Power (ac)

19.1 INTRODUCTION

- The discussion about power in the previous chapters only included the ***average power*** delivered to ac network.
- We will examine the total power equation and introduce two additional types of power: ***apparent power*** and ***reactive power***.

The power at any instant is always defined as: $\boldsymbol{p = v \cdot i}$

When *v* and *i* are sinusoidal:

$$\boldsymbol{v = V_m \sin(\omega t + \theta)}$$
$$\boldsymbol{i = I_m \sin(\omega t)}$$

Load purely resistive $\boldsymbol{\theta = 0°}$
Load purely inductive $\boldsymbol{\theta = 90°}$
Load purely capacitive $\boldsymbol{\theta = -90°}$

Load primarily inductive: $\boldsymbol{\theta} > 0°$ (*v* leads *i*)
Load primarily capacitive: $\boldsymbol{\theta} < 0°$ (*i* leads *v*)

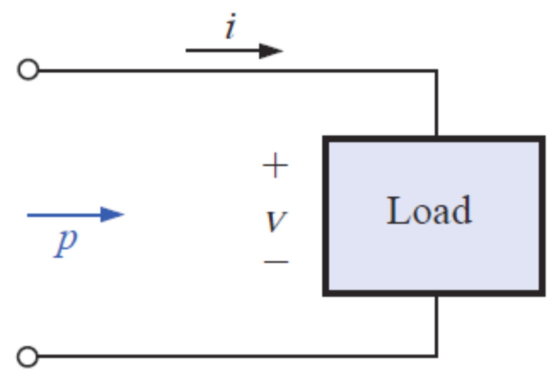


FIG. 19.1
Defining the power delivered to a load.

$$P = V_m I_m \, sin(\omega t + \theta) \, sin(\omega t)$$

Using the trigonometric identity: $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$ two times results in:

$$p = VI\cos\theta(1 - \cos 2\omega t) + VI\sin\theta(\sin 2\omega t)$$

Where: V and I are the rms values, $V = V_m/\sqrt{2}$ and $I = I_m/\sqrt{2}$

If Equation (19.1) is expanded to the form

$$p = \underbrace{VI\cos\theta}_{\text{Average}} - \underbrace{VI\cos\theta}_{\text{Peak}} \cos \underbrace{2\omega t}_{2x} + \underbrace{VI\sin\theta}_{\text{Peak}} \sin \underbrace{2\omega t}_{2x}$$

Three terms:

1- ***Average power***: independent of time

2- ***The other two terms***: vary at a frequency of (**2ω**)
Peak values having similar format:
($VI\cos\theta$ and $VI\sin\theta$)

19.2 RESISTIVE CIRCUIT

$\theta = 0° \implies P_R = VI(1 - \cos 2\omega t)$

$$p_R = VI - VI\cos 2\omega t$$

- VI is the average term
- $-VI\cos 2\omega t$ is a negative cosine wave with frequency twice the frequency of the voltage and current

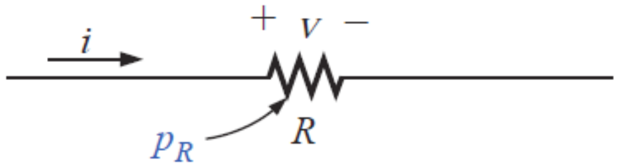


FIG. 19.2
Determining the power delivered to a purely resistive load.

T_1 = period of input quantities

T_2 = period of power curve p_R

The power curve is always positive. $\implies$

the total power delivered to a resistor will be dissipated in the form of heat.

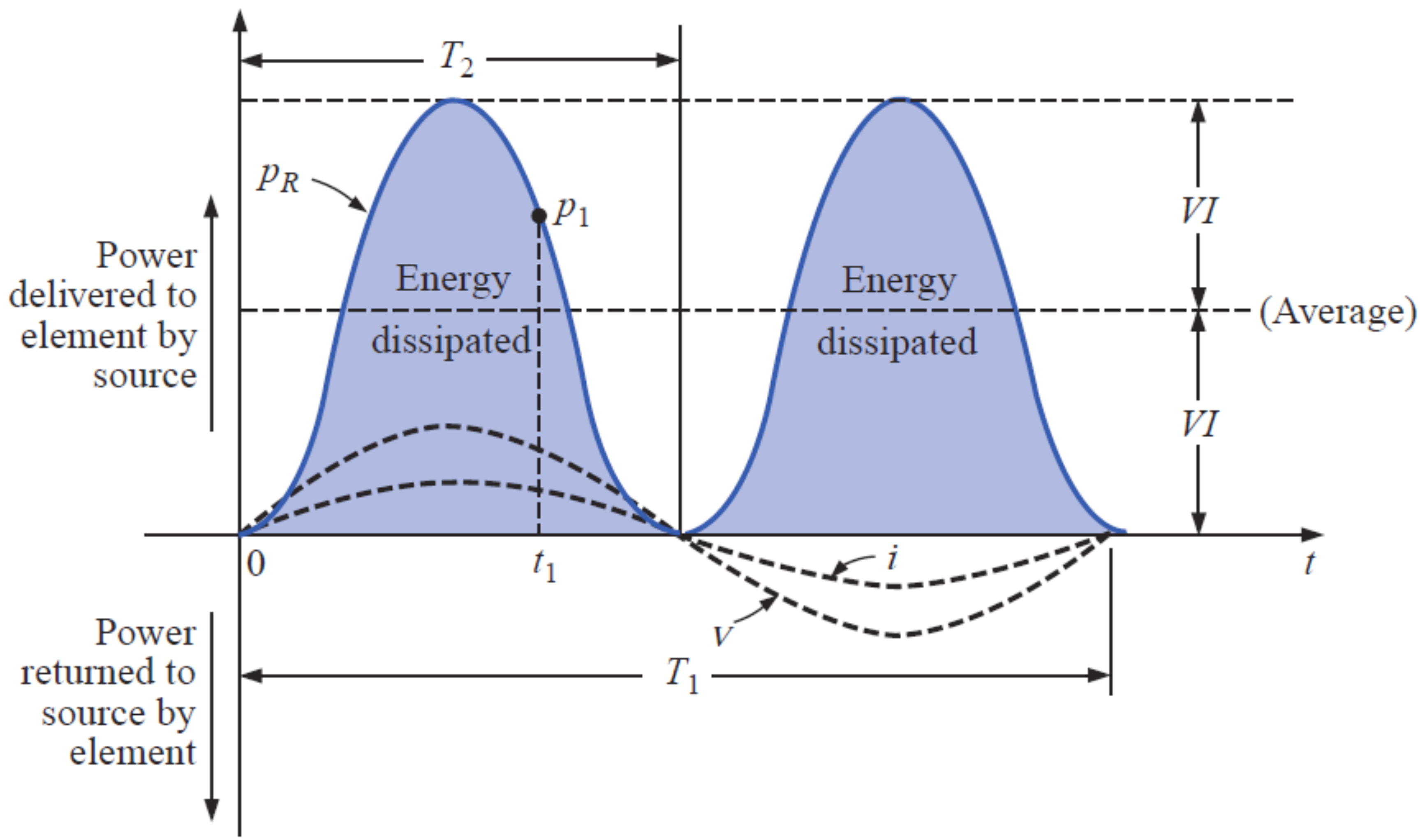


FIG. 19.3

Power versus time for a purely resistive load.

- The power returned to the source is represented by the portion of the curve below the axis, which is zero in this case.
- The power p_1 dissipated by the resistor at time t_1 can be found by simply substituting the time t_1 into the equation of the power, as indicated in Fig. 19.3.
- The **average (real) power** is *VI:*

$$P = VI = \frac{V_m I_m}{2} = I^2 R = \frac{V^2}{R} \qquad \text{(watts, W)}$$

The energy dissipated by the resistor W_R over any period of time t is:

$$W_R = Pt = VIt \qquad \text{(Joule, J)}$$

For one cycle $t = T_1$:

$$W_R = VIT_1 = \frac{VI}{f_1} \qquad \text{(Joule, J)}$$

19.3 APPARENT POWER

From what we have studied it seems ***apparent*** that the power delivered to the load is simply:

$$P = VI$$

We found: the power factor $\cos\theta$ of the load has a significant effect on the power dissipated.

The product VI is not the power delivered, but it is a useful power rating in the study of ac networks: it is called the ***apparent power*** and is represented by the symbol S.

