
Frequently Used Discrete Probability Distributions

Q7) According to a study published by a group of University of Massachusetts sociologists, about two thirds of the 20 million persons in this country who take Valium are women. Assuming this figure to be a valid estimate, find the probability that on a given day the fifth prescription written by a doctor for Valium is

a. The first prescribing Valium for a woman.

$f_x(4) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^4 = \frac{2}{243}$

b. The third prescribing Valium for a woman.

$f_y = \binom{Y-1}{y-1}\left(\frac{2}{3}\right)^Y\left(\frac{1}{3}\right)^{Y-1}$ $, y = 3,4,5$

$f_y = \binom{4}{2}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 = \frac{16}{81}$

Q9) From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that

a. All 4 will fire?

$\frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \frac{1}{6}$

b. At most 2 will not. fire?

$\frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}} + \frac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \frac{29}{30}$

Q12) On average a certain intersection results in 3 traffic accidents per month. For any given month at this intersection. What is the probability that:

a. Exactly 5 accidents will occur?

$P(X = 5) = \frac{e^{-3}3^5}{5!} = 0.1008$

b. Less than 3 accidents will occur?

$P(X < 3) = \sum_{x=0}^{2}\frac{e^{-\lambda}\lambda^x}{x!} = 0.4232$

c. At least 2 accidents will occur?

$P(X \geq 2) = \sum_{x=2}^{99}\frac{e^{-\lambda}\lambda^x}{x!} = 0.8009$

Q19) Suppose X has a geometric distribution with p=0.8. Compute the probability of the following events.

a. X > 3 $P(X > 3) = \sum_{x=3}^{99}(0.8)(0.2)^{x-1} = 0.008$

b. 4 ≤ X ≤ 7 $P(4 \leq X \leq 7) = \sum_{x=4}^{7}(0.8)(0.2)^{x-1} = 0.007$

c. 3 ≤ X ≤ 5 $P(3 \leq X \leq 5) = \sum_{x=3}^{5}(0.8)(0.2)^{x-1} = 0.0384$

Q22) Let X be uniformly distributed on 0,1,…,99. Calculate

a. $P(X \geq 25) = \sum_{x=25}^{99}\frac{1}{100} = 0.75$

b. $P(2.6 < X < 12.2) = \sum_{x=3}^{12}\frac{1}{100} = 0.1$

c. $P(8 < X \leq 10)\ or\ 2 < X \leq 32) = P(8 < X \leq 10) + P(2 < X \leq 32) - P(8 < X \leq 10) = 0.3$

d. $P(25 \leq X \leq 30) = \sum_{x=25}^{30}\frac{1}{100} = 0.6$

Q23) If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it.

$P(X = 10) = \binom{9}{2}(0.4)^3(0.6) = 0.064$

Q28) A fair die is rolled 4 times. Find

a. The probability of obtaining exactly one 6. $P(X=1)=\binom{4}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3=0.386$

b. b.The probability of obtaining no 6. $P(X=0)=\binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4=0.386$

c. c.The probability of obtaining at least one 6. $P(X=1\ or\ X=2\ or\ X=3\ or\ X=4)=1-P(x=0)=1-\binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4=0.518$

Q30) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals, (a) exactly 3, (b) more than 2, individuals will suffer.

$a.\,P(X=3)=\frac{e^{-2}2^3}{3!}=0.180$

$b.\,P(X>2)=1-P(X\le 2)=1-\left(\frac{e^{-2}2^0}{0!}+\frac{e^{-2}2^1}{1!}+\frac{e^{-2}2^2}{2!}\right)=0.323$

Q31) Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items.

$\lambda=np=100(0.02)=2$

$P(X=3)=\frac{e^{-2}2^3}{3!}=0.18$