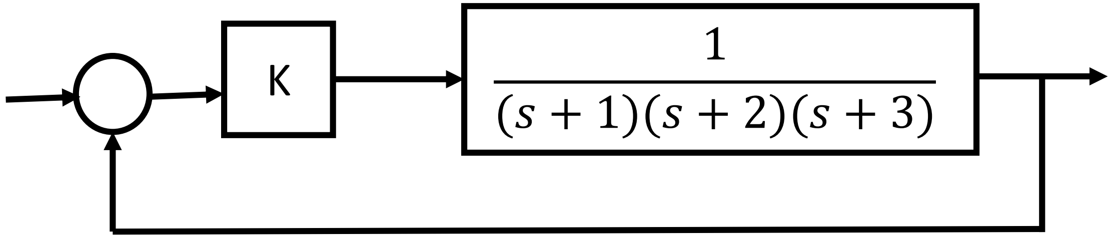
Exercise 1 (Root Locus)

Sketch the root locus for the system shown in Figure



1. Locate the open-loop poles and zeros on the complex plane
2. Find the asymptotes.
3. Determine the intersection with the $j\omega$ axis,
4. Determine the breakaway and break-in points

Solution 1_1 (Root Locus)

1. Locate the open-loop poles and zeros on the complex plane. Root loci exist on the negative real axis between -1 and between -2 and -3.
2. The number of open-loop poles is 3 and the number of finite zeros is 0. This means that we have three branches ending at infinity and there are three asymptotes in the complex region of the s plane (three infinite zeros).

The angles of asymptotes: $$\varphi_A = \pm \frac{(2q+1)\pi}{nb(poles) - nb(zeros)} \; for\; q = 0,1,2 \; \rightarrow \varphi_A = 60^o, 180^o, 300^o$$

Point of joint of asymptotes: $$\sigma_A = \frac{(poles) - (zeros)}{nb(poles) - nb(zeros)} = \frac{(-3-2-1)-(0)}{(3)-(0)} = -2$$

3. Determine the breakaway and break-in points. The characteristic equation for the system is:

$$1 + \frac{K}{(s+1)(s+2)(s+3)} = 0 \; \rightarrow K = -(s+1)(s+2)(s+3) = -[S^3 + 6S^2 + 11\,S + 6] \rightarrow \frac{dK}{ds} = -[3S^2 + 12s + 11] = 0$$

we have two roots for this equation: **-1.4226** and -2.5773 the second root is not valid (not on the root locus branch)

Solution 1_2 (Root Locus)