It's unit is simply: *Volt-Ampere* (VA)

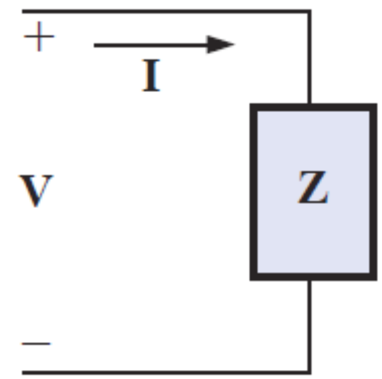


FIG. 19.4

Defining the apparent power to a load.

$$\boxed{S = VI} \quad \text{(volt-amperes, VA)}$$

or, since $$V = IZ \quad \text{and} \quad I = \frac{V}{Z}$$

$$\boxed{S = I^2 Z} \quad \text{(VA)}$$

$$\boxed{S = \frac{V^2}{Z}} \quad \text{(VA)}$$

The average power to the load of Fig. 19.4 is

$$P = VI \cos \theta$$

However, $$S = VI$$

Therefore, $$\boxed{P = S \cos \theta} \quad \text{(W)}$$

and the power factor of a system F_p is

$$F_p = \cos\theta = \frac{P}{S} \qquad \text{(unitless)}$$

For a purely resistive circuit:

$$P = VI = S$$

$$F_p = \cos\theta = \frac{P}{S} = 1$$

In general: power equipments are rated in: (***VA***) or (***kVA***)

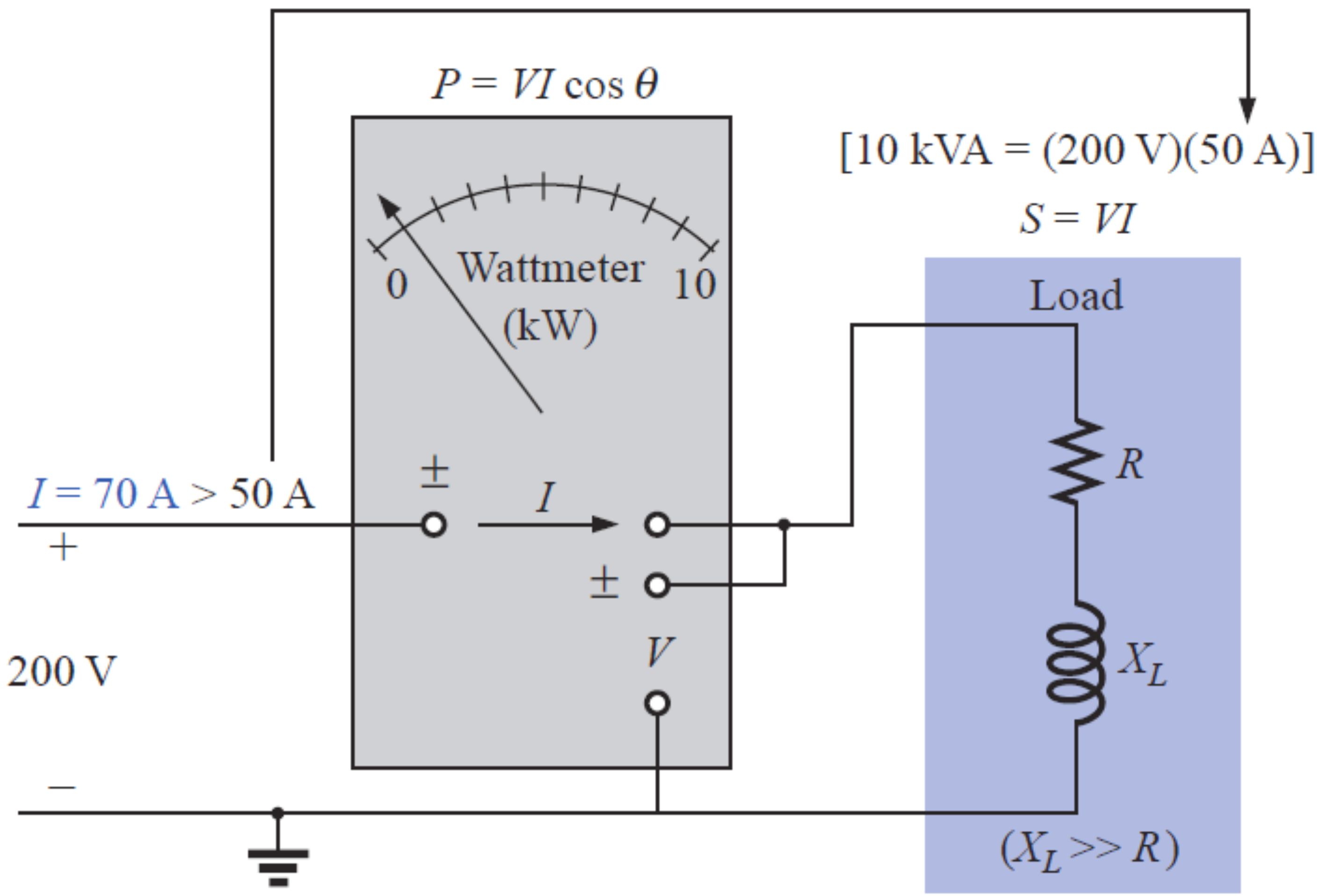


FIG. 19.5

Demonstrating the reason for rating a load in kVA rather than kW.

19.4 INDUCTIVE CIRCUIT AND REACTIVE POWER

v leads i by 90°: $\theta = 90°$, $\Longrightarrow$

$$p_L = VI\cos(90°)(1 - \cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$
$$= 0 + VI\sin 2\omega t$$

$$p_L = VI\sin 2\omega t$$

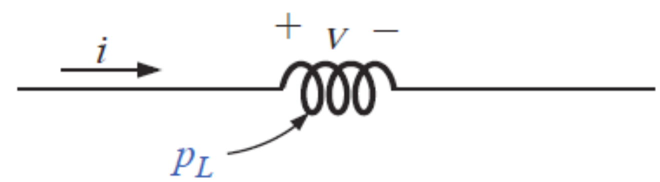


FIG. 19.6

Defining the power level for a purely inductive load

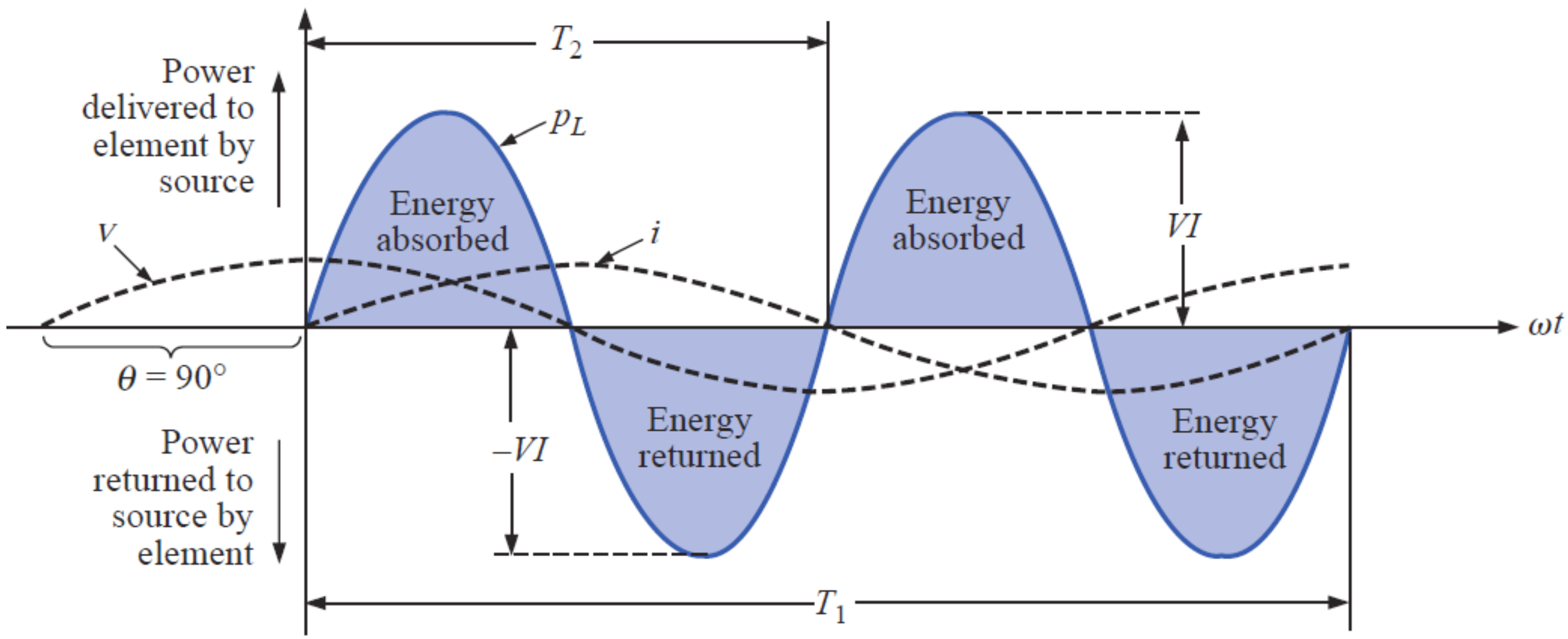


FIG. 19.7

The power curve for a purely inductive load.

- It is a sinewave with frequency twice that of the voltage or current and a peak value equal to *VI*.
- There is no average value

$$T_1 = \text{period of either input quantity}$$

$$T_2 = \text{period of } p_L \text{ curve}$$

Over one cycle: area above the horizontal axis = The area below the axis
power delivered to inductor = power returned by inductor

The net flow of power to the pure (ideal) inductor is zero over a full cycle, and no energy is lost in the transaction.

The peak value of the power curve ($V \cdot I$) is defined as the ***reactive power***.

