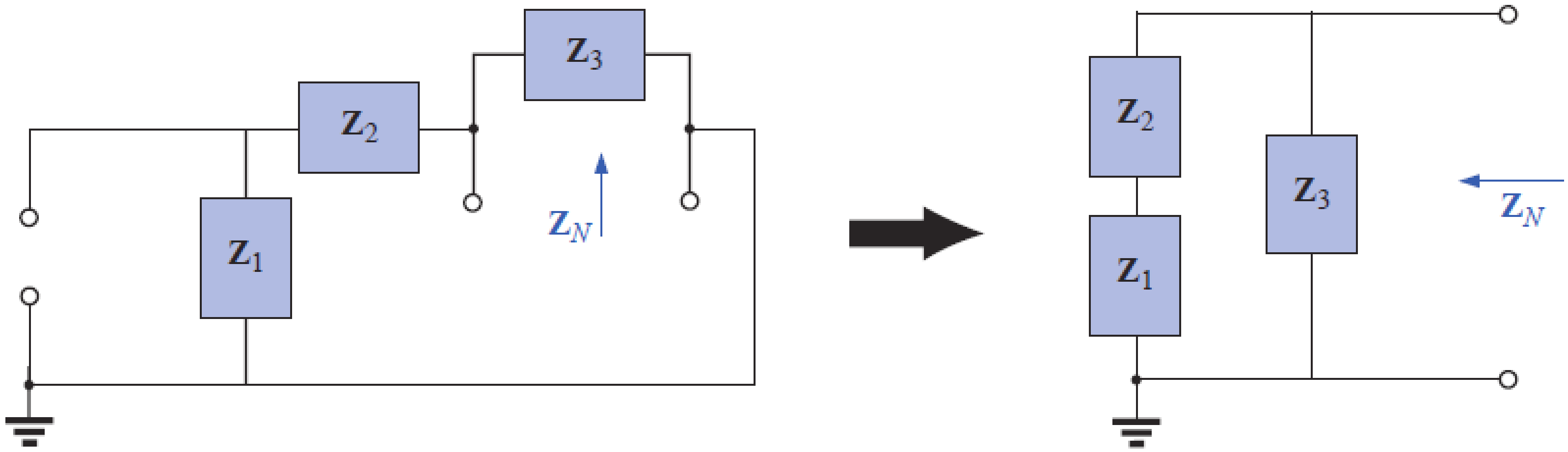
Step 3 (Fig. 18.68):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2)}{\mathbf{Z}_3 + (\mathbf{Z}_1 + \mathbf{Z}_2)}$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = 2\ \Omega - j\,4\ \Omega + 1\ \Omega = 3\ \Omega - j\,4\ \Omega = 5\ \Omega \angle -53.13^\circ$$

$$\mathbf{Z}_N = \frac{(5\ \Omega \angle 90^\circ)(5\ \Omega \angle -53.13^\circ)}{j\,5\ \Omega + 3\ \Omega - j\,4\ \Omega} = \frac{25\ \Omega \angle 36.87^\circ}{3 + j\,1} = \frac{25\ \Omega \angle 36.87^\circ}{3.16 \angle +18.43^\circ}$$

$$\mathbf{Z}_N = 7.91\ \Omega \angle 18.44^\circ = \mathbf{7.50\ \Omega + j\,2.50\ \Omega}$$

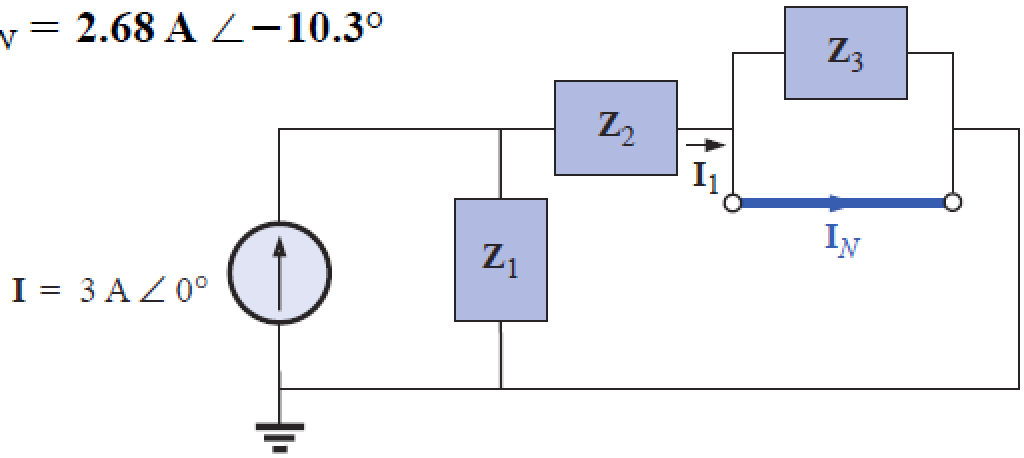


Step 4 (Fig. 18.69):