4. Determine the intersection with the $j\omega$ axis,
we use Routh-Hurwitz stability criterion, we have:

$$1 + \mathrm{G(s)H(s)} = 0 \rightarrow S^3 + 6S^2 + 11\,S + (6 + K) = 0$$

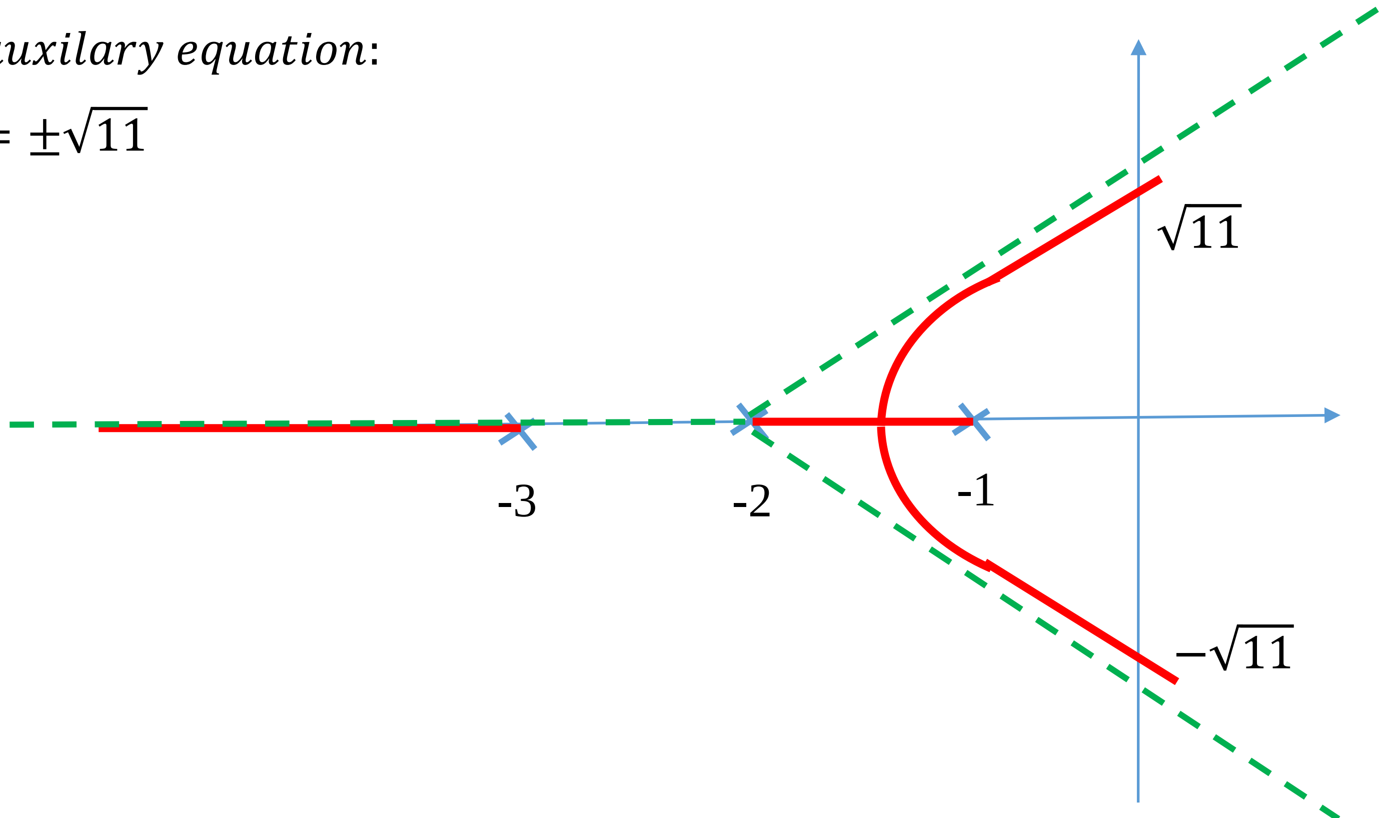
s^3	1	11	0
s^2	6	6+K	0
s^1	$\frac{66-6-K}{6}$	0	0
s^0	6+K	0	0

The line with odd power and find k to have complete zeros row

$$\frac{66-6-K}{6} = 0 \rightarrow \mathrm{K} = 60 \rightarrow \textit{the above row is auxilary equation}:$$

$$6s^2 + 66 = 0 \rightarrow s = \pm j\sqrt{11} \rightarrow \omega_{1,2} = \pm\sqrt{11}$$

- For K>60 the system will be unstable.



Solution 1_3 (Root Locus Using MATLAB)