In general: ***reactive power*** ≡ $V \cdot I \cdot \sin\theta$

A factor appearing in the general form of the power:

$$P = VI\cos\theta\,(1 - \cos 2\omega t) + VI\sin\theta\,(\sin 2\omega t)$$

The symbol of reactive power is ***Q*** unit ***V****olt-****A****mpere* ***R****eactive* (VAR)

$$\boxed{Q = VI\sin\theta}$$ (volt-ampere reactive, VAR)

Inductor: $\theta = 90° \implies$

$$Q_L = VI \quad \text{(VAR)}$$

or, since $V = IX_L$ or $I = V/X_L$,

$$Q_L = I^2 X_L \quad \text{(VAR)}$$

or

$$Q_L = \frac{V^2}{X_L} \quad \text{(VAR)}$$

Apparent power: $S = VI$

Average power: $P = VI\cos\theta = 0$

Then:

$$F_p = \cos\theta = \frac{P}{S} = \frac{0}{VI} = 0$$

The energy stored by the inductor during half-cycle is:

$$W_L = \left(\frac{2VI}{\pi}\right) \times \left(\frac{T_2}{2}\right) \qquad \boxed{W_L = \frac{VIT_2}{\pi}} \qquad \boxed{W_L = \frac{VI}{\pi f_2}} \qquad \text{(J)}$$

$$W_L = \frac{VI}{\pi(2f_1)} = \frac{VI}{\omega_1}$$

However,

$$V = IX_L = I\omega_1 L$$

so that

$$W_L = \frac{(I\omega_1 L)I}{\omega_1}$$

and

$$\boxed{W_L = LI^2} \qquad \text{(J)}$$

The energy stored or released by the inductor during half- cycle.

19.5 CAPACITIVE CIRCUIT

i leads v by 90°: $\theta = -90°, \Rightarrow$

$$p_C = VI\cos(-90°)(1 - \cos 2\omega t) + VI\sin(-90°)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

$$p_C = -VI\sin 2\omega t$$

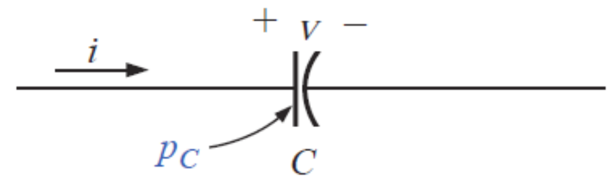
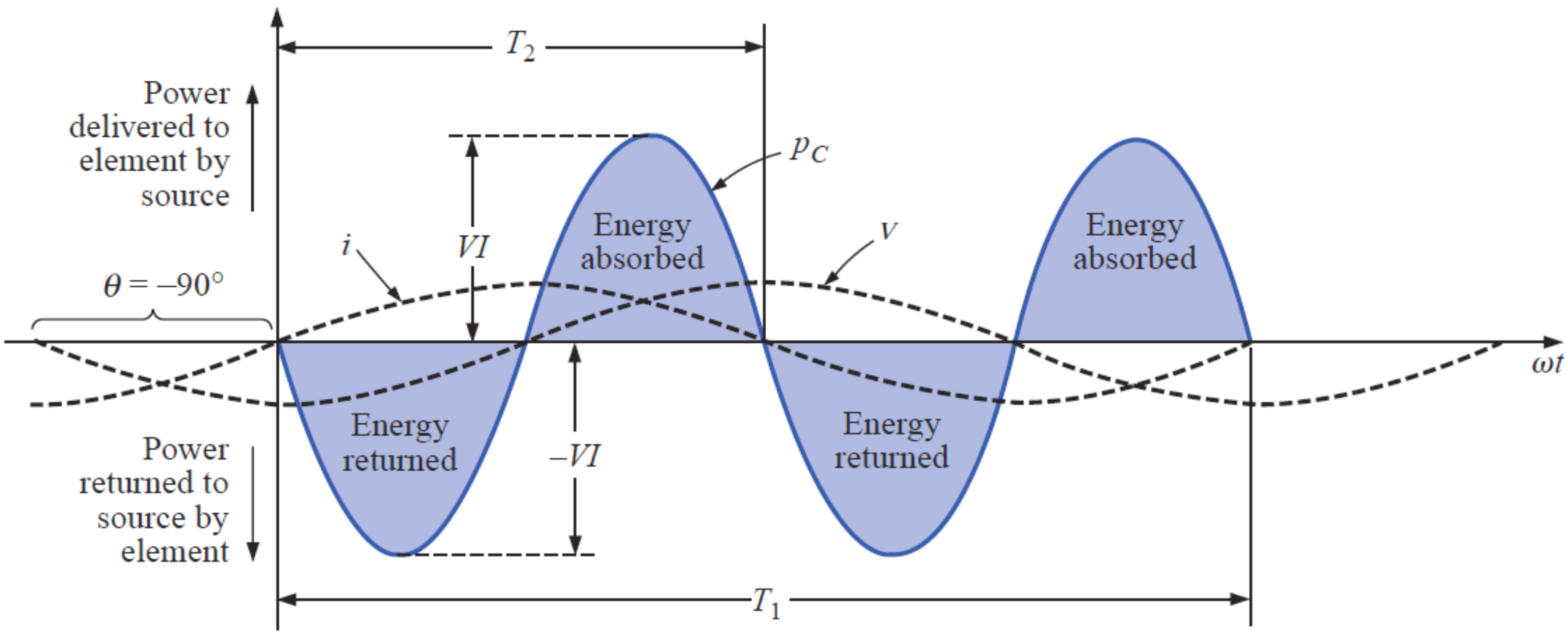


FIG. 19.8
Defining the power level for a purely capacitive load.



- It is a negative sinewave with frequency twice that of the voltage or current and a peak value equal to *VI*.
- There is no average value

$$T_1 = \text{period of either input quantity}$$
$$T_2 = \text{period of } p_L \text{ curve}$$

Over one cycle: area above the horizontal axis = The area below the axis
power delivered to Capacitor = power returned by Capacitor

The net flow of power to the pure (ideal) capacitor is zero over a full cycle, and no energy is lost in the transaction.

The reactive power associated with the capacitor is again the peak value of P_C curve

$$Q_C = VI \qquad \text{(VAR)}$$

since $V = IX_C$ and $I = V/X_C$, the reactive power

$$Q_C = I^2 X_C \qquad Q_C = \frac{V^2}{X_C} \qquad \text{(VAR)}$$

Apparent power: $S = VI$

Average power: $P = VI\cos\theta = 0$

Then:

$$F_p = \cos\theta = \frac{P}{S} = \frac{0}{VI} = 0$$

The energy stored by the capacitor during the positive half-cycle is:

$$\boxed{W_C = CV^2} \qquad \text{(J)}$$

19.6 THE POWER TRIANGLE

The three quantities ***average power***, ***apparent power***, and ***reactive power*** can be related in the vector domain by:

$$\mathbf{S} = \mathbf{P} + \mathbf{Q}$$

with

$$\mathbf{P} = P \angle 0^\circ \qquad \mathbf{Q}_L = Q_L \angle 90^\circ \qquad \mathbf{Q}_C = Q_C \angle -90^\circ$$

S is called the *phasor power*,

Inductive Load:

$$\boldsymbol{S} = P + jQ_L$$

The 90° shift in Q_L from P give another name for the reactive power: ***quadrature power***.

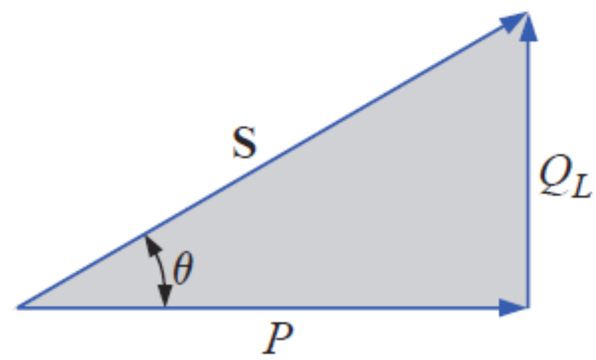
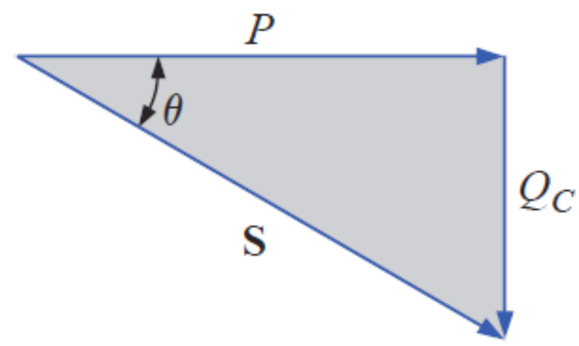


FIG. 19.10
Power diagram for inductive loads.