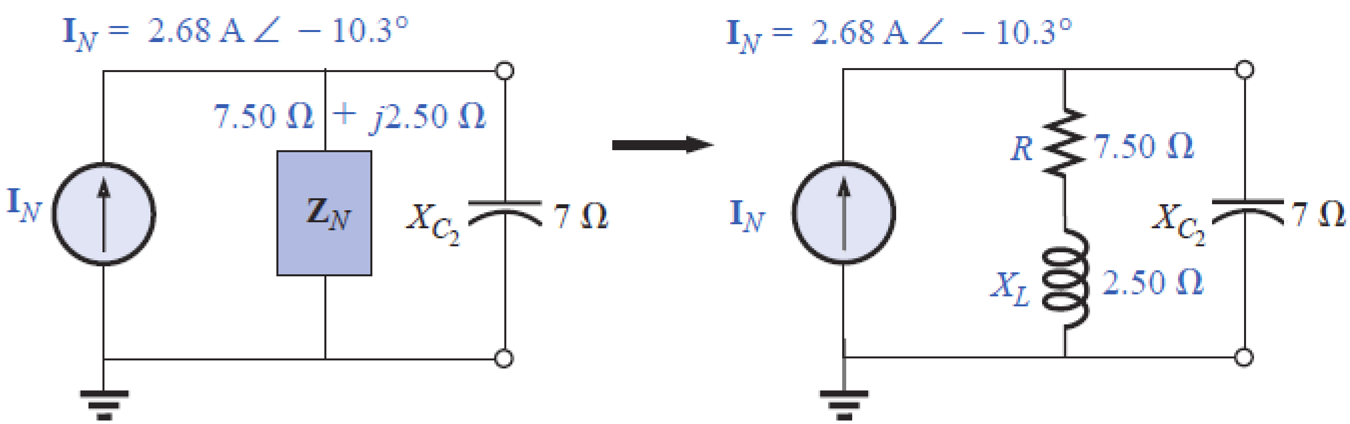
$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(current divider rule)}$$

$$= \frac{(2\,\Omega - j\,4\,\Omega)(3\text{ A})}{3\,\Omega - j\,4\,\Omega} = \frac{6\text{ A} - j\,12\text{ A}}{5\,\angle -53.13^\circ} = \frac{13.4\text{ A}\,\angle -63.43^\circ}{5\,\angle -53.13^\circ}$$

$$\mathbf{I}_N = \mathbf{2.68\text{ A}\,\angle -10.3^\circ}$$



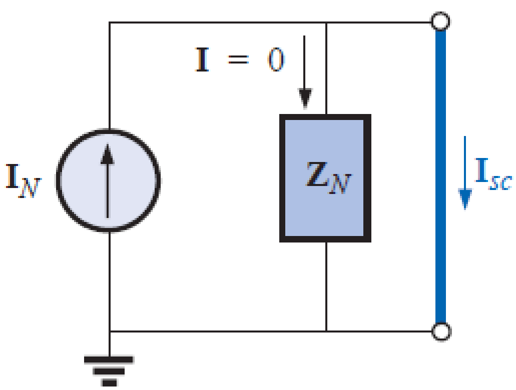
Step 5: The Norton equivalent circuit is shown in Fig. 18.70.



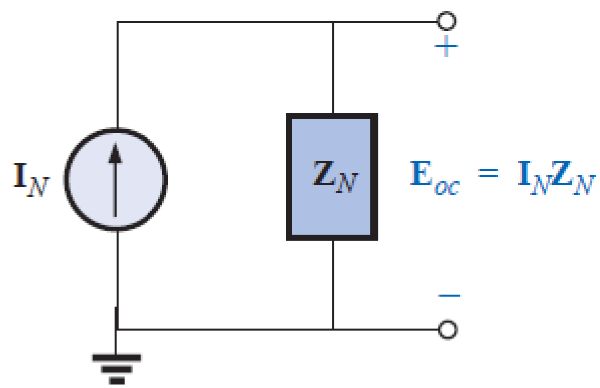
A new Approach for Norton's theorem

This new approach can be used for any circuit; however, it is especially useful for circuits with dependent sources controlled by variables within the circuit to be analyzed.