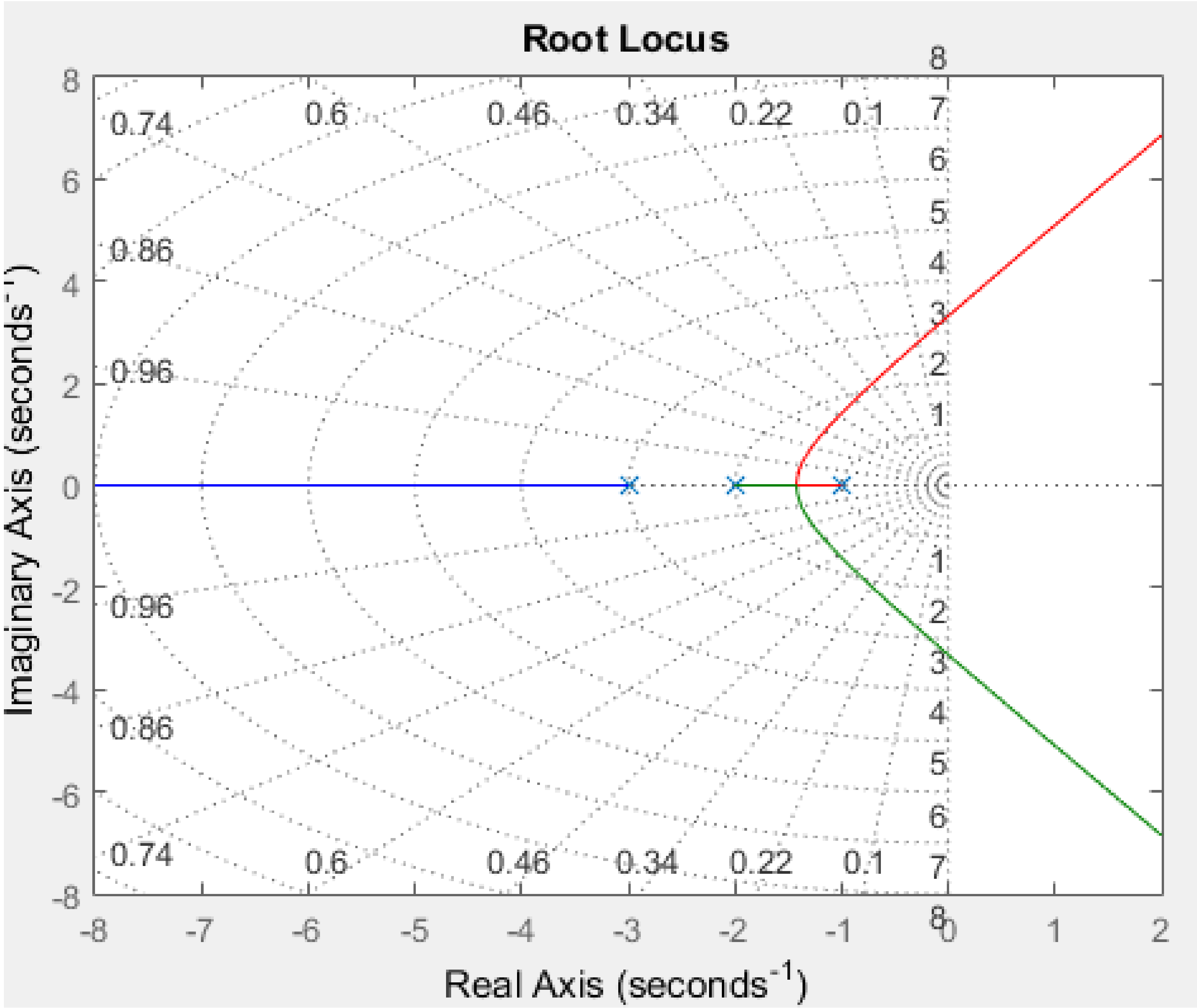
```
clear all;clc;

s=tf('s');

sys=1/((s+1)*(s+2)*(s+3)); sgrid;

rlocus(sys);grid on;

axis([-8 2 -8 8]);
```



Exercise 2 (Lag Compensator)

A unity feedback system with the forward transfer function G(s) is operating with a closed-loop step response that has 15% overshoot. Do the following:

$$G(s) = \frac{K}{s(s+7)}$$

a. Evaluate the steady-state error for a unit ramp input.

b. Design a lag compensator to improve the steady-state error by a factor of 20.

c. Evaluate the steady-state error for a unit ramp input to your compensated system.

d. Evaluate how much improvement in steady-state error was realized.

Solution 2_1 (Lag Compensator)

a. Uncompensated system analysis: The uncompensated system error. The root locus for the uncompensated system is shown in Figure. A damping ratio of 0.517 is represented by a radial line drawn on the s-plane at 121.1°.