Capacitive Load: $$\boldsymbol{S} = P - jQ_C$$	 **FIG. 19.11** *Power diagram for capacitive loads.*

If Load has both Capacitive and inductive elements:

The reactive component of the power triangle is the difference between Q_L and Q_C:

1. $Q_L > Q_C$ Power triangle similar to the case of *inductive load*
2. $Q_C > Q_L$ Power triangle similar to the case of *capacitive load*

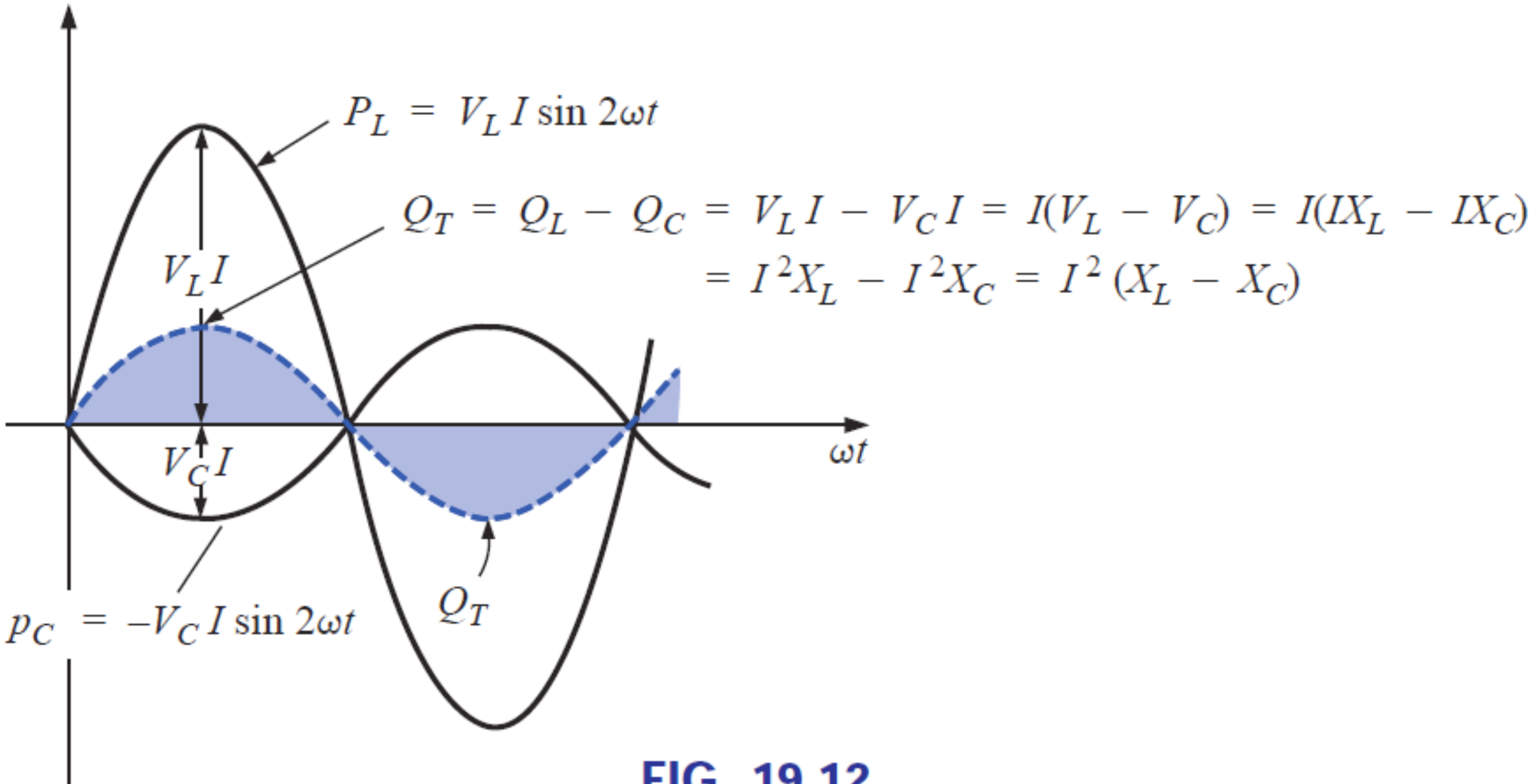


FIG. 19.12

Demonstrating why the net reactive power is the difference between that delivered to inductive and capacitive elements.

Series R-L-C Load:

If we multiply each vector in the impedance diagram by I^2:

We obtain the power triangle:

$$S^2 = P^2 + Q^2$$

It is particularly interesting that the equation

$$\mathbf{S} = \mathbf{V}\mathbf{I}^*$$

Provides the vector form of the apparent power.

V is the phasor voltage across the system
$\mathbf{I}^*$ is the complex conjugate of the phasor current

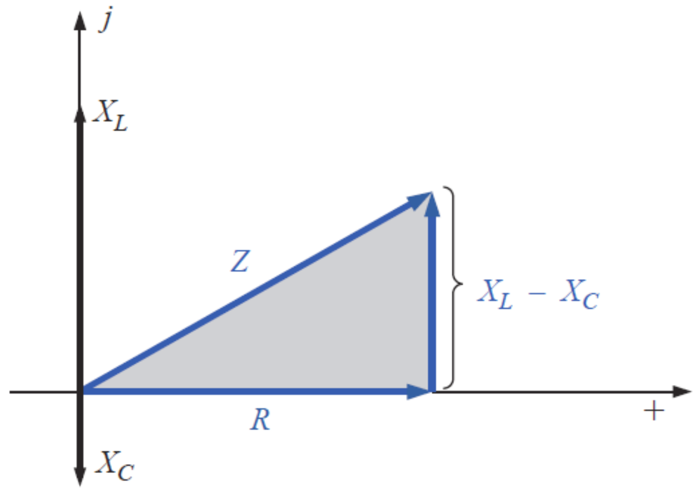


FIG. 19.13

Impedance diagram for a series R-L-C circuit.

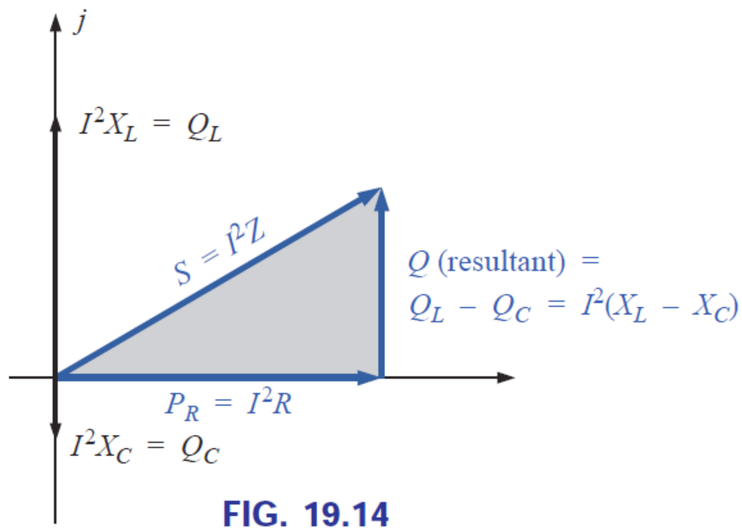


FIG. 19.14

The result of multiplying each vector of Fig. 19.13 by I^2 for a series R-L-C circuit.

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0^\circ}{3\ \Omega + j\,4\ \Omega} = \frac{10\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = 2\text{ A}\angle -53.13^\circ$$

The real power $P = I^2R = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$

the reactive power is $Q_L = I^2X_L = (2\text{ A})^2(4\ \Omega) = 16\text{ VAR }(L)$

$$\mathbf{S} = P + j\,Q_L = 12\text{ W} + j\,16\text{ VAR }(L) = 20\text{ VA}\angle 53.13^\circ$$

$$\mathbf{S} = \mathbf{VI}^* = (10\text{ V}\angle 0^\circ)(2\text{ A}\angle +53.13^\circ) = 20\text{ VA}\angle 53.13^\circ$$

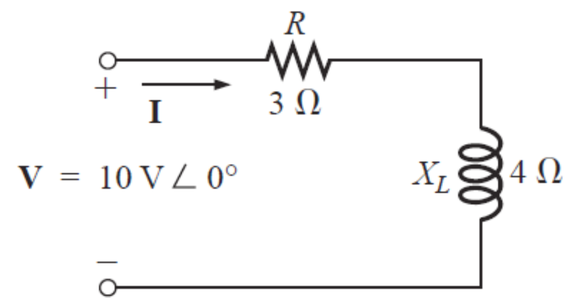
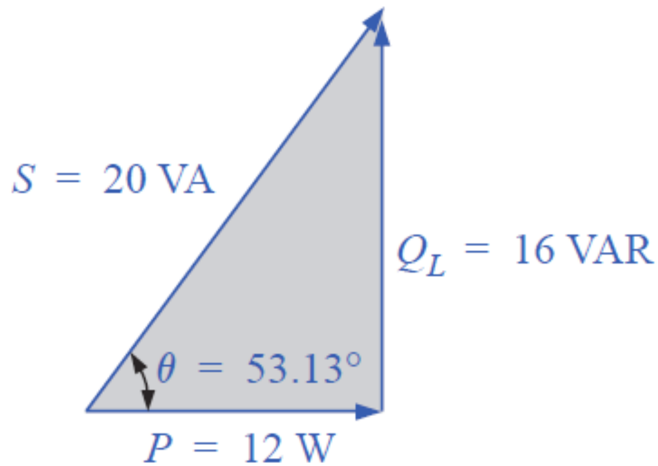