1. ***Step 1***: find the short circuit current. The Norton's equivalent current will be equal to the short circuit current.
2. ***Step 2***: find the open circuit voltage. The Norton's equivalent impedance will be equal to the ratio between the open circuit voltage and short circuit current.



$$\mathbf{I}_{sc} = \mathbf{I}_N$$



$$\mathbf{E}_{oc} = \mathbf{I}_N \mathbf{Z}_N$$

$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_N} \Longrightarrow \mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}$$

EXAMPLE 18.17 Using each method described for dependent sources, find the Norton equivalent circuit for the network of Fig. 18.75.

Solution:

$\mathbf{I}_N$ For each method, $\mathbf{I}_N$ is determined in the same manner. From Fig.18.76, using Kirchhoff's current law, we have

$$0 = \mathbf{I} + h\mathbf{I} + \mathbf{I}_{sc}$$

or

$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I}$$

Applying Kirchhoff's voltage law gives us

$$\mathbf{E} + \mathbf{I}R_1 - \mathbf{I}_{sc}R_2 = 0$$

and

$$\mathbf{I}R_1 = \mathbf{I}_{sc}R_2 - \mathbf{E}$$

or

$$\mathbf{I} = \frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}$$

so

$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I} = -(1 + h)\left(\frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}\right)$$

or

$$R_1\mathbf{I}_{sc} = -(1 + h)\mathbf{I}_{sc}R_2 + (1 + h)\mathbf{E}$$

$$\mathbf{I}_{sc}[R_1 + (1 + h)R_2] = (1 + h)\mathbf{E}$$

$$\mathbf{I}_{sc} = \frac{\mathbf{(1 + h)E}}{\mathbf{R_1 + (1 + h)R_2}} = \mathbf{I}_N$$

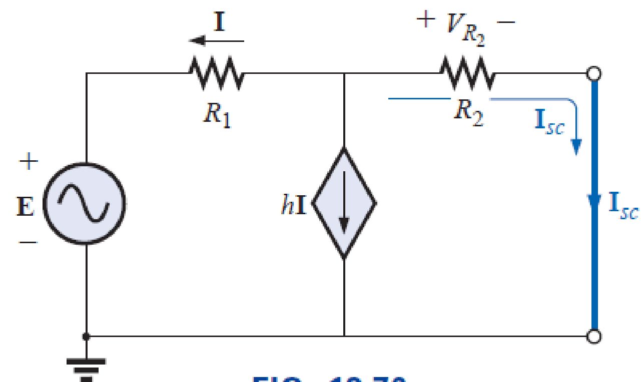


FIG. 18.76
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.75.

Z_N

Method 1: $\mathbf{E}_{oc}$ is determined from the network of Fig. 18.77. By Kirchhoff's current law,

$$0 = \mathbf{I} + h\mathbf{I} \quad \text{or} \quad \mathbf{I}(h + 1) = 0$$

For h, a positive constant $\mathbf{I}$ must equal zero to satisfy the above. Therefore,

$$\mathbf{I} = 0 \quad \text{and} \quad h\mathbf{I} = 0$$

and

$$\mathbf{E}_{oc} = \mathbf{E}$$

with

$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}}{\dfrac{(1+h)\mathbf{E}}{R_1 + (1+h)R_2}} = \frac{R_1 + (1+h)R_2}{(1+h)}$$

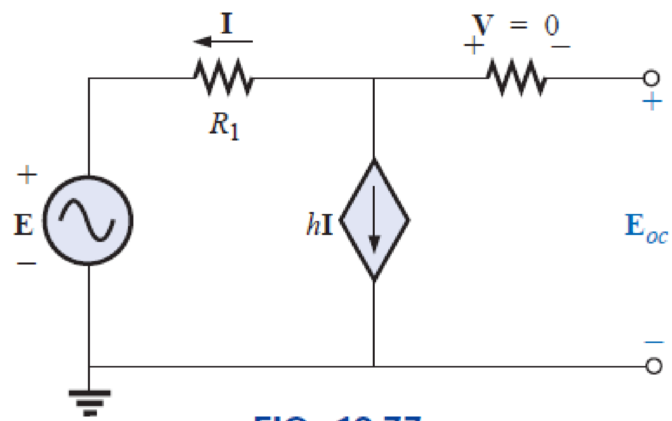


FIG. 18.77

Determining $\mathbf{E}_{oc}$ *for the network of Fig. 18.75.*

EXAMPLE 18.18 Find the Norton equivalent circuit for the network configuration of Fig. 18.56.

