Solution of Quiz II, math 280, first semester 1441

Question 1 $g(x) = x2^x - 1$, continuous and

$g(0) = -1$ and $g(1) = 1$.
By intermediate value theorem, there exists
$x \in (0,1)$ s.t. $g(x) = 0$.

Question 2

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x} = \lim_{x \to 0} \frac{x^2 \sin\left(\frac{1}{x^2}\right)}{x}$$

$$= \lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

For $x \neq 0$
$\left| x \sin\left(\frac{1}{x}\right) \right| \leq |x|$ and $\lim_{x \to 0} |x| = 0$

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x} = 0$$

Question 3. $\lim_{x \to 1} f(x) = \frac{1}{2}$, For $\varepsilon = \frac{1}{2}$ there exists $\delta > 0$, $|x-1| < \delta \Rightarrow |f(x) - \frac{1}{2}| < \frac{1}{2}$
$\Rightarrow 0 < f(x) < \frac{3}{2}$

for all $x \in (1-\delta, 1+\delta)$.

Question 4 For $x, y \in \mathbb{R}$

$$\left| \frac{1}{1+|y|} - \frac{1}{1+|x|} \right| = \frac{\big||x| - |y|\big|}{(1+|x|)(1+|y|)}$$

$$\leq \big| |x| - |y| \big| \leq |x-y|$$

Let $\varepsilon > 0$, put $\delta = \varepsilon$, then for all $x, y \in \mathbb{R}$ s.t. $|x-y| < \delta \Rightarrow$

$$|f(x) - f(y)| < \varepsilon$$

then f is uniformly continuous on $\mathbb{R}$.

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