FIG. 19.15
Demonstrating the validity of Eq. (19.29).

$$P = VI\cos\theta \quad P = S\cos\theta$$

$$F_p = \cos\theta = \frac{P}{S}$$

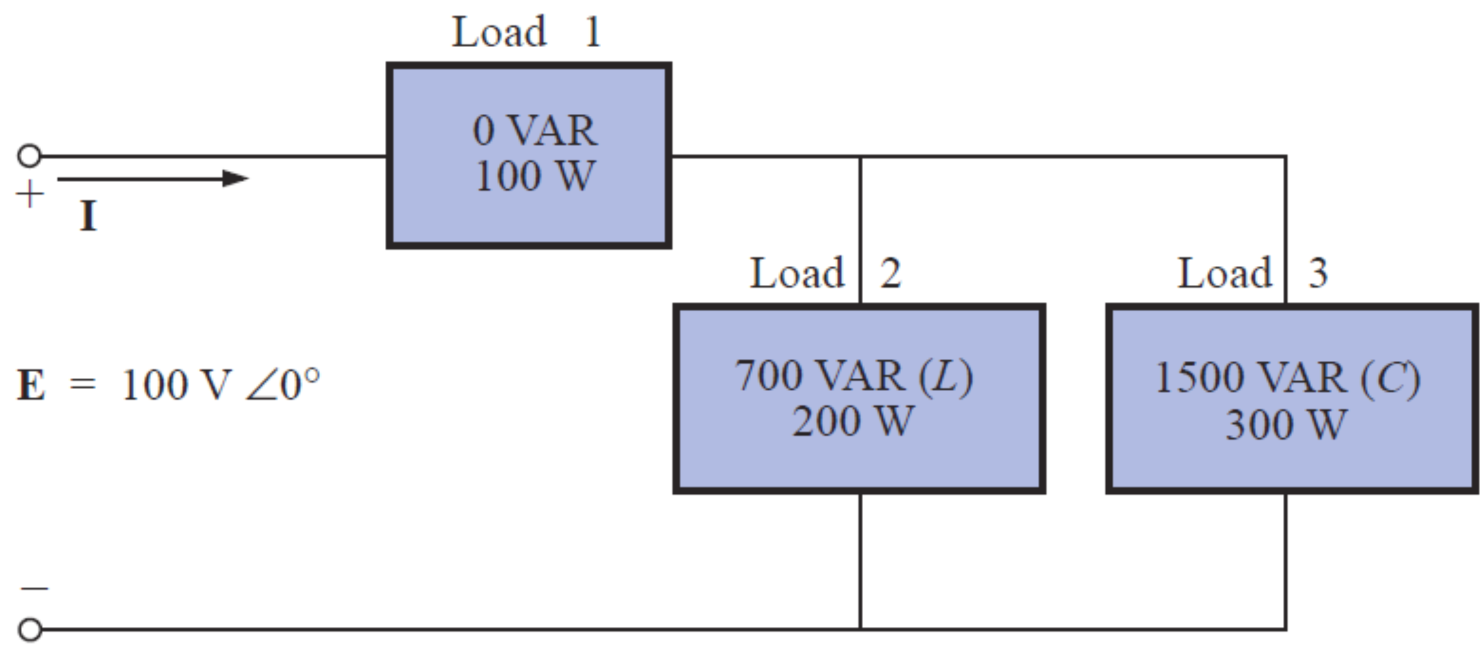


19.7 THE TOTAL P, Q, AND S

The total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of any system can be found using the following procedure:

1. ***Find the real power and reactive power for each branch of the circuit.***
2. ***The total real power of the system (P_T) is then the sum of the average power delivered to each branch.***
3. ***The total reactive power (Q_T) is the difference between the reactive power of the inductive loads and that of the capacitive loads.***
4. ***The total apparent power is*** $S_T = \sqrt{P_T^2 + Q_T^2}$.
5. ***The total power factor is*** P_T / S_T.

EXAMPLE 19.1 Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p of the network in Fig. 19.17. Draw the power triangle and find the current in phasor form.



Solution: Construct a table such as shown in Table 19.1.

TABLE 19.1

Load	W	VAR	VA
1	100	0	100
2	200	700 (L)	$\sqrt{(200)^2 + (700)^2} = 728.0$
3	300	1500 (C)	$\sqrt{(300)^2 + (1500)^2} = 1529.71$
	$P_T = \mathbf{600}$ Total power dissipated	$Q_T = \mathbf{800\ (C)}$ Resultant reactive power of network	$S_T = \sqrt{(600)^2 + (800)^2} = \mathbf{1000}$ (Note that $S_T \neq$ sum of each branch: $1000 \neq 100 + 728 + 1529.71$)

Thus,

$$F_p = \frac{P_T}{S_T} = \frac{600\ \text{W}}{1000\ \text{VA}} = \mathbf{0.6\ leading\ (C)}$$

The power triangle is shown in Fig. 19.18.

Since $S_T = VI = 1000$ VA, $I = 1000$ VA/100 V $= 10$ A; and since θ of $\cos\theta = F_p$ is the angle between the input voltage and current:

$$\mathbf{I} = \mathbf{10\ A\ \angle{+53.13^\circ}}$$

The plus sign is associated with the phase angle since the circuit is predominantly capacitive.

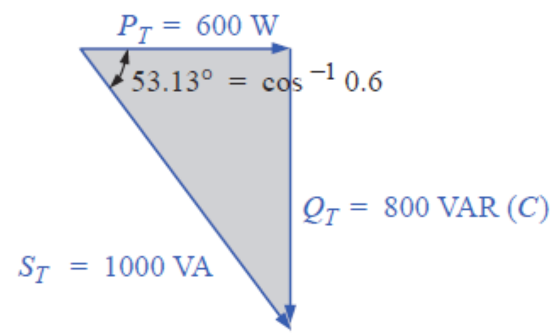


FIG. 19.18
Power triangle for Example 19.1.

EXAMPLE 19.2

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor F_p for the network of Fig. 19.19.

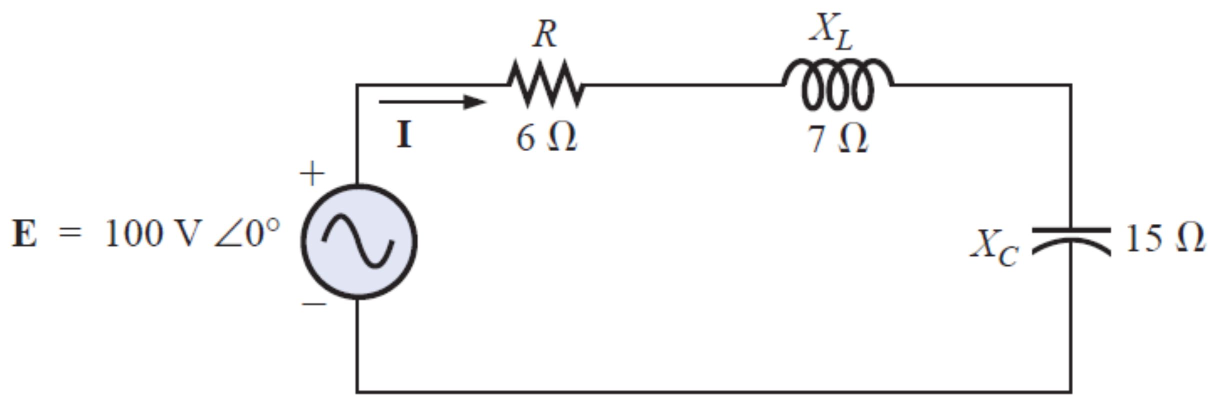


FIG. 19.19
Example 19.2.

b. Sketch the power triangle.
c. Find the energy dissipated by the resistor over one full cycle of the input voltage if the frequency of the input quantities is 60 Hz.
d. Find the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve for each if the frequency of the input quantities is 60 Hz.