$$15\% overshoot \rightarrow \xi = 0.517 \rightarrow poles = -3.5 \pm j5.82 \;\; with \;\; K = 45.8$$

$$K_v = \lim_{s \to 0} sG(s) = \frac{K}{7} = 6.54 \;\rightarrow e(\infty) = \frac{1}{K_v} = 0.1527$$

b. Lag compensator design

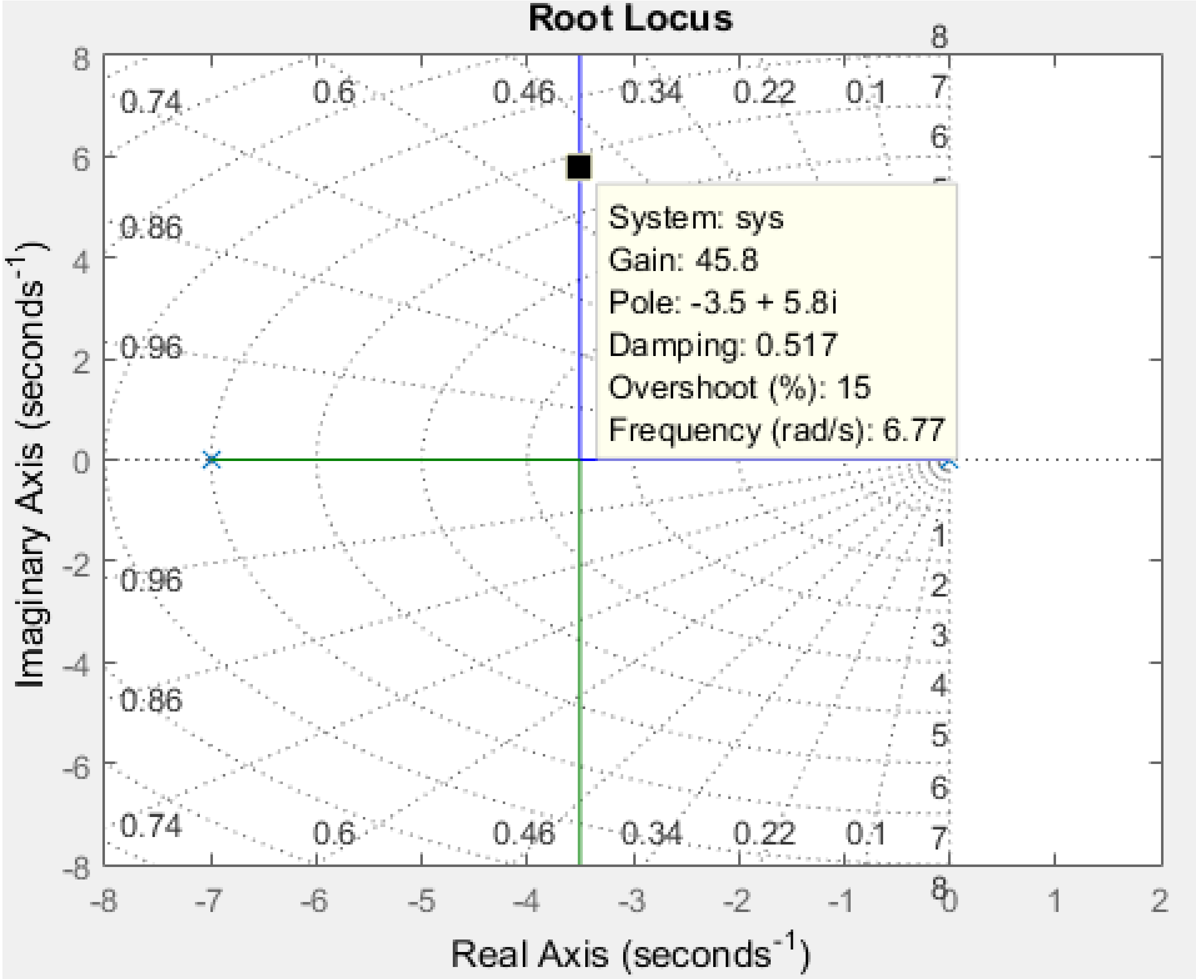
The uncompensated system error was 0.1527 with K=46.1, a factor improvement of 20:

$$e(\infty) = \frac{0.1527}{20} = 0.007635, since \;\; e(\infty) = \frac{1}{K_v} \Rightarrow K_{vN} = 130.98$$