Solution: By source conversion,

$$\mathbf{I}_N = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}} = \frac{\dfrac{-k_2 R_2 \mathbf{V}_i}{R_1 - k_1 k_2 R_2}}{\dfrac{R_1 R_2}{R_1 - k_1 k_2 R_2}}$$

and

$$\mathbf{I}_N = \frac{-k_2 \mathbf{V}_i}{R_1}$$

which is $\mathbf{I}_{sc}$ as determined in Example 18.13, and

$$\mathbf{Z}_N = \mathbf{Z}_{Th} = \frac{R_2}{1 - \dfrac{k_1 k_2 R_2}{R_1}}$$

For $k_1 \cong 0$, we have

$$\mathbf{Z}_N = R_2 \quad \text{and} \quad \mathbf{I}_N = \frac{-k_2 \mathbf{V}_i}{R_1} \qquad k_1 = 0$$

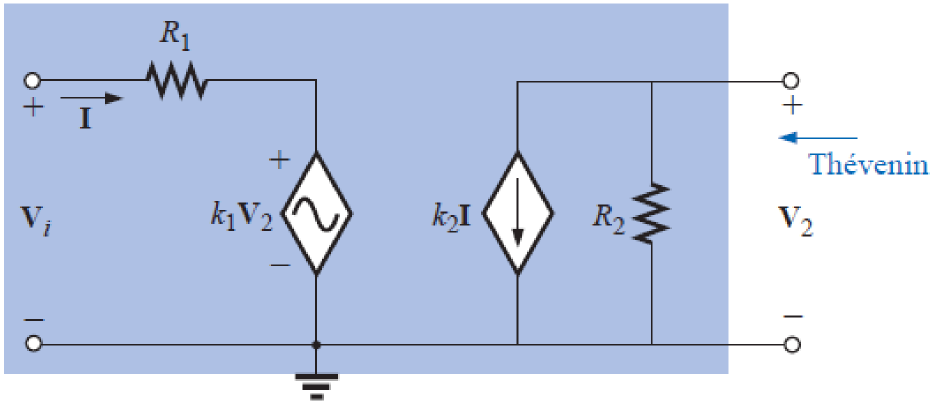


FIG. 18.56
Example 18.13: Transistor equivalent network.

18.5 MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that

Maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.

$$Z_L = Z_{Th} \quad \text{and} \quad \theta_L = -\theta_{Th_Z}$$

$$R_L = R_{Th} \quad \text{and} \quad \pm j\, X_{\text{load}} = \mp j\, X_{Th}$$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive, as indicated in Fig. 18.80:

$$\mathbf{Z}_T = (R \pm j\,X) + (R \mp j\,X)$$

$$\boxed{\mathbf{Z}_T = 2R}$$

Since the circuit is purely resistive: The power factor under maximum power transfer is 1:

$$\boxed{F_p = 1}$$

(maximum power transfer)