Solutions:

a.
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0^\circ}{6\ \Omega + j\,7\ \Omega - j\,15\ \Omega} = \frac{100\text{ V}\angle 0^\circ}{10\ \Omega\angle -53.13^\circ}$$
$$= 10\text{ A}\angle 53.13^\circ$$
$$\mathbf{V}_R = (10\text{ A}\angle 53.13^\circ)(6\ \Omega\angle 0^\circ) = 60\text{ V}\angle 53.13^\circ$$
$$\mathbf{V}_L = (10\text{ A}\angle 53.13^\circ)(7\ \Omega\angle 90^\circ) = 70\text{ V}\angle 143.13^\circ$$
$$\mathbf{V}_C = (10\text{ A}\angle 53.13^\circ)(15\ \Omega\angle -90^\circ) = 150\text{ V}\angle -36.87^\circ$$
$$P_T = EI\cos\theta = (100\text{ V})(10\text{ A})\cos 53.13^\circ = \mathbf{600\ W}$$
$$= I^2R = (10\text{ A})^2(6\ \Omega) = \mathbf{600\ W}$$
$$= \frac{V_R^2}{R} = \frac{(60\text{ V})^2}{6} = \mathbf{600\ W}$$
$$S_T = EI = (100\text{ V})(10\text{ A}) = \mathbf{1000\ VA}$$
$$= I^2Z_T = (10\text{ A})^2(10\ \Omega) = \mathbf{1000\ VA}$$
$$= \frac{E^2}{Z_T} = \frac{(100\text{ V})^2}{10\ \Omega} = \mathbf{1000\ VA}$$
$$Q_T = EI\sin\theta = (100\text{ V})(10\text{ A})\sin 53.13^\circ = \mathbf{800\ VAR}$$
$$= Q_C - Q_L$$
$$= I^2(X_C - X_L) = (10\text{ A})^2(15\ \Omega - 7\ \Omega) = \mathbf{800\ VAR}$$

$$Q_T = \frac{V_C^2}{X_C} - \frac{V_L^2}{X_L} = \frac{(150\text{ V})^2}{15\ \Omega} - \frac{(70\text{ V})^2}{7\ \Omega}$$

$$= 1500\text{ VAR} - 700\text{ VAR} = \mathbf{800\text{ VAR}}$$

$$F_p = \frac{P_T}{S_T} = \frac{600\text{ W}}{1000\text{ VA}} = \mathbf{0.6\text{ leading }(C)}$$

b. The power triangle is as shown in Fig. 19.20.

c. $$W_R = \frac{V_R I}{f_1} = \frac{(60\text{ V})(10\text{ A})}{60\text{ Hz}} = \mathbf{10\text{ J}}$$

d. $$W_L = \frac{V_L I}{\omega_1} = \frac{(70\text{ V})(10\text{ A})}{(2\pi)(60\text{ Hz})} = \frac{700\text{ J}}{377} = \mathbf{1.86\text{ J}}$$

$$W_C = \frac{V_C I}{\omega_1} = \frac{(150\text{ V})(10\text{ A})}{377\text{ rad/s}} = \frac{1500\text{ J}}{377} = \mathbf{3.98\text{ J}}$$

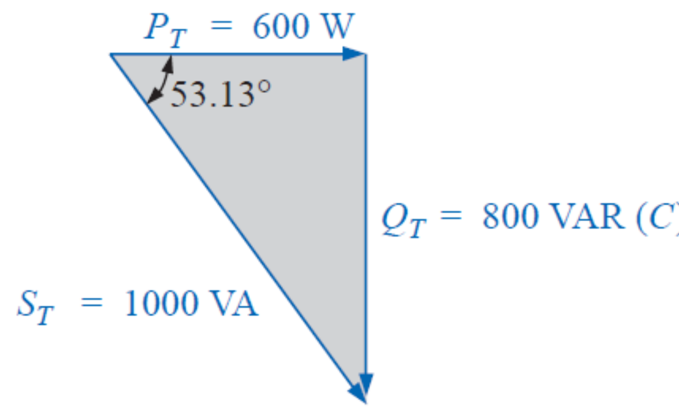


FIG. 19.20
Power triangle for Example 19.2.

EXAMPLE 19.3 For the system of Fig. 19.21,

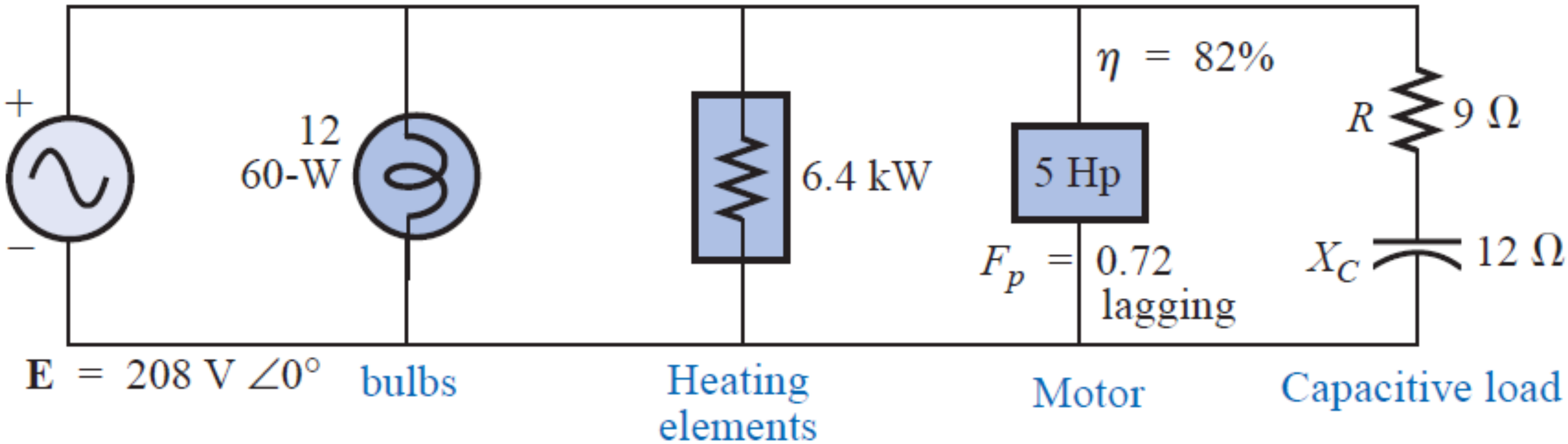


FIG. 19.21
Example 19.3.

a. Find the average power, apparent power, reactive power, and F_p for each branch.
b. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of the system. Sketch the power triangle.
c. Find the source current *I*.

Solutions:

a. *Bulbs:*

Total dissipation of applied power

$$P_1 = 12(60\text{ W}) = \mathbf{720\ W}$$
$$Q_1 = \mathbf{0\ VAR}$$
$$S_1 = P_1 = \mathbf{720\ VA}$$
$$F_{p_1} = \mathbf{1}$$

Heating elements:

Total dissipation of applied power

$$P_2 = \mathbf{6.4\ kW}$$
$$Q_2 = \mathbf{0\ VAR}$$
$$S_2 = P_2 = \mathbf{6.4\ kVA}$$
$$F_{p_2} = \mathbf{1}$$

Motor:

$$\eta = \frac{P_o}{P_i} \longrightarrow P_i = \frac{P_o}{\eta} = \frac{5(746\text{ W})}{0.82} = \mathbf{4548.78\ W} = P_3$$

$$F_p = \mathbf{0.72\ lagging}$$

$$P_3 = S_3 \cos\theta \longrightarrow S_3 = \frac{P_3}{\cos\theta} = \frac{4548.78\text{ W}}{0.72} = \mathbf{6317.75\ VA}$$

Also, $\theta = \cos^{-1} 0.72 = 43.95°$, so that

$$Q_3 = S_3 \sin\theta = (6317.75\text{ VA})(\sin 43.95°)$$
$$= (6317.75\text{ VA})(0.694) = \mathbf{4384.71\ VAR\ (\mathit{L})}$$

Capacitive load:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{208\text{ V}\angle 0°}{9\ \Omega - j\,12\ \Omega} = \frac{208\text{ V}\angle 0°}{15\ \Omega\angle -53.13°} = 13.87\text{ A}\angle 53.13°$$

$$P_4 = I^2R = (13.87\text{ A})^2 \cdot 9\ \Omega = \mathbf{1731.39\ W}$$

$$Q_4 = I^2X_C = (13.87\text{ A})^2 \cdot 12\ \Omega = \mathbf{2308.52\ VAR\ (\mathit{C})}$$

$$S_4 = \sqrt{P_4^2 + Q_4^2} = \sqrt{(1731.39\text{ W})^2 + (2308.52\text{ VAR})^2}$$
$$= \mathbf{2885.65\ VA}$$

$$F_p = \frac{P_4}{S_4} = \frac{1731.39\text{ W}}{2885.65\text{ VA}} = \mathbf{0.6\ leading}$$

b. $P_T = P_1 + P_2 + P_3 + P_4$

$= 720\text{ W} + 6400\text{ W} + 4548.78\text{ W} + 1731.39\text{ W}$

$= \mathbf{13{,}400.17\ W}$

$Q_T = \pm Q_1 \pm Q_2 \pm Q_3 \pm Q_4$

$= 0 + 0 + 4384.71\text{ VAR }(L) - 2308.52\text{ VAR }(C)$

$= \mathbf{2076.19\ VAR\ (}\boldsymbol{L}\mathbf{)}$

$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}400.17\text{ W})^2 + (2076.19\text{ VAR})^2}$

$= 13{,}560.06\text{ VA}$

$F_p = \dfrac{P_T}{S_T} = \dfrac{13.4\text{ kW}}{13{,}560.06\text{ VA}} = \mathbf{0.988\ lagging}$

$\theta = \cos^{-1} 0.988 = 8.89°$

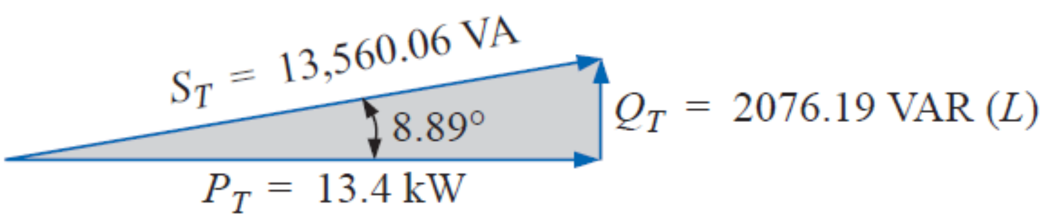


FIG. 19.22
Power triangle for Example 19.3.

c. $S_T = EI \longrightarrow I = \dfrac{S_T}{E} = \dfrac{13{,}559.89\ \text{VA}}{208\ \text{V}} = 65.19\ \text{A}$

Lagging power factor: **E** leads **I** by 8.89°, and

$$\mathbf{I} = \mathbf{65.19\ A} \angle \mathbf{-8.89^\circ}$$

19.8 POWER FACTOR CORRECTION

In power transmission system we need to minimize the magnitude of the current:

- Minimize power losses in the lines ($P = I^2R$)
- Large current require large conductors $\Longrightarrow$ more copper

Since the line voltage of a system is fixed $\Longrightarrow$ the apparent power is related to the current level

Smaller apparent power $\Longrightarrow$ smaller current drawn from the supply

➡ Minimum current is drawn when: $S = P$ and $Q_T = 0$

Note the effect of decreasing levels of Q_T on the length (magnitude) of S. For the same real power.