The improvement in K_v from the uncompensated system to the compensated system is the required ratio of the compensator zero to the compensator pole:

$$\frac{z_c}{p_c} = \frac{K_{vN}}{K_v} = \frac{130.98}{6.54} = 20.03$$ Arbitrarily selecting $p_c = 0.01$

$$\Rightarrow z_c = 20.03\, p_c \approx 0.2 \Rightarrow G_{Lag}(s) = \frac{s+0.2}{s+0.01}$$



Solution 2_2 (Lag Compensator)

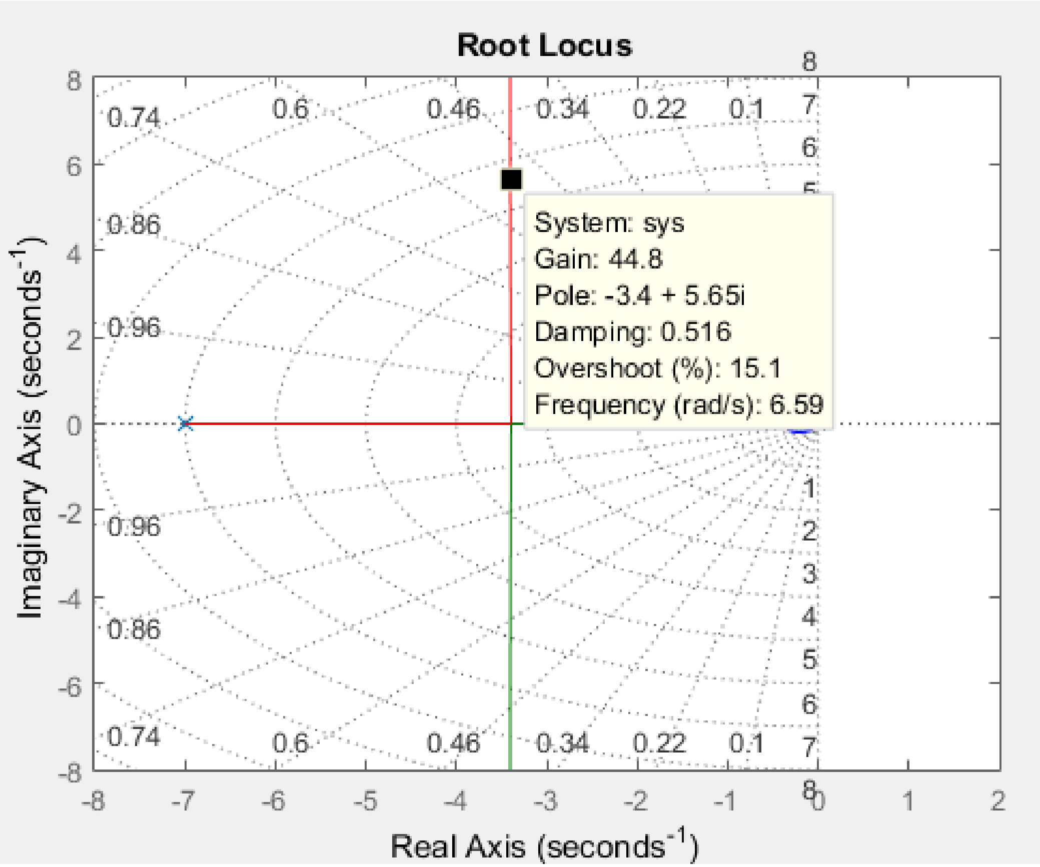
c. Error evaluation for the compensated system

$$G_N(s) = \frac{K\,(s+0.2)}{s(s+0.01)(s+7)}$$

$$15\%overshoot \rightarrow \xi = 0.517 \rightarrow poles = -3.4 \pm j5.65 \;\; with \;\; K = 44.8$$

$$K_v = \lim_{s\to 0} sG_N(s) = \frac{K\;0.2}{(7)(0.01)} = 128 \;\; \rightarrow e_N(\infty) = \frac{1}{K_v} = 0.0078$$

$$e_N(\infty) = \frac{1}{K_v} = 0.0078$$



d. Realized improvement in steady-state error

$$\frac{e(\infty)}{e_N(\infty)} = \frac{0.1527}{0.0078} = 19.58$$

Exercise 3 (Lead Compensator)