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2R}$$

$$P_{\max} = I^2 R = \left(\frac{E_{Th}}{2R}\right)^2 R$$

$$\boxed{P_{\max} = \frac{E_{Th}^2}{4R}}$$

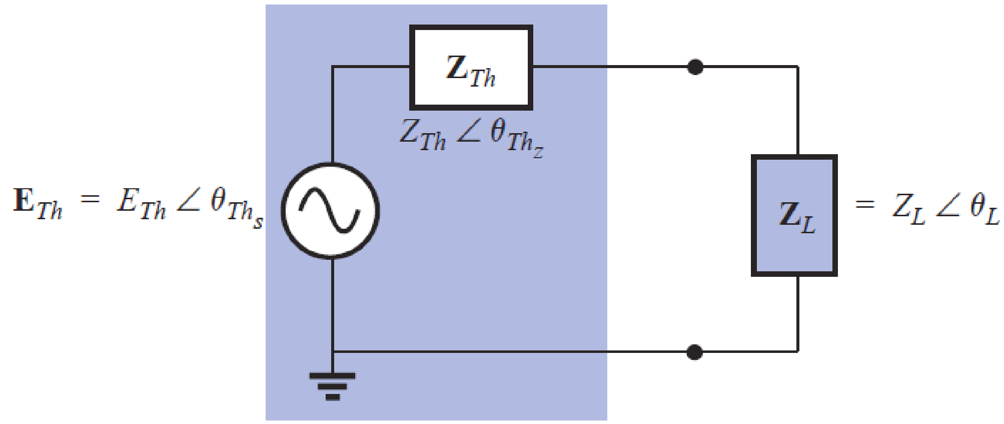


FIG. 18.79
Defining the conditions for maximum power transfer to a load.

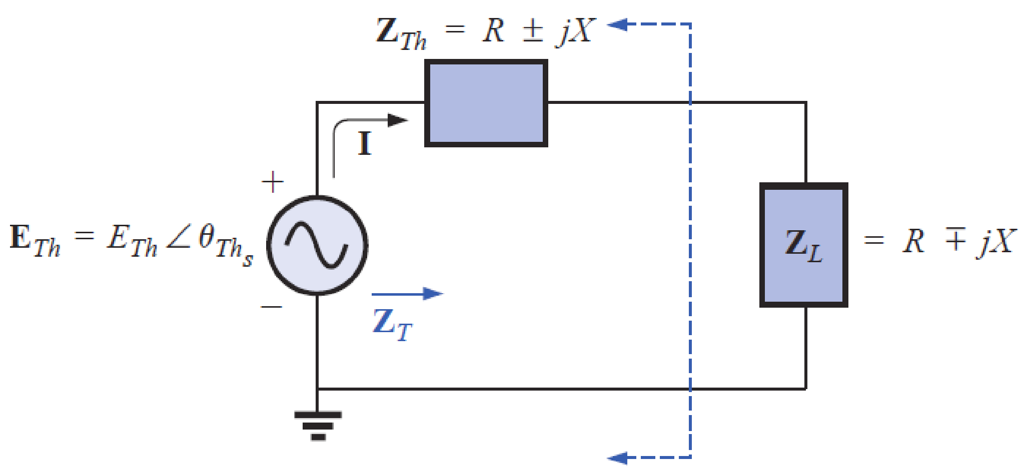


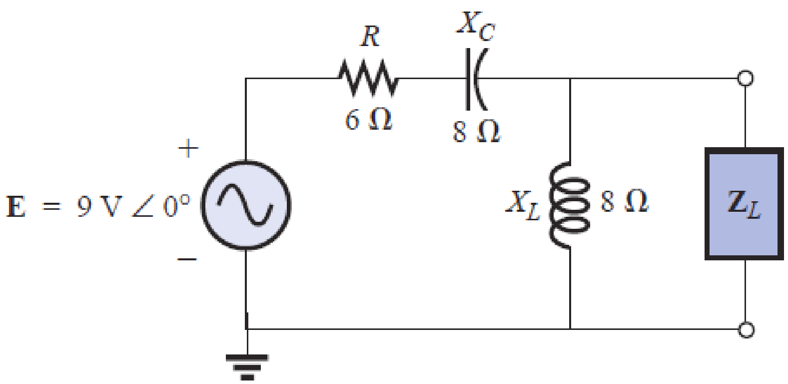
FIG. 18.80
Conditions for maximum power transfer to $\mathbf{Z}_L$.