The power factor angle: $\theta \longrightarrow 0$

The power factor: $F_P \longrightarrow 1$

The network becoming more and more resistive at the input terminal.

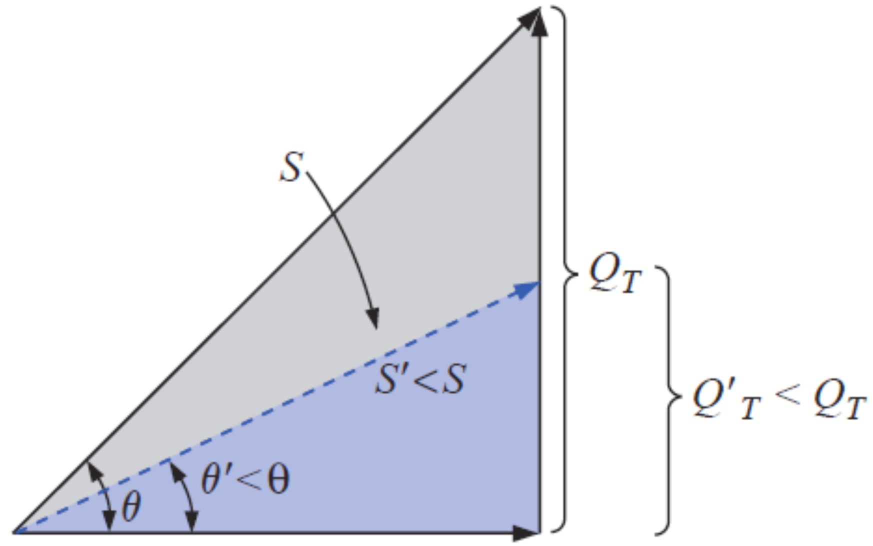


FIG. 19.24
Demonstrating the impact of power-factor correction on the power triangle of a network.

The process of introducing reactive element to bring the power factor closer to unity is called ***power-factor correction***.

Most loads are inductive $\Longrightarrow$ the process involve introducing capacitive elements.

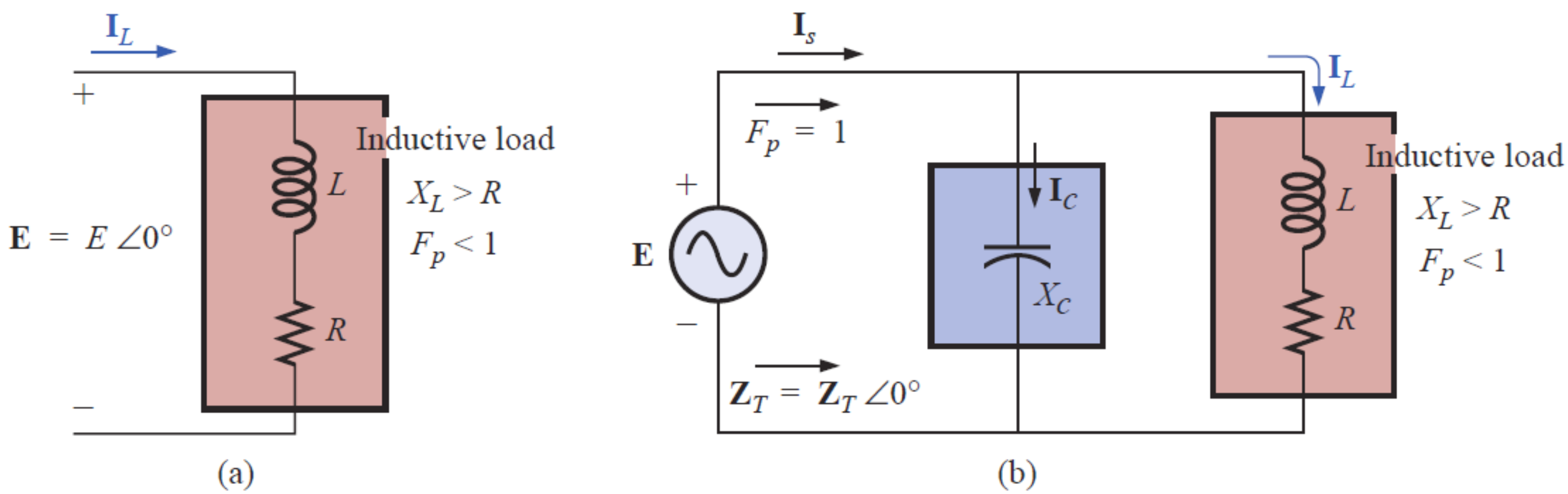


FIG. 19.25

Demonstrating the impact of a capacitive element on the power factor of a network.

In the two circuits the Inductive load receive the same current in both cases: there is no difference for the load.

Solving for the source current in Fig. 19.25(b):

$$\begin{aligned}\mathbf{I}_s &= \mathbf{I}_C + \mathbf{I}_L \\ &= j\,I_C(I_{\text{mag}}) + I_L(R_e) + j\,I_L(I_{\text{mag}}) \\ &= I_L(R_e) + j\,[I_L(I_{\text{mag}}) + I_C(I_{\text{mag}})]\end{aligned}$$

If X_C is chosen such that $|I_C(I_{mag})| = |I_L(I_{mag})|$ Then:

$$\mathbf{I}_s = I_L(R_e) + j\,(0) = I_L(R_e)\angle 0°$$

EXAMPLE 19.5 A 5-hp motor with a 0.6 lagging power factor and an efficiency of 92% is connected to a 208-V, 60-Hz supply.

a. Establish the power triangle for the load.
b. Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.
c. Determine the change in supply current from the uncompensated to the compensated system.
d. Find the network equivalent of the above, and verify the conclusions.

Solutions:

a. Since 1 hp = 746 W,

$$P_o = 5 \text{ hp} = 5(746 \text{ W}) = 3730 \text{ W}$$

and P_i (drawn from the line) $= \dfrac{P_o}{\eta} = \dfrac{3730 \text{ W}}{0.92} = 4054.35 \text{ W}$

Also, $F_P = \cos\theta = 0.6$

and $\theta = \cos^{-1} 0.6 = 53.13°$

Applying $\tan\theta = \dfrac{Q_L}{P_i}$

we obtain $Q_L = P_i \tan\theta = (4054.35 \text{ W}) \tan 53.13° = 5405.8 \text{ VAR } (L)$

and

$$S = \sqrt{P_i^2 + Q_L^2} = \sqrt{(4054.35 \text{ W})^2 + (5405.8 \text{ VAR})^2} = 6757.25 \text{ VA}$$

The power triangle appears in Fig. 19.26.

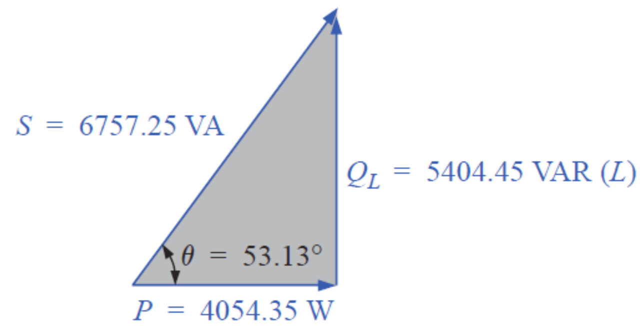


FIG. 19.26
Initial power triangle for the load of Example 19.5.

b. A net unity power-factor level is established by introducing a capacitive reactive power level of 5405.8 VAR to balance Q_L. Since

$$Q_C = \frac{V^2}{X_C}$$

then $$X_C = \frac{V^2}{Q_C} = \frac{(208\ \text{V})^2}{5405.8\ \text{VAR}\ (C)} = 8\ \Omega$$

and $$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60\ \text{Hz})(8\ \Omega)} = \mathbf{331.6\ \mu F}$$

c. **At 0.6F_p,**

$$S = VI = 6757.25\ \text{VA}$$

and $$I = \frac{S}{V} = \frac{6757.25\ \text{VA}}{208\ \text{V}} = \mathbf{32.49\ A}$$

At unity F_p,

$$S = VI = 4054.35\ \text{VA}$$

and $$I = \frac{S}{V} = \frac{4054.35\ \text{VA}}{208\ \text{V}} = \mathbf{19.49\ A}$$

producing a 40% reduction in supply current.

d. For the motor, the angle by which the applied voltage leads the current is

$$\theta = \cos^{-1} 0.6 = 53.13^\circ$$

and $P = EI_m \cos\theta = 4054.35$ W, from above, so that

$$I_m = \frac{P}{E\cos\theta} = \frac{4054.35\text{ W}}{(208\text{ V})(0.6)} = \mathbf{32.49\ A} \qquad \text{(as above)}$$

resulting in $\mathbf{I}_m = 32.49\text{ A} \angle -53.13^\circ$

Therefore,

$$\mathbf{Z}_m = \frac{\mathbf{E}}{\mathbf{I}_m} = \frac{208\text{ V}\angle 0^\circ}{32.49\text{ A}\angle -53.13^\circ} = 6.4\ \Omega\angle 53.13^\circ$$

$$= 3.84\ \Omega + j\,5.12\ \Omega$$

as shown in Fig. 19.27(a).