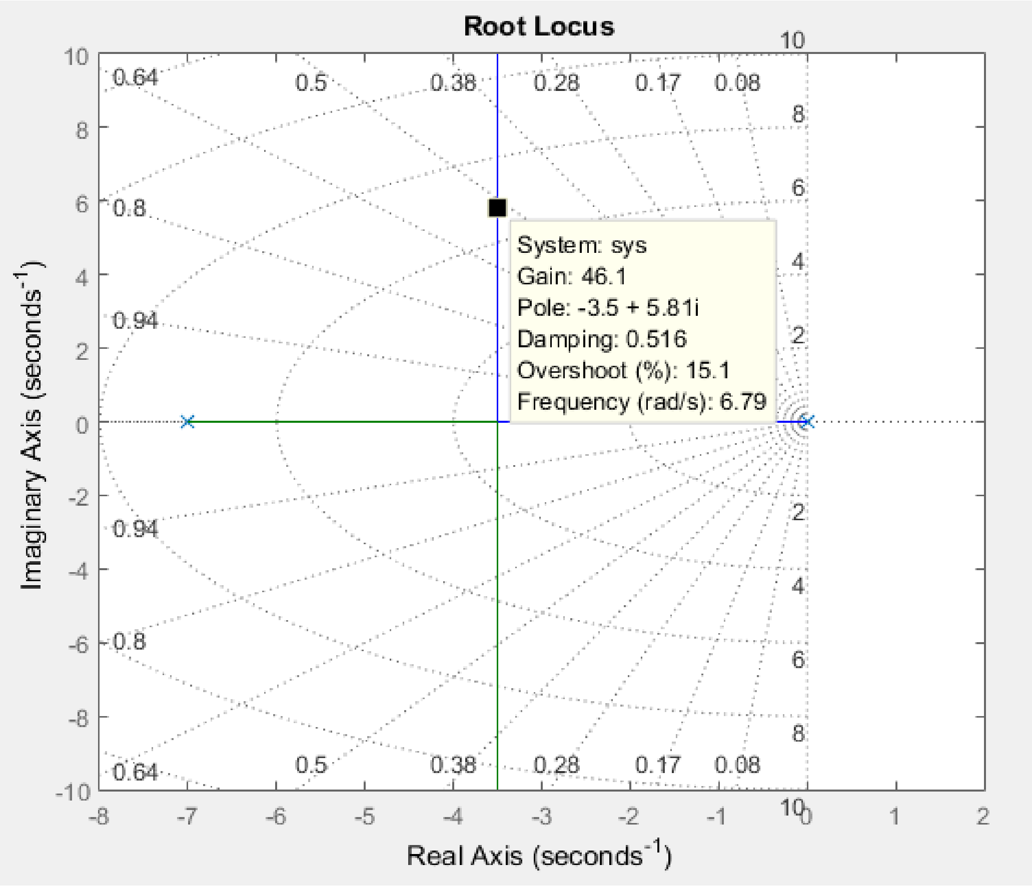
A unity feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Do the following:

a. Evaluate the settling time.
b. Design a lead compensator to decrease the settling time by three times.
Choose the compensator's zero to be at -10.

Solution 3_1 (Lead Compensator)



- **Characteristics of the uncompensated system**

System operating at 15% overshoot

15% Overshoot → (damping ratio) $\zeta = 0.517$, $\propto = 121.13^o$, → (Dominant second-order pair of poles along damping ratio line) $-3.5 \pm j5.82.$

→ (settling time, From pole's real part) $T_s = {}^4/_{3.5} = 1.143\ sec$

- Design point

→ (Threefold reduction in settling time) $T_{sN} = {}^{1.143}/_3 = 0.381\ sec$ → (real part of the desired pole location) $-\zeta\omega_n = {}^{-4}/_{T_{sN}} = -10.499$

→ (Imaginary part of the desired pole location) $\omega_d = -10.499 \tan(121.13^0) = 17.38$

Solution 3_2 (Lead Compensator)

- **Lead compensator Design.**

Place the zero on real axis at -10 (arbitrarily as possible solution).
sum the angles (this zero and uncompensated system's poles and zeros),

resulting angle → $\theta = -130.86^0$

the angular contribution required from the compensator pole → $\theta_{pc} = -180^0 + 130.86^0 = 49.14^0$