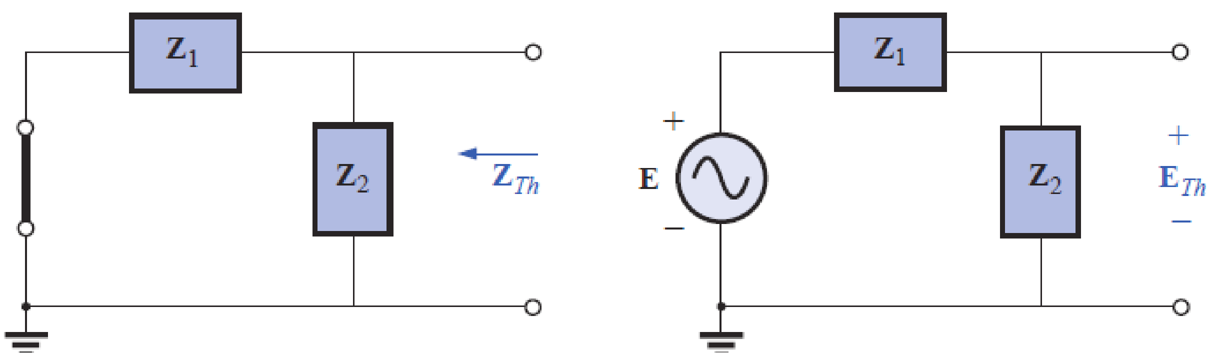
EXAMPLE 18.19 Find the load impedance in Fig. 18.81 for maximum power to the load, and find the maximum power.

Solution: Determine $\mathbf{Z}_{Th}$ [Fig. 18.82(a)]:

$$\mathbf{Z}_1 = R - j\,X_C = 6\ \Omega - j\,8\ \Omega = 10\ \Omega\ \angle{-53.13^\circ}$$

$$\mathbf{Z}_2 = +j\,X_L = j\,8\ \Omega$$





(a) (b)

FIG. 18.82

Determining (a) $\mathbf{Z}_{Th}$ and (b) $\mathbf{E}_{Th}$ for the network external to the load in

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13°)(8\ \Omega\ \angle 90°)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = \frac{80\ \Omega\ \angle 36.87°}{6\ \angle 0°}$$

$$= 13.33\ \Omega\ \angle 36.87° = 10.66\ \Omega + j\,8\ \Omega$$

and $$\mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87° = \mathbf{10.66\ \Omega - j\,8\ \Omega}$$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.82(b)], as follows:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(8\ \Omega\ \angle 90°)(9\ \text{V}\ \angle 0°)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90°}{6\ \angle 0°} = 12\ \text{V}\ \angle 90°$$

Then $$P_{\max} = \frac{E_{Th}^2}{4R} = \frac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \frac{144}{42.64} = \mathbf{3.38\ W}$$

EXAMPLE 18.20 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

Solution: First we must find $\mathbf{Z}_{Th}$ (Fig. 18.84).

$$\mathbf{Z}_1 = +j\,X_L = j\,9\,\Omega \qquad \mathbf{Z}_2 = R = 8\,\Omega$$

Converting from a Δ to a Y (Fig. 18.85), we have

$$\mathbf{Z'}_1 = \frac{\mathbf{Z}_1}{3} = j\,3\,\Omega \qquad \mathbf{Z}_2 = 8\,\Omega$$

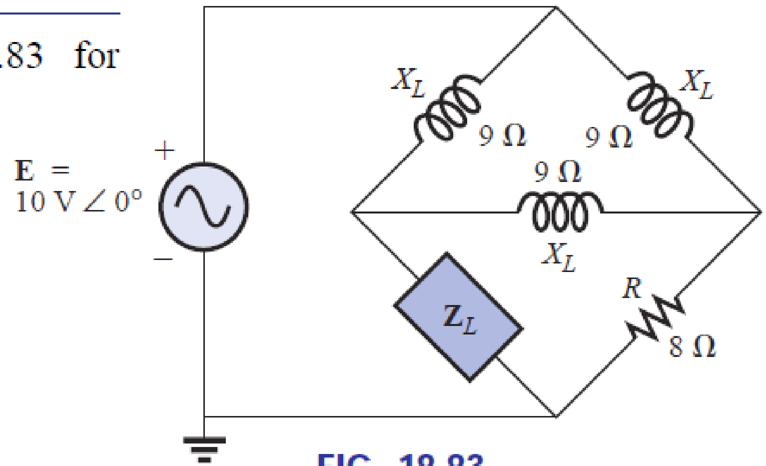


FIG. 18.83
Example 18.20.