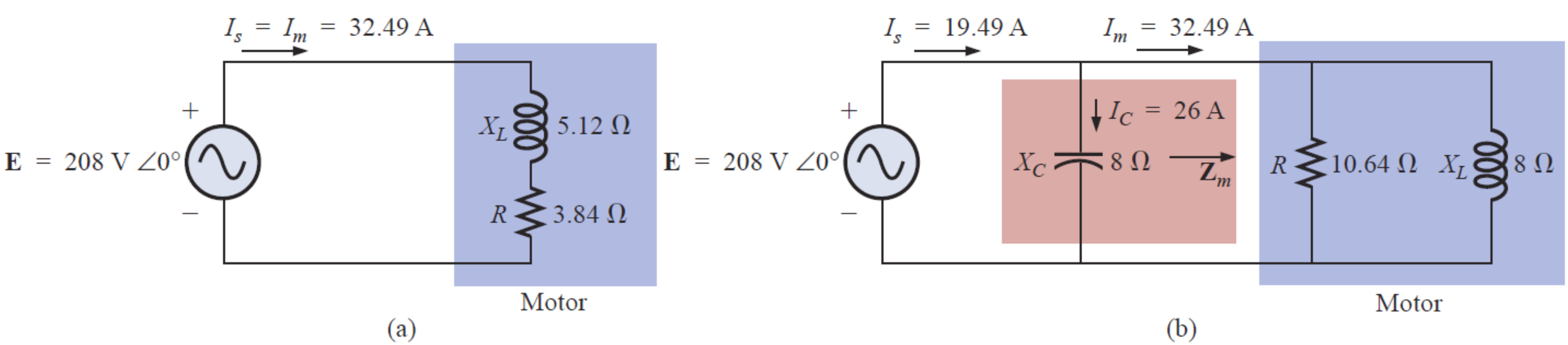


FIG. 19.27
Demonstrating the impact of power-factor corrections on the source current.

The equivalent parallel load is determined from

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{6.4\ \Omega\ \angle 53.13^\circ}$$
$$= 0.156\ \text{S}\ \angle -53.13^\circ = 0.094\ \text{S} - j\,0.125\ \text{S}$$
$$= \frac{1}{10.64\ \Omega} + \frac{1}{j\,8\ \Omega}$$

as shown in Fig. 19.27(b).

It is now clear that the effect of the 8-Ω inductive reactance can be compensated for by a parallel capacitive reactance of 8 Ω using a power-factor correction capacitor of 332 μF.

Since

$$\mathbf{Y}_T = \frac{1}{-j\,X_C} + \frac{1}{R} + \frac{1}{+j\,X_L} = \frac{1}{R}$$

$$I_s = EY_T = E\left(\frac{1}{R}\right) = (208\ \text{V})\left(\frac{1}{10.64\ \Omega}\right) = \mathbf{19.54\ A} \qquad \text{as above}$$

In addition, the magnitude of the capacitive current can be determined as follows:

$$I_C = \frac{E}{X_C} = \frac{208\ \text{V}}{8\ \Omega} = \mathbf{26\ A}$$

EXAMPLE 19.6

a. A small industrial plant has a 10-kW heating load and a 20-kVA inductive load due to a bank of induction motors. The heating elements are considered purely resistive ($F_p = 1$), and the induction motors have a lagging power factor of 0.7. If the supply is 1000 V at 60 Hz, determine the capacitive element required to raise the power factor to 0.95.

b. Compare the levels of current drawn from the supply.

Solutions:

a. For the induction motors,

$$S = VI = 20 \text{ kVA}$$

$$P = S\cos\theta = (20 \times 10^3 \text{ VA})(0.7) = 14 \times 10^3 \text{ W}$$

$$\theta = \cos^{-1} 0.7 \cong 45.6°$$

and

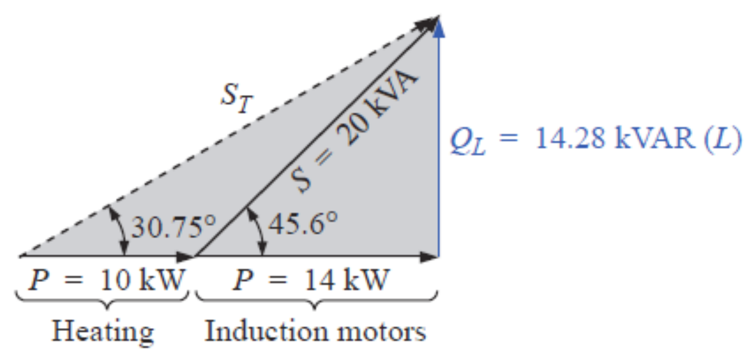
$$Q_L = VI\sin\theta = (20 \times 10^3 \text{ VA})(0.714) = 14.28 \times 10^3 \text{ VAR } (L)$$

The power triangle for the total system appears in Fig. 19.28. Note the addition of real powers and the resulting S_T:

$$S_T = \sqrt{(24 \text{ kW})^2 + (14.28 \text{ kVAR})^2} = 27.93 \text{ kVA}$$

with

$$I_T = \frac{S_T}{E} = \frac{27.93 \text{ kVA}}{1000 \text{ V}} = \mathbf{27.93 \text{ A}}$$



The desired power factor of 0.95 results in an angle between S and P of

$$\theta = \cos^{-1} 0.95 = 18.19°$$

changing the power triangle to that of Fig. 19.29:

$$\text{with } \tan\theta = \frac{Q'_L}{P_T} \longrightarrow Q'_L = P_T \tan\theta = (24 \times 10^3\ \text{W})(\tan 18.19°)$$
$$= (24 \times 10^3\ \text{W})(0.329) = 7.9\ \text{kVAR}\ (L)$$

The inductive reactive power must therefore be reduced by

$$Q_L - Q'_L = 14.28\ \text{kVAR}\ (L) - 7.9\ \text{kVAR}\ (L) = 6.38\ \text{kVAR}\ (L)$$

Therefore, $Q_C = 6.38$ kVAR, and using $Q_C = \dfrac{E^2}{X_C}$

we obtain
$$X_C = \frac{E^2}{Q_C} = \frac{(10^3\ \text{V})^2}{6.38 \times 10^3\ \text{VAR}} = 156.74\ \Omega$$

and
$$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60\ \text{Hz})(156.74\ \Omega)} = \mathbf{16.93\ \mu F}$$

b. $S_T = \sqrt{(24\ \text{kW})^2 + [7.9\ \text{kVAR}\ (L)]^2}$
$= 25.27$ kVA

$$I_T = \frac{S_T}{E} = \frac{25.27\ \text{kVA}}{1000\ \text{V}} = \mathbf{25.27\ A}$$

The new I_T is $I_T = 25.27\ \text{A} \angle 27.93\ \text{A}$ (original)

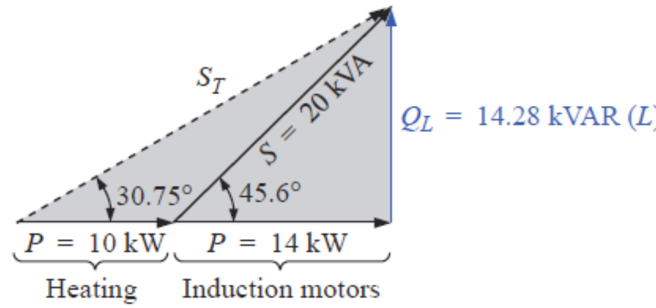


FIG. 19.28
Initial power triangle for the load of Example 19.6.

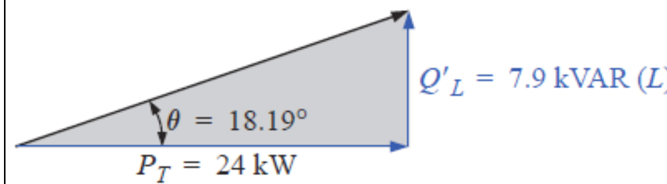


FIG. 19.29
Power triangle for the load of Example 19.6 after raising the power factor to 0.95.