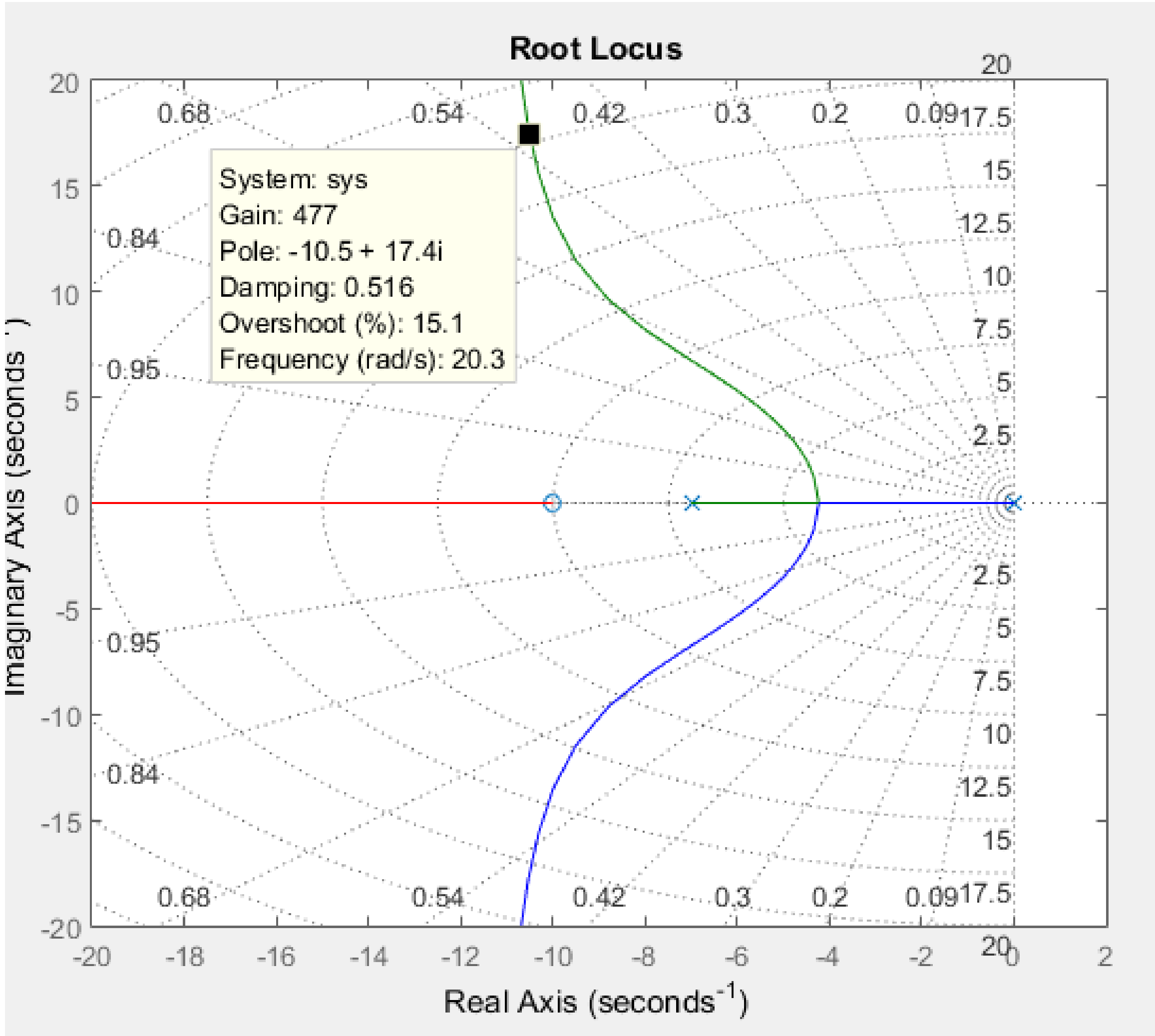
location of the compensator pole → $\dfrac{17.38}{p_c - 10.499} = \tan(49.14^0)$