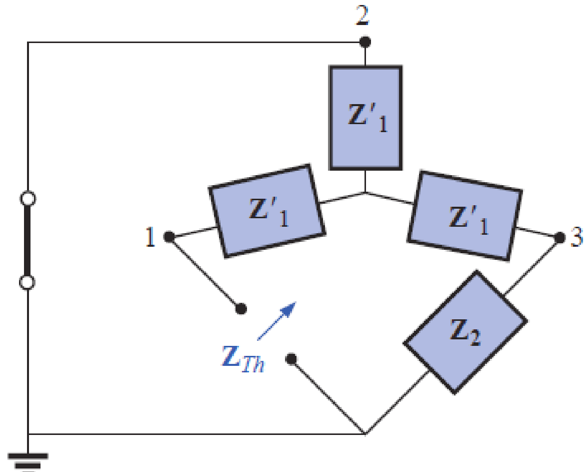
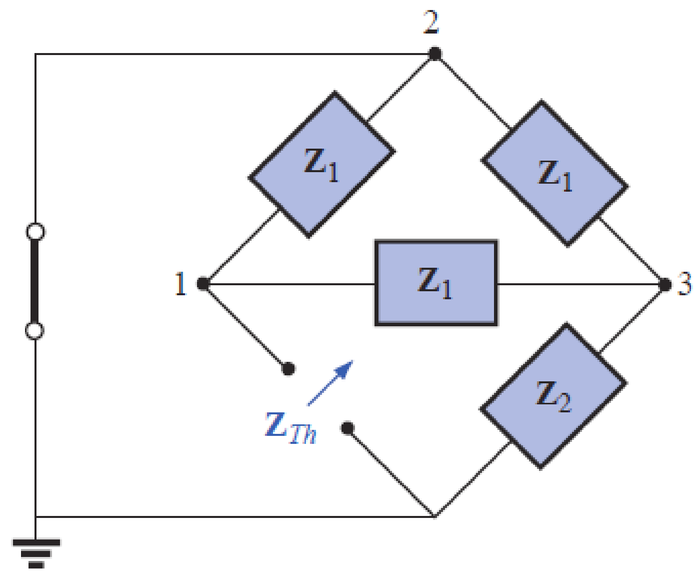


FIG. 18.85
Substituting the Y equivalent for the upper Δ configuration of Fig. 18.84.



The redrawn circuit (Fig. 18.86) shows

$$\mathbf{Z}_{Th} = \mathbf{Z}'_1 + \frac{\mathbf{Z}'_1(\mathbf{Z}'_1 + \mathbf{Z}_2)}{\mathbf{Z}'_1 + (\mathbf{Z}'_1 + \mathbf{Z}_2)}$$

$$= j\,3\ \Omega + \frac{3\ \Omega\ \angle 90^\circ(j\,3\ \Omega + 8\ \Omega)}{j\,6\ \Omega + 8\ \Omega}$$

$$= j\,3 + \frac{(3\ \angle 90^\circ)(8.54\ \angle 20.56^\circ)}{10\ \angle 36.87^\circ}$$

$$= j\,3 + \frac{25.62\ \angle 110.56^\circ}{10\ \angle 36.87^\circ} = j\,3 + 2.56\ \angle 73.69^\circ$$

$$= j\,3 + 0.72 + j\,2.46$$

$$\mathbf{Z}_{Th} = 0.72\ \Omega + j\,5.46\ \Omega$$

and $$\mathbf{Z}_L = \mathbf{0.72\ \Omega - j\,5.46\ \Omega}$$

For $\mathbf{E}_{Th}$, use the modified circuit of Fig. 18.87 with the voltage source replaced in its original position. Since $I_1 = 0$, $\mathbf{E}_{Th}$ is the voltage across the series impedance of $\mathbf{Z}'_1$ and $\mathbf{Z}_2$. Using the voltage divider rule gives us

$$\mathbf{E}_{Th} = \frac{(\mathbf{Z}'_1 + \mathbf{Z}_2)\mathbf{E}}{\mathbf{Z}'_1 + \mathbf{Z}_2 + \mathbf{Z}'_1} = \frac{(j\,3\ \Omega + 8\ \Omega)(10\ \text{V}\ \angle 0^\circ)}{8\ \Omega + j\,6\ \Omega}$$

$$= \frac{(8.54\ \angle 20.56^\circ)(10\ \text{V}\ \angle 0^\circ)}{10\ \angle 36.87^\circ}$$

$$\mathbf{E}_{Th} = 8.54\ \text{V}\ \angle -16.31^\circ$$

$$P_{max} = \frac{E_{Th}^2}{4R} = \frac{(8.54\ \text{V})^2}{4(0.72\ \Omega)} = \frac{72.93}{2.88}\ \text{W} = \mathbf{25.32\ W}$$