$\lim_{x \to 1^+} f(x)$

The Solution of mid term exam 1, math 280
first semester 1444.

Question 1

$$\sup\left(\frac{A}{B}\right) = \frac{\sup(A)}{\operatorname{Inf}(B)}$$

For all $a \in A$ and $b \in B$

$$\left.\begin{array}{l} a \le \sup A \\ b \ge \operatorname{Inf}(B) \end{array}\right\} \Rightarrow \frac{a}{b} \le \frac{\sup A}{\operatorname{Inf}(B)}$$

Hence. $\sup\left(\frac{A}{B}\right) \le \frac{\sup(A)}{\operatorname{Inf}(B)}$.

On the other hand

$\frac{a}{b} \le \sup\left(\frac{A}{B}\right)$, for all $a \in A, b \in B$.

$a \le b \sup\left(\frac{A}{B}\right)$; for all a

$\sup(A) \le b \sup\left(\frac{A}{B}\right)$ for all $b \in B$

$\Rightarrow \sup(A) \le b \sup\left(\frac{A}{B}\right)$ for all $b \in B$

$\Rightarrow \frac{\sup(A)}{\sup\left(\frac{A}{B}\right)} \le b$,

$\Rightarrow \frac{\sup(A)}{\sup\left(\frac{A}{B}\right)} \le \operatorname{Inf}$

Therefore $\sup($

Then $\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = 1$ the right

Since

$\frac{3x - 2 - 1}{x - 1} = 3.$

from the left dérivative

So if $x = 1$.

Question 2

$$\sqrt{n+1} - \sqrt{n} = \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}} \searrow$$

Then

$$\sup A = \frac{1}{\sqrt{2}+1} \quad \text{and} \quad \inf(A) = 0$$

Question 3.

$$\frac{n - \cos(n)}{n} = 1 - \frac{\cos(n)}{n}.$$

for all $n \in \mathbb{N}^*$, $\left|\frac{\cos(n)}{n}\right| \leq \frac{1}{n}$

and $\lim_{n \to \infty} \frac{1}{n} = 0$

Then $\lim_{n \to \infty} \frac{\cos(n)}{n} = 0$

Therefore. $\lim_{n \to \infty} \frac{n - \cos(n)}{n} = 1.$

Question 4

$$\left|\frac{n^2+1}{n^2} - \frac{m^2+1}{m^2}\right| = \left|\frac{1}{n^2} - \frac{1}{m^2}\right| = \left|\frac{1}{n} + \frac{1}{m}\right| \left|\frac{1}{n} - \frac{1}{m}\right|$$

$$\leq 2 \left|\frac{1}{n} - \frac{1}{m}\right| \leq 2\left(\frac{1}{n} + \frac{1}{m}.\right)$$

$\left(\frac{1}{n}\right)$ converges to 0, By Cauchy.

for $\varepsilon > 0$, there exists $N \in \mathbb{N}$, $m, n \geq N$

$$\left|\frac{1}{n} - \frac{1}{m}\right| < \varepsilon$$

This ~~shows~~, achieving the result.