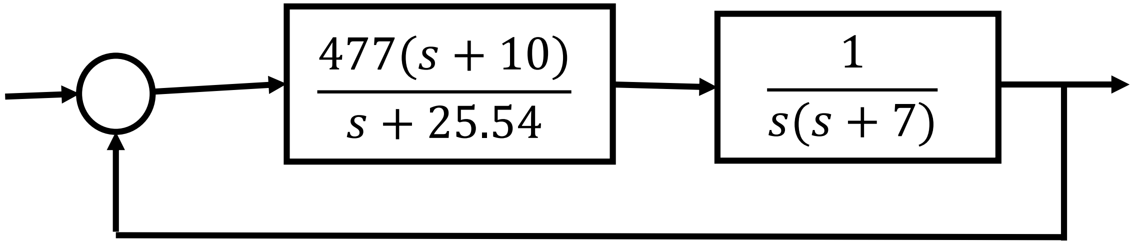
From geometry in fig(a)

compensator pole → $p_c = -25.54$

$$G_{Led}(s) = \frac{s+10}{s+25.54}$$

With gain K = 477





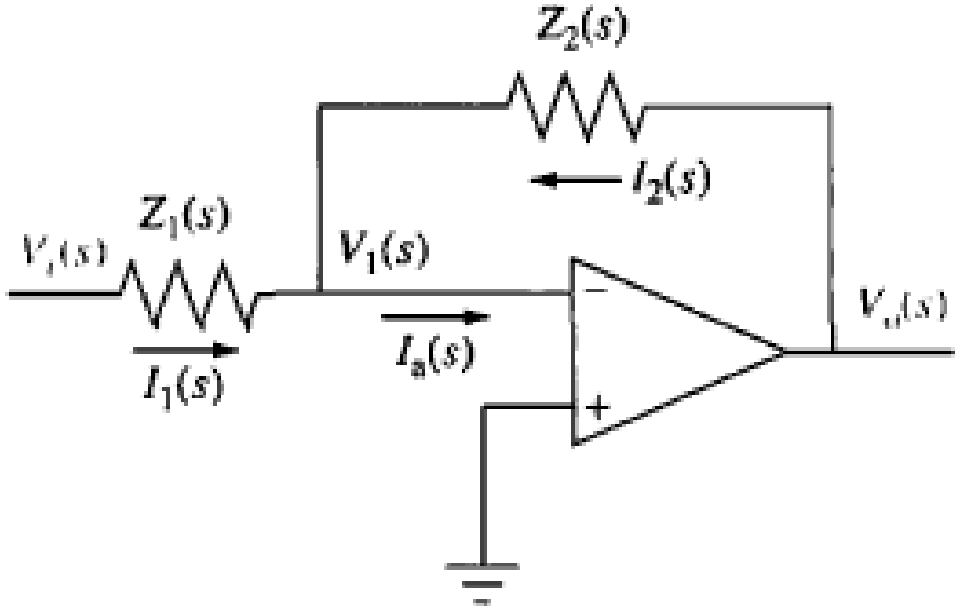
Exercise 4 (Implementation)

Implement the compensator $G_c(s)$

$$G_c(s) = \frac{(s+0.1)(s+5)}{s}$$

Choose a passive realization if possible.

Solution 4 (Implementation)



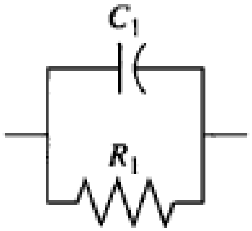
The transfer function of the controller $G_c(s) = \frac{(s+0.1)(s+5)}{s}$

$$G_c(s) = \frac{s^2 + 5.1s + 0.5}{s} = 5.1 + s + \frac{0.5}{s}$$

$G_c(s)$ is a PID controller and thus requires active realization. From table

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

PID controller



$$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$$

Matching the coefficients:

$$\frac{R_2}{R_1}+\frac{C_1}{C_2} = 5.1$$

$$R_2C_1 = 1$$

$$\frac{1}{R_1C_2} = 0.5$$

If we choose $C_1 = 10\mu F$ and $C_2 = 100\ \mu F \rightarrow R_2 = 100k\Omega$ and $R_1 = 20k\Omega$